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Article

Reliability-based optimization planning of aerodromes runway orientation for crosswind

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Abstract: The Wind Rose is the conventional method applied to define the best runway orientation, which is based on a frequentist analysis of probability and on a significant data amount. The method presents questionable precision about results. Hence, this paper aims to provide an alternative method based on the axiomatic definition of probability, which uses the First Order Reliability Method (FORM) in order to achieve the reliability indexes associated to the able runway orientations. The proposed method was developed in ForTran language and applied to a case study in order to its evaluation and exemplification. Results showed that the best orientation was similar for both conventional and proposed method, however, the calculated probabilities were significantly different. Finally, this study contributes to the advancement of this probabilistic approach for cases that data cannot be exclusively described by a bivariate normal distribution and when relevant correlations occur between variables.

Keywords: Aerodromes; Air transport; Airport planning; Runway project; Reliability-based optimization

1. Introduction

One of the basic particulars that have to be defined during a design concept of an aerodrome is the runway orientation. The main analysis factor in this definition is called wind coverage, or usability factor, that is obtained from a statistical data analysis of direction and intensity of the local wind. According to FAA (2012), other factors such as great wind variability, predominance and nature of wind gusts, wind turbulences, topography and geographical relief, land use and occupation, obstacles, water presence, snow, ice, among others, may also influence the analysis, but these are not considered here.

The definition of the best runway orientation, mainly based on local wind analysis, is a subject widely addressed in the literature. For example, in 1958, Lenhard Jr. and Foard (1958) presented a method to determine the probability of combine occurrence of crosswind and runway wind components at an airport from Prestwick, Scotland. The method consists of plotting the wind frequency in a polar coordinate diagram, which is divided into sixteen directional sectors (22.5° each) and into five speed classes (wind rose). In this case, interpolations are done based in each diagram section areas. This method was an extension of the technique proposed earlier by Tennenbaum (1948).

Later, Falls and Brown (1972) described two processes to obtain the best runway orientation, one defined as empirical and another as theoretical. On the one hand, the empirical process consists of setting a conventional wind rose and manual calculations. On the other hand, the theoretical process makes use of computed statistics from fitting a normal bivariate distribution into a data sampling of wind components. Both methods achieved the same result for the best runway orientation however different probabilities were acquired. The authors pointed out that some imprecision may emerge due to the difficulties in fitting data with a normal bivariate distribution. In other words, this theoretical method should be applied only in cases that this type of distribution fit the wind data.

It is worthwhile to note that, accordingly with structured searches in the literature, there is an interval with no, or almost that, studies focused on this subject – from Falls and Brown (1972) to Mosa and Mumayiz (2000). This interruption, presumably, occurs due to the efficiency of the Wind Rose method, which may had been considered satisfactory and practical at that moment. The regulatory agencies adopting this method in the advisory circulars, during that time, is the best proof of that. Additionally, another probably reason for this event is the computational advancements in last decades that provided new opportunities of analysis.

Consequently, the employment of computational resources in this subject has been increased, as studies developed by Mosa and Mumayiz (2000), Jia et al. (2004), Sarsan and Ateia (2011), Chang (2013 and 2015), Bellasio (2014), Laat and Roling (2014), and Oktal and Yildirim (2019). In general, the currently studies aimed to propose a computational model in order to define the best runway orientation. For such propose, some frameworks have been used, such as spreadsheets, VBA (Virtual Basic for Applications), GIS (Geographic Information System), existing programs (as WindRose PRO3), and other.

Except for Falls and Brown (1972) and Laat and Roling (2014), concerned to the so-called theoretical process, the studies have based the development of models on the conventional Wind Rose method. However, working with the trial and error methodology and visual estimation procedure makes the accuracy of the method questionable. According to Applequist (2012), a well-known methodological bias exists in the creation of wind roses. Generally, wind directions are reported at discrete increments (usually 10°) while the analyses are carried out by dividing the total possibilities (360°) into sixteen equally spaced intervals, with 22.5° amplitude each. Then, this practice may induce errors in analysis. For example, for a uniform wind distribution, this preparation may introduce a positive bias of 33% at primary directions (N, S, E, W) and negative bias of 11% at the others (NNE, NE, …, NNW).

In addition, note that the Wind Rose method deals with *a posteriori* calculated probabilities, from a collection of significant data amount. In other words, Wind Rose uses the frequentist definition of probability. According to Melchers and Beck (2017), this definition is often used to interpret probability. However, in practice, it may be considered as limited because the number of observations will never reach the infinite, when the actual probability value is achieved. Then, another way to obtain the probability associated with an interesting event is from use of the axiomatic definition of probability, as demonstrated by Falls and Brown (1972) and Laat and Roling (2014). Yet these studies were limited to a condition, which supposes that a normal bivariate distribution type could fit the wind components.

Thus, this paper aims to propose a computational method in order to determine the best runway orientation for aerodromes, based on probabilistic analysis, or reliability analysis, of the frequency, direction, and intensity of winds from a local of interest. The methodology is based on the axiomatic definition of probability and makes use of an analytical method called FORM (First Order Reliability Method), which is not limited by the assumed distribution type for random variables. The method is applied to a collection of wind data provided by the International Airport of São José dos Campos – Professor Urbano Ernesto Stumpf (SBSK), located into São José dos Campos city, São Paulo, Brazil.

2. Conventional method – Wind Rose (FAA)

The FAA (United States Federal Aviation Administration) presents and recommends the use of a method to determine the best runway orientation, known as the Wind Rose method, widely used all around the world. This method allows the analysis of the crosswind component that has a substantial impact on the safe operation of aircraft during landing and taking off conditions. This impact significance is inversely proportional to the analyzed aircraft size. Thus, the ICAO (International Civil Aviation Organization) defines three acceptance levels for this component, 10, 13 and 20 kts. The occurrence probability of an acceptable crosswind component is defined as coverage or usability factor. Therefore, the FAA (2012) recommends that the aerodrome runway have to be orientated in order guarantee 95% or higher coverage. In other terms, it is sought the condition that an aircraft can land or take off in a prevailing wind situation with minimum exposure to crosswind.

The method consists to mainly treat a database of directional and intensity of local winds, five years or longer data collection is recommended. Intervals of direction and speed of winds are predefined based on design features. The graphical representation is so achieved by a wind rose (direction), or a polar coordinate diagram, with concentric circumferences (speeds). Thus, there will be several sectors representing each speed interval for a given directional interval. From data treatment, frequencies are quantified (by percentages of total) to each sector that composes the diagram. The runway is graphically represented by two non-overlapping parallel lines and tangents to the circumference, associated with the allowable crosswind speed to the aircraft. The wind coverage is obtained by subtracting the sum of the percentages outside the runway graphical representation of the unit. Finally, the best runway orientation is achieved from a trial and error analysis, seeking a satisfactory coverage value by rotating the runway graphical representation on the diagram.

Therefore, note that it is a probabilistic method in which data are analyzed in a pure frequentist view. This means, probabilities are determined *a posteriori* the events. This fact, together with others mentioned above, may turn the accuracy of the method questionable.

3. Probabilistic and reliability analysis

According to Melchers and Beck (2017), in the frequentist definition of probability, it is calculated *a posteriori* based on a high number of collected samples. This definition is fundamental in order to associate probabilities with the observable world, and it is usually assumed to interpret probability, but it is a limited definition. In practice, the number of samples will never reach the infinite, when the actual probability value is achieved.

Beyond the frequentist definition, there are other ones, as the classical and subjective definitions of probability. Nevertheless, none of these definitions is good in order to formulate a probability theory. The Mathematical Theory of Probability has based on the definition that the probability of an event A is a number associated with this event that obeys to three following axioms:

I. The probability is a number greater or equal to zero ($P[A] \geq 0$);

II. The probability of a right event is equal to the unit ($P[\Omega] = 1$);

III. If events A and B are mutually exclusive, the probability of the union of events is acquired by summing each event probability ($P[A \cup B] = P[A] + P[B]$).

The axiomatic definition of probability, or the definition of Kolmogorov (1933), is the development of the theory of probability from these axioms.

About the reliability theory, it is presented by Freudenthal (1947) and some engineering applications can be found in Ang and Tang (1975, 1984). In the early 1970s, Cornell (1971) proposed an index in order to measure, probabilistically, the safety of structures, like dams and foundations, called reliability index (β). The β can be defined as a geometric measure of the probability of failure (P_f), corresponding to the minimum distance between the limit state function ($g(x)$) and the origin of the standard normal space $\mathbb{Y}$, which is obtained by performing the transformation method proposed by Hasofer and Lind (1974).

In the literature, the main three analytical methods based on this transformation are the First-Order Second-Moment (FOSM), the First Order Reliability Method (FORM), and the Second Order Reliability Method (SORM). The transformation consists on mapping the random variables from the design space $\mathbb{X}$ (dimensional space) to the standard normal space $\mathbb{Y}$ (dimensionless space, where random variables assume means equal to zero and standard deviations equal to the unit).

4. Method

The proposed method was performed for a case study in order to its demonstration and evaluation. The analyzed case makes reference to the International Airport of São José dos Campos – Professor Urbano Ernesto Stumpf (SBSK), located into São José dos Campos city, São Paulo, Brazil. The airport administration provided the wind data, which were acquired from an automatic weather station. This data collection presents hourly wind information over 19 years. This significance level was evaluated by performing the chi-squared test (χ^2) of Pearson (1900).

The proposed method for performing reliability analysis can be divided into four steps, the data treatment, the determination of statistical information, the reliability analysis, and the reliability-based optimization. In the data treatment step, acquiring or storing errors are identified. The analyzed data presents information about wind direction, speed and collection frequency. The occurrence of a significant discrepancy, or lack of one or other information, for each collection period, resulted in discarding all information assumed for that period. This attitude was assumed in order to prevent possible bias in the next steps and, consequently, in the result.

Accumulated and non-accumulated histograms (frequency graphs) were generated in order to determine the statistical information for both wind directions and speeds. Then, it was able to determine the best distribution type that fits the available data, acquiring its associated statistical moments (such as means and standard deviation). The χ^2 test of Pearson (1900) was performed again in order to evaluate the distributional goodness of fit. However, the parameters were readjusted in order to obtain the maximum adherence for the assumed distribution type.

The reliability analytical method FORM was assumed to perform the analysis. The method needs the determination of the limit state function for the system or the analyzed condition. Therefore, Equation 1 presents the limit state function assumed for the system.

$$g(\boldsymbol{X}) = S_a - |S_w(\boldsymbol{X}) \cdot \sin(\theta_w(\boldsymbol{X}) - \theta_r)| \tag{1}$$

In this equation, the $g(\boldsymbol{X})$ represents the performing equation, S_a is the allowable crosswind speed, S_w is the wind speed (random variable), θ_w and θ_r are the measured angles from true north of wind direction and runway orientation, respectively. Thus, negative values for this function evaluation characterize the failure of the system (unfeasible operation for the aircraft size assumed). In the analysis, three S_a levels were investigated (10, 13, and 20 kts).

Finally, this method was programmed in ForTran language. For the reliability index evaluation (FORM), an academic reliability analysis software developed by Beck and Verzenhassi (2008), called StRAnD, was adapted and used. The Figure 1 presents the proposed methodology by a flowchart, in which the adapted StRAnD algorithm is represented by the red sector (FORM), and may be applied for many purposes, *e.g.*, Belo and Silva (2020). At the same time, the controlling algorithm and complementary routines were programmed just for this research intention.

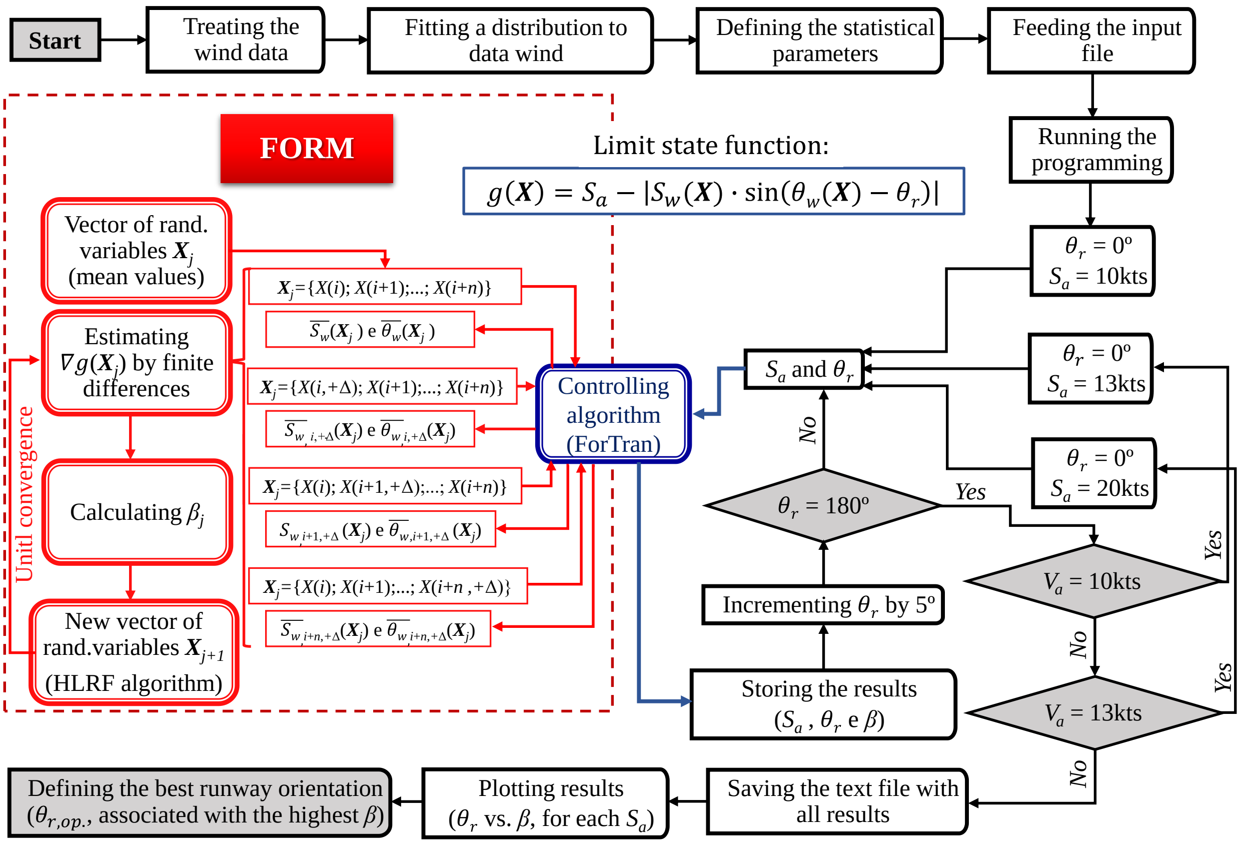

Figure 1. Flowchart of the proposed method.

Note that, in Figure 1, when running the programming, the algorithm performs searches about statistical information inside the input file, and assumes θ_r = 0° and S_a = 10kts as a starting point of analysis. Then, the control algorithm executes the FORM in order to evaluate the β for this starting point. The FORM assumes the initial vector of random variables ($\boldsymbol{X}$) with the means values of the variables that make it up (S_w and θ_w). Since this is an iterative method, the algorithm HLRF (Hasofer and Lind, 1974; and Rackwitz and Fiessler, 1978) is used in order to assume a new vector of random variables, repeating the calculations until the convergence of β. With β acquired, the algorithm will store data about the analyzed configuration and its associated result. Next, an increment of 5° in θ_r is applied, and the whole process is repeated, analyzing the all possible runway orientations (0° ≤ θ_r < 180°). At this moment, all assumed configurations for the first S_a value will have been analyzed so S_a is changed for the next assumed allowable level, and the whole process is repeated until all predefined S_a levels are analyzed. In the end, all acquired results are stored into a text file, which allows data to be plotted in a graphic view, making the definition of the best runway orientation easier ($\theta_{r,optimum}$, a configuration that achieve the highest β value). The execution time of the whole process calculation may vary with the predefined increment value of θ_r and with the number of S_a levels assumed for the analysis. However, for the presented analysis configuration, it takes about 40 seconds to evaluate the problem, using a personal computer with a common configuration.

The results acquired via the proposed method were graphically compared with results acquired via the conventional method (Wind Rose). The conventional method results were transformed into equivalent β values, using the relationship $P_f = \Phi(-\beta)$, in order to facilitate the comparison. The wind rose was generated and evaluated using the FAA (2012) application.

5. Results and discussion

The graphs presented by Figure 2 and Figure 3 were acquired from data treatment. The Figure 2(a) shows the cumulative distribution function (CDF), built with actual data of wind speed in the local, and the best fitting curve is assumed to be a clipped, or bimodal, theoretical distribution. The assumed clipped distribution makes use of the uniform and the lognormal distributions, achieving a good agreement with the measured curve (actual data). The probability density function (PDF) of the clipped distribution can be observed in Figure 2(b), where the uniform distribution makes reference to the calm condition (wind speed equal to zero) and the log-normal distribution to situations of wind occurrence (wind speed greater than zero). The input parameters of this random variable were the probability of calm condition = 0.33, to the uniform distribution, and equivalent means = 1.593 and standard deviation = 0.588, to the lognormal one.

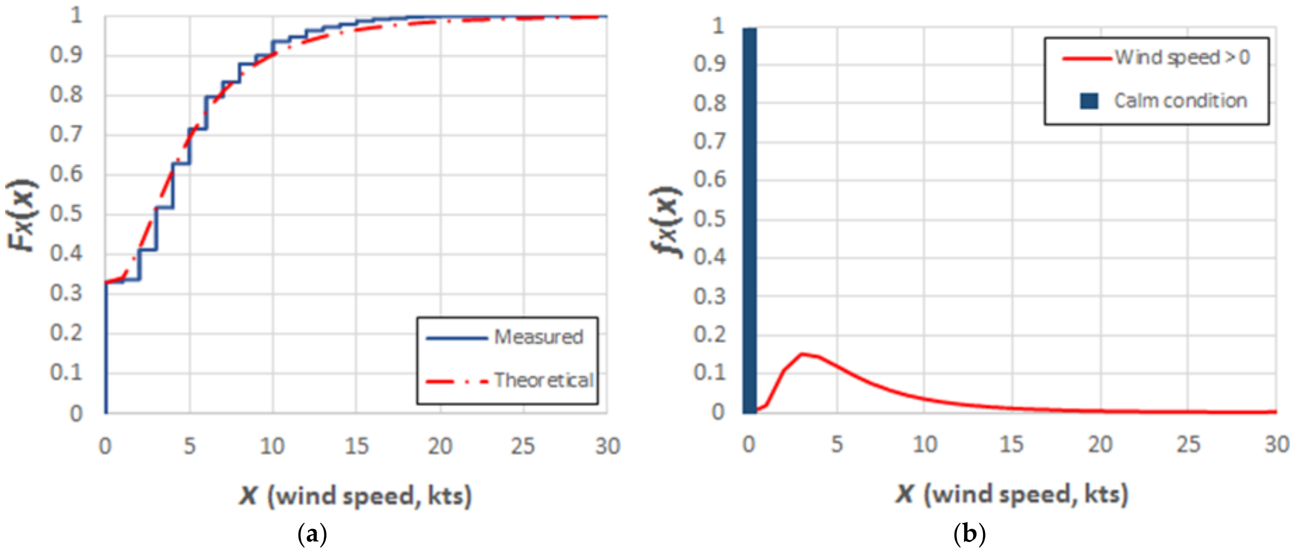

Figure 2. Probability distribution obtained with actual measurements of wind speed (kts) in the local, and the fitting curve by a theoretical distribution: (**a**) Cumulative distribution function (CDF); (**b**) Probability density function (PDF).

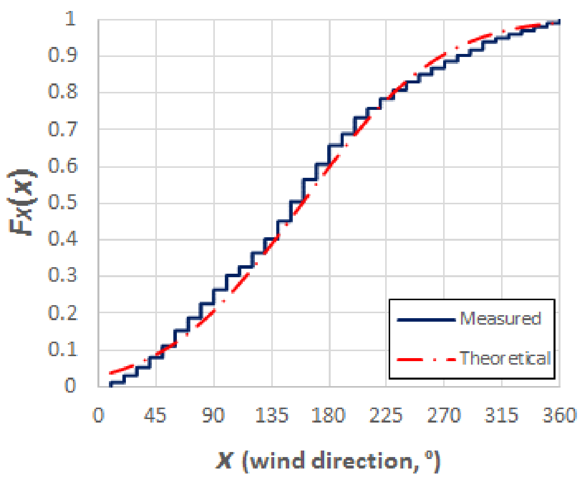

Figure 3. CDF obtained with actual measurements of wind direction (°) in the local, and the fitting curve by a theoretical distribution.

Then, Figure 3 presents the CDF for the second random variable (wind direction). The best-fitting curve is assumed as a normal distribution. It is noteworthy that only the wind occurrence condition (wind speed greater than zero) was considered to the curve construction. The input parameters of this variable were mean = 154.82 and standard deviation = 85.011.

As demonstrated by Hu and Du (2018), the proposed method (FORM) does not present a good consistency on the treatment of multimodal distributions because this method transforms the assumed distributions into normal equivalents, and approximates the performance function by a linear one. However, seeking to circumvent this problem, it was decided to consider the concept of conditional probability. In this case, the reliability analysis method is applied only to the wind occurrence condition, in other words, only considering the lognormal distribution for the wind speed random variable and the normal distribution for its direction. Thus, the conditionality is created, and the calculated probabilities and β makes reference to the wind occurrence condition ($S_w > 0$). The actual probabilities can be evaluated by performing Equation 2.

$$P_f = P[A|B] \cdot P[B] \tag{2}$$

In the Equation 2, P_f represents the actual probability of failure of the system, $P[A|B]$ is the probability of occurrence of a wind with cross-component greater than the allowable level (event A), since a wind speed greater than zero occurs (event B), and $P[B]$ is the occurrence probability of a wind speed greater than zero. The probability of $P[B]$ may be acquired by Figure 2(a), associating this with the value of $[1–F_X(x)]$ with x equal to zero. For this case, this value is around 0.67 (67%), therefore 33% of the analyses take place in calm conditions.

In addition, the existence of a correlation between the assumed random variables was investigated, resulting in the correlation matrix presented in Table 1, considered that the calm conditions were neglected. Thus, Figure 4 shows the results of the reliability analysis, which were obtained by applying the proposed method with all the statistical information presented before.

Table 1. Correlation matrix acquired for assumed random variables (wind direction, θ_w, and speed, S_w).

Random variables	θ_w	S_w
θ_w	1	0.112
S_w	0.112	1

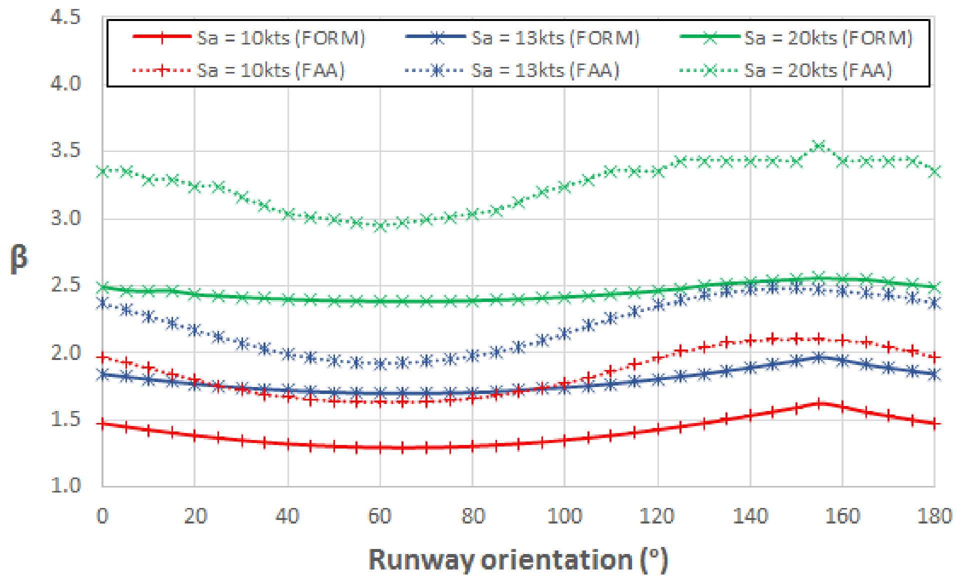

Figure 3. CDF obtained with actual measurements of wind direction (°) in the local, and the fitting curve by a theoretical distribution.

Analyzing the results presented in Figure 4, note that the optimum configuration, which results in the best runway orientation, is similar for both proposed (FORM) and conventional method, around 155°. However, the β values calculated via FORM present significant lower values than the obtained ones via the conventional method, in other words, greater probabilities of failure (undesired condition) are achieved by the proposed method. The comparison between these values is presented in Table 2.

Table 2. Correlation matrix acquired for assumed random variables (wind direction, θ_w, and speed, S_w).

Sa = 10kts				Sa = 13kts				Sa = 20kts			
FORM		FAA		FORM		FAA		FORM		FAA	
P_f	β	P_f	β	P_f	β	P_f	β	P_f	β	P_f	β
5.23%	1.623	1.77%	2.104	2.46%	1.967	0.67%	2.473	0.53%	2.557	0.02%	3.540

This fact has been already reported by the study developed by Falls and Brown (1972), which proposed the use of a process defined as theoretical, very similar to the FOSM methodology. These are limited to data that can be fitted by a normal distribution for both the wind direction and speed. Therefore, the contribution of this paper is highlighted, because the proposed method does not present the previously mentioned limitations, mainly about the distribution type assumed for the random variables of the problem (wind direction and speed). The advances of this paper are in the probabilistic analysis filed in order to define the best runway orientation for aerodromes, based on the axiomatic definition of probability.

6. Conclusions

This paper aimed to propose a computational method in order to determine the best runway orientation for aerodromes based on probabilistic analysis, or reliability analysis, of the frequency, direction, and intensity of winds from a local of interest. The methodology is based on the axiomatic definition of probability, differing from the conventional approach (Wind Rose) that is purely frequentist, calculated *a posteriori*.

The proposed method has been satisfactory in the definition of the best runway orientation, achieving the same orientations indicated by the conventional method. However, the β values acquired via proposed method were significantly lower than that obtained via the conventional method, in other words, higher P_f values were calculated by the proposed method (assuming the FORM) when compared with the Wind Rose method. As mentioned, this discrepancy may be associated with the applied method that may achieve results with questionable accuracy.

Finally, it is worth noting the progress provided by this paper, which in addition to enables a probabilistic analysis based on the axiomatic definition of probability, also exceed some limitations imposed by previous studies about this probabilistic issue, as the limitation on fitting data exclusively by a bivariate normal distribution. The proposed method also allows taking into account the correlation between the random variables, when observed. By this way, analyses that require a greater rigor in the calculated probabilities are recommended to make use of this proposed method instead of the conventional one.

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Conflicts of Interest: The authors declare no conflict of interest.

References

1. FAA. Advisory Circular AC 150/5300-13A. U.S. Department of Transportation, Federal Aviation Administration, 2012.
2. LENHARD JR., R.W.; FOARD, J.M. Determining the availability of existing runways. Bulletin American Meteorological Society, 39(5), p.253-257, 1958, doi: https://doi.org/10.1175/1520-0477-39.5.253
3. TENNENBAUM, J.S. The usability of aerodromes. Aircraft Engineering and Aerospace Technology, 20(7), p.194-199, 1948, doi: https://doi.org/10.1108/eb031652
4. FALLS, L.W.; BROWN, S.C. Optimum runway orientation relative to crosswinds. NASA Technical Note TND-6930, National Aeronautics and Space Administration, Washington, DC, USA, 1972.
5. MOUSA, R.M.; MUMAYIZ, S.A. Optimization of runway orientation. Journal of Transportation Engineering, 126(3), p.228-236, 2000, doi: https://doi.org/10.1061/(ASCE)0733-947X(2000)126:3(228)

6. JIA, X.; CHUNG, D.; HUANG, J.; PETRILLI, M.; THE, L. ARO: Geographic information systems-based system for optimizing airport runway orientation. Journal of Transportation Engineering, 130(5), p.555-559, 2004, doi: https://doi.org/10.1061/(ASCE)0733-947X(2004)130:5(555)
7. SARSAM, S.I.; ATEIA, H.A. Development of a computer program for airport runway location, orientation, and length design in Iraq. In.: Proceedings of the ASCE Transportation and Development Institute Congress, p.291-300, 2011, doi: https://doi.org/10.1061/41167(398)29
8. CHANG, S.-W. Orientations optimization for two runway configurations. In.: Proceedings of the Eastern Asia Society for Transportation Studies, 9, p.1-10, 2013.
9. CHANG, S.-W. Crosswind-based optimization of multiple runway orientations. Journal of Advanced Transportation, 49, p.1-9, 2015, doi: https://doi.org/10.1002/atr.1247
10. BELLASIO, R. Analysis of wind data for airport runway design. Journal of Airline and Airport Management, 4(2), p.97-116, 2014, doi: http://dx.doi.org/10.3926/jairm.26
11. LAAT, J.P.; ROLING, P.C. Runway location and orientation suitability analysis. In.: Proceedings of the 14th AIAA Aviation Technology, Integration, and Operation Conference, p.2153-2162, 2014, doi: https://doi.org/10.2514/6.2014-2153
12. OKTAL, H.; YILDIRIM, N. Method for identifying optimum runway orientation. U.S. Patent Application No. 15/828,343, 2019.
13. APPLEQUIST, S. Wind rose bias correction. Journal of Applied Meteorology and Climatology, 51(7), p.1305-1309, 2012, doi: https://doi.org/10.1175/JAMC-D-11-0193.1
14. MELCHERS, R.E; BECK, A.T. Structural Reliability Analysis and Prediction. 3ed. New York: Wiley, 2017.
15. KOLMOGOROV, A. Über die Grenzwertsätze der Wahrscheinlichkeitsrechnung. Bulletin de l'Académie des Sciences de l'URSS. Classe des sciences mathématiques et na, 3, 363–372, 1933.
16. FREUDENTHAL, A. M. The Safety of Structural. Transactions of ASCE. Vol. 112, p. 125-180, 1947.
17. ANG, A.H-.S; TANG, W. Probability Concepts in Engineering Planning and Design. New York: John Wiley and Sons, v.1, 1975.
18. ANG, A.H-.S; TANG, W. Probability Concepts in Engineering Planning and Design. New York: John Wiley and Sons, v.2, 1984.
19. CORNELL, C.A. First order uncertainty analysis of soils deformations and stability. In.: Proceedings of the 1st International Conference on Application of Statistics and Probability in Soil and Structural Engineering, Hong Kong, China: Hong Kong University Press, p. 129-144, 1971.
20. HASOFER, A.M.; LIND, N.C. Exact and invariant second-moment code format. Journal of Engineering Mechanics, 100(1), p.111-121, 1974.
21. BENOÎT, C. Note sur une méthode de résolution des équations normales provenant de l'application de la méthode des moindres carrés à un système d'équations linéaires en nombre inférieur à celui des inconnues, (Procédé de Commandant Cholesky). Bulletin of Géodetic Sciences. 2, p.67-77, 1924, doi: https://doi.org/10.1007/BF03031308
22. PEARSON, K. On the criterion that a given system of deviations from the probable in the case of a correlated system of variables is such that it can be reasonably supposed to have arisen from random sampling. Philosophical Magazine Series 5, 50(302), 157-175, 1900, doi: https://doi.org/10.1080/14786440009463897
23. RACKWITZ, R.; FIESSLER, B. Structural reliability under combined load sequences. Computers & Structures, 9, p.489-494, 1978, doi: https://doi.org/10.1016/0045-7949(78)90046-9
24. BECK, A.T.; VERZENHASSI, C.C. Risk Optimization of a Steel Frame Communications Tower Subject to Tornado Winds. Latin American Journal of Solids and Structures, 5, 187-203, 2008
25. Belo, J.L.P., and Silva, J.L. Reliability analysis of a controlled stage-constructed and reinforced embankment on soft ground using 2D e 3D models. Frontiers Research Foundation, 5:150, 2020, doi: https://doi.org/10.3389/fbuil.2019.00150
26. HU, Z.; DU, X. Reliability methods for bimodal distribution with first order approximation. Journal of Risk and Uncertainty in Engineering Systems, Part B: Mechanical Engineering, 5(1), 011005, 2018, doi: https://doi.org/10.1115/1.4040000