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ANALYSIS OF RSVD ALGORITHM USING HIGH FREQUENCY FRFs IN DAMAGE DETECTION

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Abstract. *The Robust Singular Value Decomposition algorithm (RSVD) was used to identify damaged structure using frequency response functions (FRFs). The experimental data were obtained from a laminate beam. The routine uses robust statistics and analysis was done in a data set from healthy and damaged structure. This technique has being developed for FRFs at high frequency range, which have showed to be more sensitive to small damages. In this work are presented results for detection algorithm in different frequency ranges, effectiveness of coherence signal applied to measurement dataset, FRF signal resolution, RSVD parameters and computational costs.*

Keywords: *Singular value decomposition, Structural health monitoring, laminated structure, damage detection*

INTRODUCTION

In order to reduce conservative project and equipment maintenance costs, and due to an increasing use of new materials the health structural monitoring has gained increasing attention of the scientific community. The composite structures have a damage mechanism different from those of metals that they are replacing and failure detection need to be improved.

Changes in dynamic response of the structure are linked to a change in stiffness, mass or damping of the structure and this is the basic idea of structural health monitoring Shane and Jha (2011). Many techniques proposed to detect damage are based in changes in modal parameters, a comprehensive review on modal parameters-based damage identification can be found in Fan and Qiao (2011).

Another method is the ultrasonic guided waves, in particular Lamb waves, that is aimed at plate-like structures, Konstantinidis et al. (2006). Its ability to detect defects have been demonstrated by published results. The measurements are made using high frequency PZT sensors. Being thin and light they can be bonded or embedded on the structure. There are another methods in research, sensors and monitoring strategies, that will not mentioned here.

The use of modal analysis on structural health monitoring is a natural consequence of its use on dynamic of structures. However, a major concern is that its validity is limited to linear structures (Kerschen et al., 2005). Vanlanduit (Vanlanduit et al. (2005)) said that methods based on modal parameters are sensitive to changes in environment, operational conditions and structural uncertainties. The proper orthogonal decomposition (POD) method has been proposed aiming to remove the influence of these external factors (Vanlanduit et al., 2005; Shane and Jha, 2011).

The proper orthogonal decomposition (POD) is a powerful multi-variate statistical method for data analysis aimed at obtaining low-order approximate descriptions of a high-dimensional process (Wu et al. (2003)). The POD is an orthogonal transformation that decorrelates the signal components and maximizes variance. By this way the POD method reduce a large number of interdependent variables to a much smaller number of uncorrelated variables while retaining as much as possible of the variation in the original variables (Kerschen et al. (2005)). The discrete implementation of the proper orthogonal decomposition is the popular singular value decomposition (Kerschen and Golinval, 2002; Wu et al., 2003).

There are different implementations of the proper orthogonal decomposition method to structural health monitoring. Vanlanduit (2005), proposed the robust singular value decomposition method (RSVD) that uses the frequency response functions (FRFs) of displacement, force, acceleration or other output as a basic data for the method.

This work is closely related to implementation of the robust singular value decomposition method (RSVD) as published by Vanlanduit et al. (2005), which could find fault signals on a set with healthy and damaged data. In this work there are two important differences. The most important is that the method was implemented to high frequency range.

The Vanlanduit method was developed to identify small changes in the first natural frequencies. The main assumption on this paper is that small damages can be more easily found by changes in high frequency modes in other words assuming that they are more likely to present great FRF shape changes at high frequency. There are another important aspect as little affected mode could be treated as having a linear behavior. So it is possible to employ all modal analysis technique enabling gathering more information about the problem in a further stage.

The experimental data was done with PZT sensors to fulfill measurement request at high frequency. This kind of sensor could be embedded or bonded to structure, a requisite to health structural monitoring procedure (Konstantinidis et al. (2006)), allowing good repeatability of measurements, essential in this frequency range.

The original RSVD method merges damaged and healthy FRF measurement subsets without any distinction between them. In other words the method should be able to find out the damaged subset by computing its distance to original set. This procedure has two drawbacks, first all possible subset combinations must be tested so it is not feasible for large data set. Second aspect is that small sets are more susceptible to noise measurements and robust methods could not eliminate the outliers.

To avoid those problems the RSVD was applied to reduce the original measurement to a small optimal subset, those with minimal distance to its original set. In addition the health subset is supposed to be known and the other set is the set to be tested, so it needs to know a healthy dataset in advance. This procedure requires less from the detection algorithm as all distance above the acceptable level is a failure indication. The two sets are optimal and presents minimal distance from its original measurements, healthy or those to be tested, so a great distance to merged set could indicate a damage.

The RESULTS AND ANALYSIS presented in this work were made using FRF data from a metallic laminated Aluminum beam and the damaged structure was the same beam with an attached mass. The sensitivity analysis of the detection algorithm were performed using a set of FRFs from damaged and healthy structure. The measurements were taken at same PZT, for both healthy and damaged structure, the excitation PZT was also at same position. The detection algorithm was employed over different frequency range, 2 kHz to 10 kHz, and 2 kHz to 12 kHz same FRF resolution.

The effectiveness of measurement signal times its coherence aiming to select the best dataset was also verified. The coherence signal is a indication of measurement quality. If close to 1 it means that the response signal is only function of input, if it is equal 0 there are no relationship between response signal and excitation. So measurement signal multiplied by its coherence might help the detection algorithm to eliminate the outliers.

The best subset size, for damaged and healthy FRF measurement was also tested.

DAMAGE DETECTION ALGORITHM

The classical Least-Squares SVD-based damage detection can be outlined as follow. The database is obtained from frequency response function, FRF, a $M \times N$ matrix $H = [H_1, H_2, ...H_N]$ at N specified conditions, each matrix H_n has FRF data from each P specified positions. $N = N_u + N_d$ where N_u is the undamaged experimental FRFs from healthy structure and N_d is from damaged ones. The damaged database $N_D = \sum N_d$ must be equal or greater than $N_U = \sum N_u$. The sequence steps of the damage detection algorithm are presented below.

1- Computing the SVD of the $M \times N$ matrix $H = USV^H$;
2- Putting all singular values $s_{k+1}, ..s_N$, from $S = diag(s_1, s_2, .., s_N)$, bellow the noise level equals zero, $S1 = diag(s_1, s_2, .., s_k, 0, 0, .., 0)$;
3- Using singular vectors U and V from original matrix synthesize matrix $H1$ of rank k $H1 =$

$US1V^H$;
4- Computing residual matrix $E1$, $E1 = H - H1$;
5- Computing the variance s of the residuals:

$$s = \frac{1}{MN-1}\sum_{i=1}^{M}\sum_{j=1}^{N}(H_{i,j} - H1_{i,j})^2 \tag{1}$$

6- Computing the variance of each measurement j (each column j):

$$s_j = \frac{1}{M-1}\sum_{i=1}^{M}(H_{i,j} - H1_{i,j})^2 \tag{2}$$

7- Estimating the relative distance d_j^{SVD} of each measurement j to subspace computed by SVD:

$$d_j^{SVD} = \frac{s_j}{s} \tag{3}$$

As the relative distance d_j^{SVD} is defined as a variance ratio it obeys a χ^2 distribution. For a χ^2 distribution with a confidence level of $(100 - \alpha) = 95\%$ and for $(N_n - 1)$ degrees of freedom up to 30, $(N_n - 1) < 30$ (Vanlanduit et al., 2005), a threshold T can be defined as

$$T = \sqrt{\frac{\chi^2_{(100-\alpha)_{(N_n-1)}}}{2}} \tag{4}$$

If $d_j^{SVD} > T$ then there are a damaged sample. For N >30 a maximal value of T for N= 30 is taken in order to make sure that the threshold is not too tight, (Vanlanduit et al., 2005).

The Robust SVD

The problem with the LS-SVD based damage detection is that it is very sensitive to outliers in measurement, Vanlanduit (2005). To avoid this, the robust method was proposed, and it does not compute the whole database H, the SVD is computed only over $N/2$ observations, the original matrix is re-synthesized and the distances and the error are computed. The solution is the small cost function from all possible combinations of the database. The steps of the RSVD routine are:

1- Construct a matrix H_R from matrix H with $L = N/2$ combination columns from N columns of the original matrix.

a- computing the SVD of the matrix $H_R = U_R S_R V_R^H$;

b- computing the extended right singular vector $V_E = U_R^H S_R^{-1} H$;

c- computing re-synthesized matrix H_S using $H_S = U_R S_E V_E^H$, $S_E(i,i) = S_R(i,i)$ $i = 1, 2, ..., L$ and $S_E(i,i) = 0$ if $i > L$;

d- Computing the residuals $E_S = H - H_S$;

e- Computing the variance s of the residuals using the median absolute deviation MAD, modifying the step 4 of SVD-based damage detection, $s = MAD(E_S)$, where:

$$MAD(E_S) = 1.4826 * median(|E_S - median(E_S)|) \tag{5}$$

f- Computing the variance for each measurement j (each column j):

$$s_j = \frac{1}{M-1}\sum_{i=1}^{L}(H_{i,j} - H_{S(i,j)})^2 \quad (6)$$

g- Estimating the relative distance d_j^{SVD} for each measurement j to subspace computed by SVD:

$$d_j^{SVD} = \frac{s_j}{s} \quad (7)$$

h- computing the cost function $\kappa = \sum_j^L |d_j|^2$;
2- The smallest cost function κ is taken as the RSVD solution, $H_B = H_S$. The variance s and d_j^{SVD} are computed following the procedures from **d** to **g** in the previous steps. As the relative distance d_j^{SVD} is defined as a variance ratio it obeys a χ^2 distribution. For a χ^2 distribution, the threshold T is the same as defined on Eq. 4 and there was employed the same confidence interval.

EXPERIMENTAL SETUP

The experimental procedure was done using dynamic response of a free-free 4 layer laminate Aluminum 2024-T4 each 0.46mm thick (0.018) bonded with epoxy resin. The laminate plate has 300mm length, 32.5mm wide and a total thickness of 2.0mm. On Fig. 1 there are 5

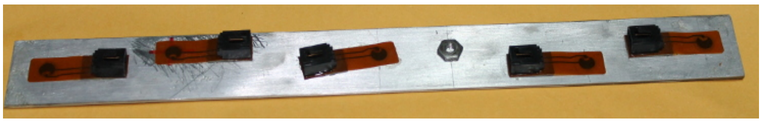


Figure 1: Laminate beam and PZT sensors. From right to left PZT 2, 3, extra mass (nut), PZTs 1, 4 and 5.

PZTs. PZT 1, as sketched on Fig. 2, was a measurement or passive sensor used as a reference signal, and PZT 3 was used to measure frequency response function, FRF, of the system at this response point. PZT 5 was employed to excite the plate. Results from sensors 2 and 4 was not used in this work. The signal from PZT 1 was taken as a reference to obtain the FRFs. Using

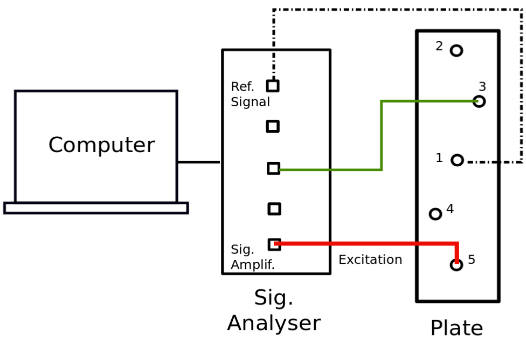


Figure 2: Measurement setup layout. PZT 5 (excitation), PZT 1 (signal reference), PZT 3 measurement.

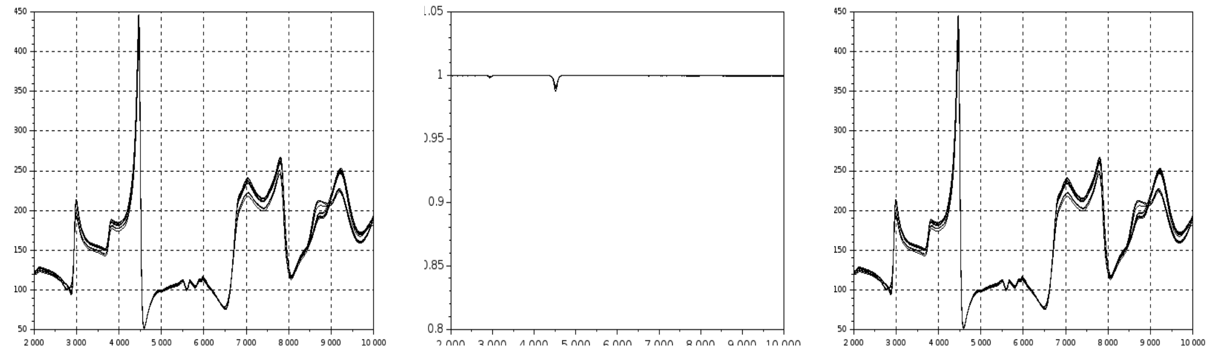
the excitation signal from PZT 5 directly there were a high noise level and low coherence in the response FRFs from PZT 3.

Experimental data acquisition was performed by the PHOTON II from LDS. Measurements were done using different excitation signal, where the white noise, uniform random, sweep sine signals from the PHOTON II were amplified and sent to PZT 5, Fig. 2. All FRFs have units Volts/Volts (response/excitation) in a frequency range from 2.0 to 32.0 kHz and 3200 line resolution. There are 39 FRFs, 19 FRFs were from healthy structure and 20 from damaged one. From those were taken FRFs from 2.0 to 10 kHz, the frequency range for the first case, Fig. 3. For second case was taken a frequency interval from 2.0 to 12 kHz.

The damaged structure was the same healthy structure with a 'nut', Fig. 1, bonded by a accelerometer wax. The PZT sensors are from Acellent Technologies and were attached by epoxy resin.

RESULTS AND ANALYSIS

As stated the analysis were performed for first case in a frequency range from 2 to 10 kHz, another case from 2 to 12 kHz. Two situations were also explored from the experimental results. The first one is the behavior of the detection algorithm using the product between FRF response magnitude and its coherence signal, Fig. 3.



(a) **(b)** **(c)**

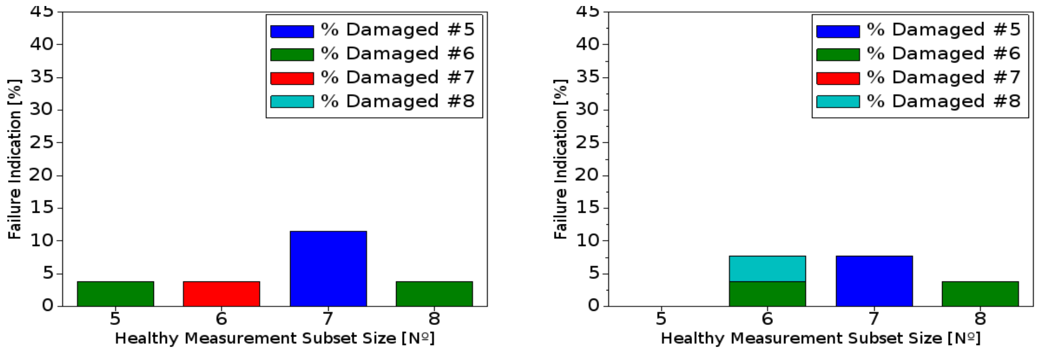
Figure 3: First case, (a) FRFs, (b) coherence of signal and (c) FRF times coherence, damaged data from 2 to 10 kHz.

The coherence signal is a indication of the measurement quality, good measurement quality means coherence equal 1, worthless measurement coherence near 0. On Fig.3(a) there are plotted the measured FRFs, the product between FRF magnitude and its coherence is on Fig.3(c), those graphics are not distinct from each other. This procedure has been used in previous works aiming to avoid bad or noisily measurements and help the algorithm to reject the outliers. The second analysis was performed directly on FRF set for both frequency range.

First Frequency range, 2 to 10kHz

The healthy structure database had 19 measurements. Before using the RVSD routine to compare healthy and damaged structure measurements the RVSD routine was also used as a technique to select 4 best subsets from original data. From the original set was taken subsets with 8, 7, 6 and 5 best measurement combinations. The damaged structure database had 20 measurements, from those 4 best dataset combinations were also taken from original, subsets with 8, 7, 6, 5 FRFs.

The best subset means that from original FRF measurement the RSVD routine select a subset that will re-synthesized matrix H_S which will give the smallest relative distance, d^{SVD},



(a) **(b)**

Figure 4: Failure indication for First Case, (a) results for FRFs times Coherence, and (b) measured FRFs.

to original data as described in Section 2. This procedure was used as a technique to avoid outliers in the reference data for healthy and damaged sets. The Vanlanduit (Vanlanduit et al, 2005) work suggests a similar number of samples for healthy and damaged structure to prevent the number of samples biasing results. The RVSD routine is time consuming for all possible subset combinations in a big database, so best subgroups were searched for before comparing healthy and damaged measurements.

The Blue Color Bar, Fig.4, represents the percentage of failure indications using 5 FRFs for damaged subset, the Green Bar means percentage of failure indications using 6 FRFs damaged data, Red Bar is perceptual for 7 FRFs and magenta for 8 FRF subset. For each healthy FRF subset, on horizontal axis, there are results from combinations with 4 damaged subset (8 magenta, 7 red, 6 green, 5 blue) and for each healthy subset there are 26 possible failure indications (8+7+6+5=26).

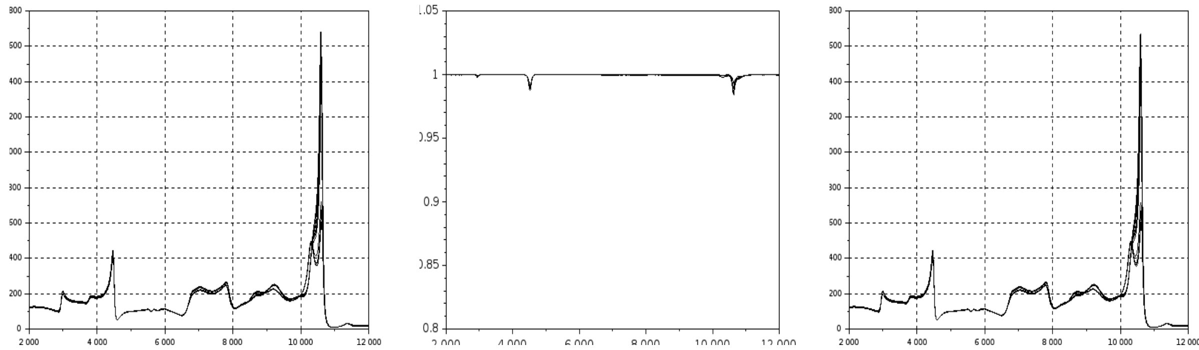
The failure percentage presented, for each healthy subset combination, was computed using this maximum value. This reference value under represents the algorithm efficiency for each combination and for small size sets. So if there is 100% failure indication for 5 damaged set case this mean 19% of total possible indications, for 6 damaged set 23%, 7 means 27% and 8 equals 31%. Although it permits a global view and a unbiased comparison as it uses the same reference.

On Fig. 4 there are no remarkable differences between the two cases and a weak failure indication. There is a small trend indicating positive failure for small damaged sets, Blue and Green bar, in both cases and for all healthy combinations.

Frequency range from 2 to 12kHz

On Fig. 5 there are the measured FRFs, damaged dataset, and its coherence signal. On Fig. 6 one could found the RSVD algorithm results for frequency range from 2 to 12kHz. There are a remarkable difference from the previous case on Fig. 4. There are also differences between the two cases, FRFs times Coherence, Fig. 6 (a), and measured FRFs, Fig. 6 (b).

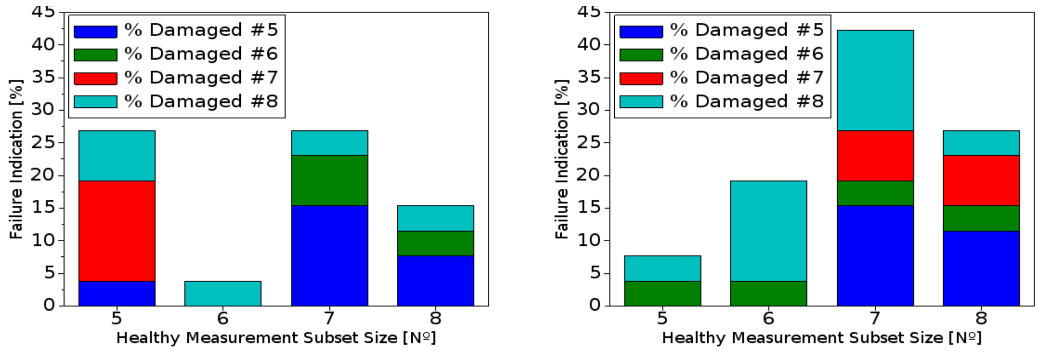
As one could expected, Fig. 6, there are an expressive positive failure indication for both cases, the only unexpected aspect is related to better performance for whose FRF data that was not multiplied by its coherence signal, Fig. 6(b). The FRF measurement has used a good resolution and the coherence signal is close to 1 so there is no expectation of a great difference on



(a) **(b)** **(c)**

Figure 5: Frequency range from 2 to 12kHz (a) FRFs, (b) coherence of signal and (c) FRF times coherence, healthy measurement.

results. Those results, Fig. 6, (a) and (b), have showed a noticeable RSVD algorithm sensitivity to data.



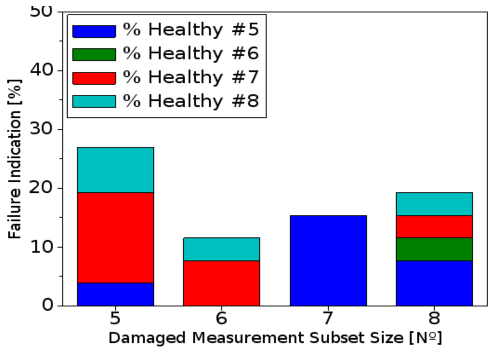
(a) **(b)**

Figure 6: Frequency range from 2 to 12kHz, (a) FRFs times coherence signal, and (b) measured FRFs.

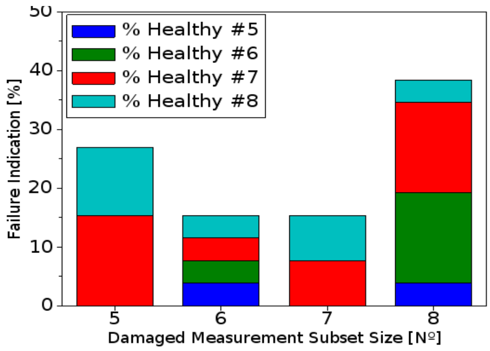
The failure indication has gone toward the healthy FRFs large sets, specially for measured FRF showed on Fig. 6(b). The 8 FRF damaged subset had more expressive participation on results in Fig. 6(b). Small sets specially 5 FRF damaged one has been effective in all cases, this is another indication that small sets presents good results supposing that more random data could be eliminated.

Verifying if sample size was biasing results the reference on horizontal axis were changed, Fig. 7. The vertical bars are the failure indication from combination of testing subset, or damaged, in horizontal axis and the healthy subset. In Fig. 7 Blue Bar represents the failure indication for combinations between the damaged set and 5 healthy measurement subset, Green Bar for combination with 6 measurement healthy sample, Red Bar for combinations with 7 healthy subset, and Magenta Bar for 8.

Figure 7, as one can observe on Fig. 6, has showed more failure indication for using measured FRFs than those for Coherence times FRFs. In Fig. 7(a) and (b), there are a similar failure indication for the damaged sample size in both cases. Observing Fig. 6 and Fig. 7 it is difficult to state that there are a subset size that biased the results, there is not a trend. There is a random behavior for sample size combinations and more results are needed.



(a)



(b)

Figure 7: Frequency range from 2 to 12kHz. Damaged sets on horizontal axis. (a) FRFs times coherence signal, and (b) measured FRFs.

So one can conclude that the subset size is not so important than data quality. Another important conclusion is that FRF times coherence was not effective to improve the RSVD algorithm, this procedure had improved results for the early results, maybe due to bad quality signal.

CONCLUSIONS

The results presented good agreement with the hypothesis that FRF at high frequency ranges are better to identify small damages. Results using FRF times coherence procedure are not effective as those using FRF data only, but indicates RSVD sensitivity to data quality. Those somewhat random results for each data size also supports this statement. Using the 'RSVD' algorithm to reduce the dataset to a small set seems to work providing good results at acceptable computational costs.

REFERENCES

C. Shane and R. Jha. Proper orthogonal decomposition based algorithm for detecting damage location and severity in composite beams. *Mechanical Systems and Signal Processing*, 25: 1062–1072, 2011.

W. Fan and P. Qiao. Vibration-based damage identification methods: A review and comparative study. *Structural Health Monitoring*, 10(1):83–111, 2011.

G. Konstantinidis, B. W. Drinkwater, and P. D. Wilcox. The temperature stability of guided wave structural health monotoring system. *Smart Materials and Structures*, 15:967–976, 2006.

G. Kerschen, J.C. Golival, A. F. Vakakis, and L. A. Bergman. The method of proper orthogonal decomposition for dynamical characterization and order reduction of mechanical systems: an overview. *Nonlinear Dynamics*, 41:147–169, 2005.

S. Vanlanduit, E. Parloo, B. Cauberghe, P. Guillaume, and P. Verboven. A robust singular value decomposition for damage detection under changing operating condtions and structural uncertainties. *Journal of Sound and Vibration*, 284:1033–1050, 2005.

C. Wu, Y. Liang, W. Lin, H. Lee, and S.P. Lim. Letter to the editor: A note on equivalence of proper orthogonal decomposition methods. *Journal of Sound and Vibration*, 265:1103–1110, 2003.

G. Kerschen and J. C. Golinval. Physical interpretation of the proper orthogonal modes using the singular value decomposition. *Journal of Sound and Vibration*, 249(5):849–865, 2002.