INFLUENCE OF THE PIEZOELECTRIC CONSTITUTIVE AND MECHANICAL DUFFING-TYPE NONLINEARITIES ON VIBRATION-BASED ENERGY HARVESTING

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Abstract. *Vibration-based energy harvesting has the main objective to convert available environment mechanical energy into electrical energy. Piezoelectric materials are usually employed to promote the mechanical-electrical conversion. This work investigates the influence of nonlinear effects in piezoelectric vibration-based energy harvesting. Mechanical nonlinearities are treated considering Duffing-type mechanical oscillator. Piezoelectric nonlinearities are analyzed assuming quadratic constitutive coupling model. Energy harvesting performance is evaluated for different system characteristics being monitored by the input and the generated power. Numerical simulations are carried out considering different kinds of responses, including periodic and chaotic regimes.*

Keywords: *Smart Materials, Nonlinear Dynamics, Chaos, Energy Harvesting, Piezoelectricity*

CILAMCE 2016
Proceedings of the XXXVII Iberian Latin-American Congress on Computational Methods in Engineering
Suzana Moreira Ávila (Editor), ABMEC, Brasília, DF, Brazil, November 6-9, 2016

1 INTRODUCTION

The concept of sustainability is increasing the importance of energy harvesting that is the process by which available environmental energy can be captured and stored. Among the main types of energy it should be highlighted solar power, electrical noise, wind energy. Vibration based energy harvesting is another alternative that can be applied in several situations. The main objective of this kind of process is to convert mechanical energy into electrical energy. Piezoelectric materials are usually employed to promote mechanical-electrical conversion. Machines, civil structures, airplanes, oil drilling, human movements, transportations, are some examples of applications of this idea. The energy harvested by the piezoelectric can be useful for application as small circuits like batteries and wireless network.

Nonlinear energy harvesting systems have been developed to obtain better performances over a broad frequency range providing more power than linear systems. In this regard, nonlinear effects are of special interest. Different nonlinearities can be incorporated in the mechanical, electrical and constitutive system. In order to increase the power generated by the piezoelectric.

Several research efforts have been dealing with nonlinear mechanical system. Ramlan et al. (2010) showed the benefits of using a nonlinear oscillator in an energy harvesting device. The use of Duffing-type oscillators with monostable and bistable harvesters has been studied for many authors (Mann & Sims, 2009; Sebald et al., 2011; Erturk and Inman, 2011; Leadenham and Erturk, 2015). Stanton et al. (2009) validated a nonlinear energy harvester capable of bidirectional hysteresis using a piezoelectric beam with a permanent magnet end mass. Either the softening or the hardening responses were obtained, allowing frequency response to be extended bidirectionally. Stanton et al. (2010) and Cottone (2009) investigated bistable configurations of energy harvesting system using an experimental approach. De Paula et al. (2015) investigated random aspects on vibration-based energy harvesting of a piezomagnetoelastic structure. A comparison between linear, nonlinear bistable and nonlinear monostable systems subjected to random excitations showed an enhancement of harvested power in a bistable system. Betts et al. (2012) presented a nonlinear device through an arrangement of bistable composites combined with piezoelectric elements for broadband energy harvesting of ambient vibrations. Results showed that it is possible to improve the power harvested over conventional devices.

Regarding nonlinear constitutive effects, Crawley and Anderson (1990) discussed nonlinear aspects related to piezoelectric coupling constants showing that there is a significant dependence of strains. Triplett and Quinn (2009) investigated the nonlinear coupling behavior of piezoelectric materials and also some aspects related to the mechanical nonlinearities in vibration-based energy harvesting. Stanton et. al. (2010) proposed a model for the nonlinear piezoelectric response of an electro-elastic energy harvester using a quadratic dependence of piezoelectric coupling coefficient on the induced strain. Experimental tests are performed showing a good agreement between numerical and experimental data. Silva et al. (2013) investigated the influence of hysteretic behavior of piezoelectric coupling comparing results with linear models. Results suggest that there is an optimum hysteretic behavior that can increase the harvested power output of the energy harvesting systems. Silva et al. (2015) showed a comparison among experimental data and models including distinct nonlinear piezoelectric couplings. The inclusion of nonlinear terms in the energy harvester models can be used to reduce discrepancies predicted by linear models. Moreover, nonlinear aspects as dynamical jumps are associated with dramatic changes of system responses.

This article analyzes nonlinear effects considering both mechanical and piezoelectric aspects. An archetypal model for vibration-based energy harvesting system is considered by assuming a mechanical system coupled to an electric circuit by a piezoelectric element. A Duffing-type oscillator that can be monostable and bistable represents mechanical nonlinearity. Constitutive nonlinearity is investigated considering a piezoelectric element described by a quadratic equation. Input and output powers are monitored evaluating the energy harvesting system. Periodic and chaotic responses are evaluated in order to verify system performance.

2 ENERGY HARVESTING SYSTEM

A vibration-based energy harvesting system consists of a mechanical system connected to an electrical circuit by a piezoelectric element, Fig. 1. Mechanical system is an oscillator with a mass m that presents a displacement y; the base excitation is represented by $u=u(t)$ while z represents the mass displacement relative to the base. In addition, the oscillator has a linear viscous damping with coefficient b, and a restitution element that provides a force $\mathcal{F}(z)$. Electro-mechanical coupling is provided by a piezoelectric element with coupling coefficient $\widehat{\Theta}$. This element is connected to an electric circuit represented by an electrical resistance R_l and capacitance C; V is the voltage across the piezoelectric element.

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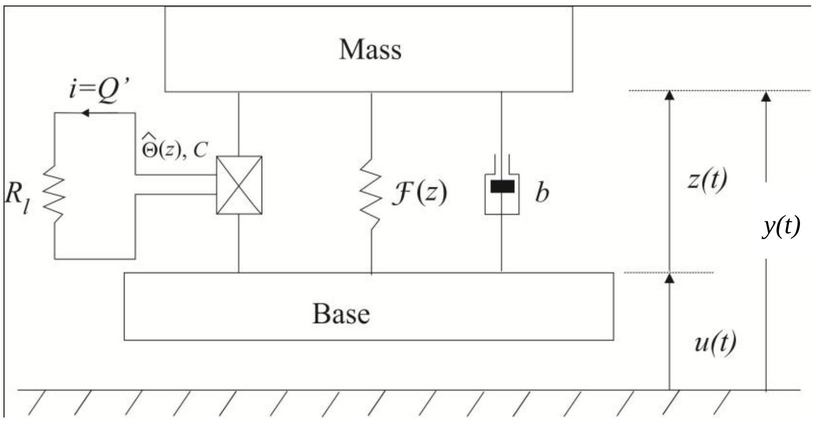

Figure 1. Archetypal model of the vibration-based energy harvesting system.

The dimensionless mathematical model for the energy harvesting system is as follows, where a spatial and electrical new coordinates as $x = z/l$, $v = V/\hat{V}$ is employed with l is a reference length and $\hat{V}$ is a reference voltage and $\phi = (l/C\hat{V})\widehat{\Theta}$ is the piezoelectric coupling term.

$$x'' + 2\zeta x' + f - \epsilon\phi v = \gamma \sin(\bar{\omega}\tau) \quad (1)$$

$$\phi x' + v' + {}^{v}/_{\rho} = 0 \quad (2)$$

where ζ is the dissipation parameter, ρ is a time constant, f is the restitution force, γ is the forcing amplitude and $\bar{\omega}$ is the forcing frequency. In addition, $(\blacksquare') \equiv d(\blacksquare)/d\tau$ is the derivative with respect to nondimensional time.

A Duffing-type restitution force that represents the general behavior being expressed by the following equation:

$$f(x) = \beta x + \alpha x^3 \tag{3}$$

And piezoelectric coupling nonlinearity is given by:

$$\phi = \theta(1 + \xi_1|x| + \xi_2 x^2) \tag{4}$$

Note that when β <0, the system has a bistable aspect and when β >0 the system is nonlinear monostable. The monostable system oscillates around one equilibrium point and the bistable around one or three. Linear system is characterized by β >0 and α=0. Values of α define different types of nonlinearity. Figure 2 (a) shows the corresponding restitution force for different values of these parameters The general behavior of the nonlinear electro-mechanical coupling can be observed in Fig.2 (b) for different parameters. There is a strong dependence between the piezoelectric coupling and strain as showed in Crawley and Anderson (1990). This nonlinearity has a significant influence on energy hasvesting response.

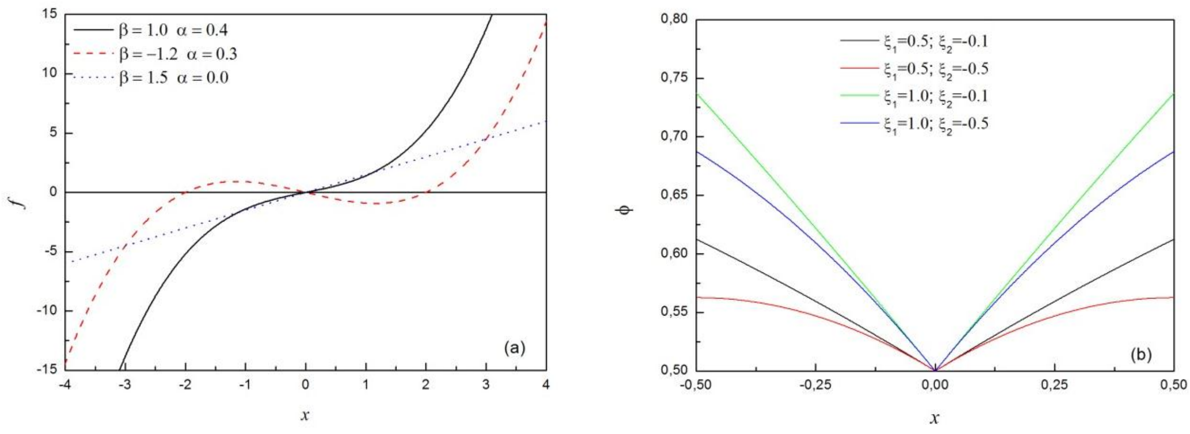

Figure 2. (a) Duffing type system: force for different values of α and β. (b) Electro-mechanical behavior as function of induced strain.

The system can be evaluated considering the output and input powers, or in other words, electrical and mechanical powers. The definitions are as follows, $P_{out} = \sqrt{\frac{1}{\tau}\int_0^\tau (v^2/\rho)^2 d\tau}$ and $P_{in} = \sqrt{\frac{1}{\tau}\int_0^\tau (x'(\delta \sin(\bar{\omega}\tau)))^2 d\tau}$.

The electrical instantaneous power of the energy harvesting system is defined as $P = v^2/\rho$, and average power, defined as $P_{avg} = \frac{1}{T}\int_0^T P(t)d\tau$, where $T = 2\pi$. The conversion efficiency, $\eta = P_{out}/P_{in}$, establishes a relation between electrical and mechanical powers.

3 NUMERICAL SIMULATIONS

Numerical simulations are carried out to explore the energy harvesting system dynamics considering different nonlinearities and excitations. The fourth order Runge-Kutta method implemented in FORTRAN 9.0 is applied in all simulations considering time steps smaller than 10^{-4}.

System analysis considers a Duffing type restitution force for different parameters α e β considering monostable and bistable characteristics. Constitutive nonlinearities of the piezoelectric element are also investigated by comparing with a linear constitutive model ($\xi_1 = \xi_2 = 0$). The main focus of the analysis is the average power and efficiency of the system. Some parameters are adopted in all simulations: $\zeta = 0.01, \epsilon = 0.1, \theta = 0.5$ and $1/\rho = 0.05$.

3.1 Influence of the Forcing Frequency

Initially, the system efficiency is evaluated changing the forcing frequency with $\gamma = 0.1$ (amplitude forcing) and adopting different parameters for the restitution force ($\beta = 1.0, \alpha = 0.4$ and $\beta = -1.2, \alpha = 0.3$) with a linear piezoelectric constitutive model ($\xi_1 = \xi_2 = 0$). There is a special interest on the resonant conditions. Nevertheless, nonlinearities are associated with complex responses that need to be properly investigated.

Figure 3 shows the system efficiency considering different characteristics of the mechanical system and variations of the forcing frequency. The second one is related to jumps, pointing to a typical nonlinear resonance.

In general, for these sets of parameters, monostable systems (associated with positive β) have better efficiency than the bistable ones (associated with negative β). Note that the maximum efficiency occurs at different frequencies, and this conclusion is more evident for bistable systems. Note that a better efficiency occurs under the resonance condition for both systems, but the monostable system has a larger band of frequencies with high performance.

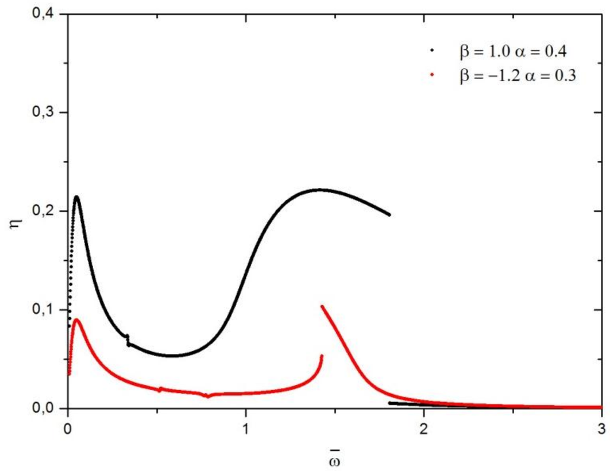

Figure 3. Efficiency versus forcing frequency for linear piezoelectric coupling ($\xi_1 = \xi_2 = 0$). Monostable, $\beta = 1.0$ and $\alpha = 0.4$ (black line) and bistable, $\beta = -1.2$ and $\alpha = 0.3$ (red line).

Figure 4 presents a comparison between input and output powers for monostable and bistable systems. Note that monostable systems have better efficiency than the bistable ones for these sets of parameters. Besides, monostable systems have hardening trend while the bistable ones present softening behavior.

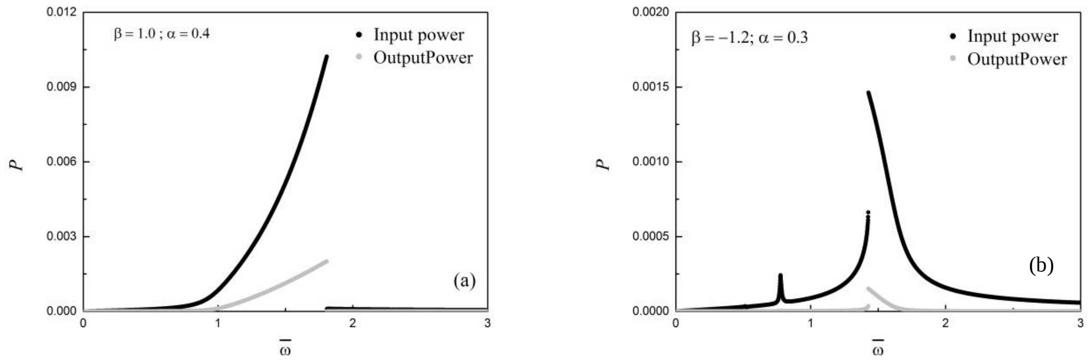

Figure 4. Input power and output power for different parameters. (a) $\beta = 1.0, \alpha = 0.4$ and (b) $\beta = -1.2, \alpha = 0.3$

Energy harvesting system response is now investigated assuming nonlinear piezoelectric couplings ($\xi_1 = 0.5$ and $\xi_2 = -0.1$). Figure 5 shows a significant increase in efficiency for the nonlinear piezoelectric constitutive model compared with linear constitutive model. Details can be observed in Fig.6, where the monostable and bistable results are displayed together, comparing the effect of linear and nonlinear piezoelectric couplings.

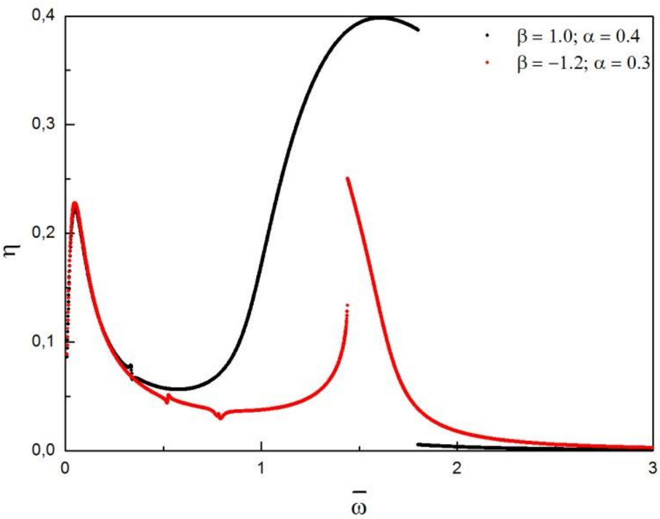

Figure 5. Efficiency versus forcing frequency with $\xi_1 = 0.5$ and $\xi_2 = -0.1$.

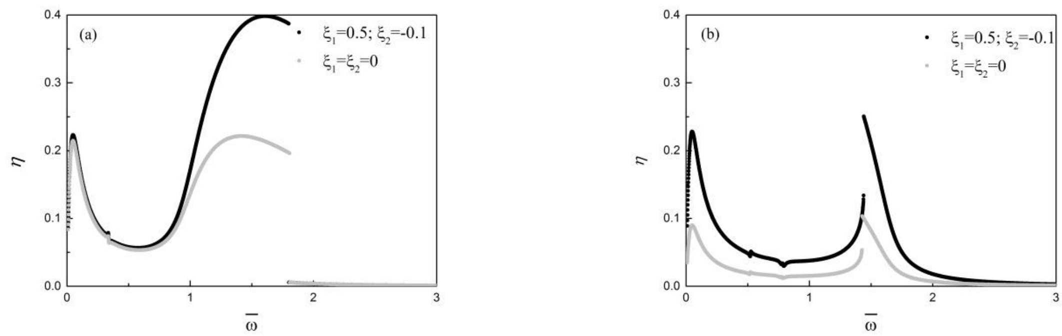

Figure 6. Efficiency versus forcing frequency comparing linear and nonlinear piezoelectric couplings with (a) $\beta = 1.0, \alpha = 0.4$ and (b) $\beta = -1.2, \alpha = 0.3$.

3.2 Influence of the Forcing Amplitude

The influence of the forcing amplitude for different excitation conditions are now investigated and exploited. Basically, it is considered the system response under a constant forcing frequency, varying the forcing amplitude. Initially, a monostable system ($\beta > 0$) is treated considering linear piezoelectric coupling ($\xi_1 = \xi_2 = 0$) with $\varpi = 1.5$. Figure 7 presents bifurcation diagrams considering power and average power for different values of the excitation forcing amplitude and adopts the previous parameter response as initial conditions. Note that for γ = 0.05, it is possible to collect a maximum power around 0.002282. Similar situation can be observed for γ = 4.23 where 0.118328 in terms of maximum power and 0.05254 in terms of average power.

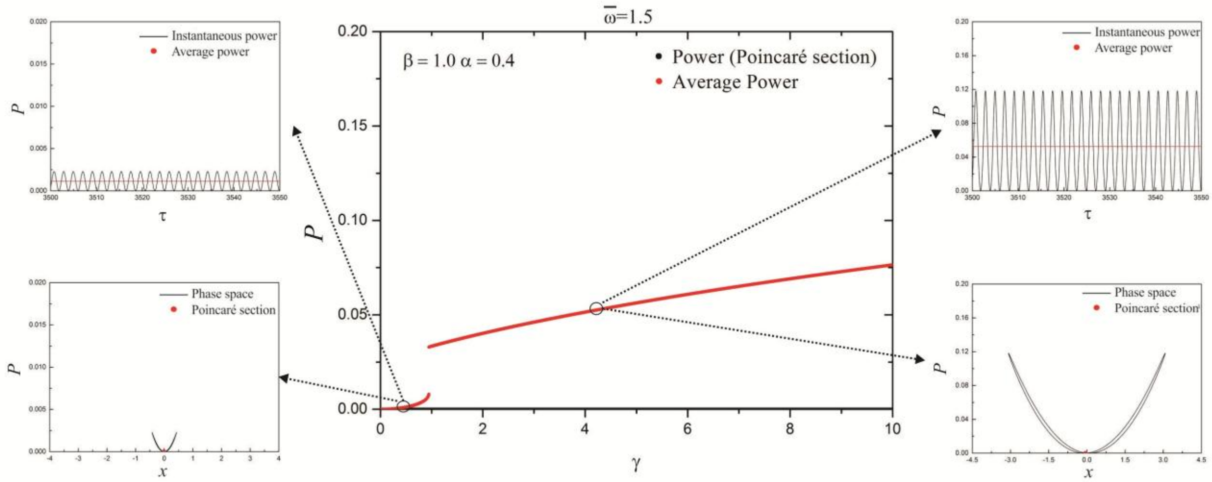

Figure 7. Power energy (P) and average power energy for different values of forcing amplitude $\xi_1 = \xi_2 = 0$ using different initial conditions for each parameter (monostable system).

The system response is now investigated assuming linear and nonlinear piezoelectric couplings using $\bar{\omega} = 1.5$. This frequency is close to resonant conditions of both cases. Figure 8 presents bifurcation efficiency diagrams. Once again, nonlinear piezoelectric coupling tends to increase the efficiency. It is noticeable a general low efficiency and a jump close to $\gamma = 1.0$.

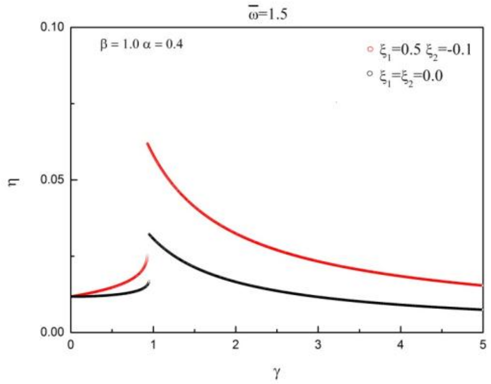

Figure 8. Efficiency for different values of forcing amplitude for linear ($\xi_1 = \xi_2 = 0$) and nonlinear ($\xi_1 \neq 0$ and $\xi_2 \neq 0$) monostable model.

Bistable system ($\beta < 0$) is investigated considering linear coupling terms ($\xi_1 = \xi_2 = 0$) with $\bar{\omega} = 1.5$. Figure 9 present bifurcation diagrams considering power and average power for different values of the excitation forcing amplitude and adopts the previous parameter response as initial conditions. Chaotic regions are observed in regions associated with cloud of points, being related to good performance in terms of generated power and therefore, are of special interest in terms of harvested energy. The maximum power value varies as follows: 0.173352 (γ = 0.74), 0.23520 (γ = 1.944), and 0.281445 (γ = 4.966).

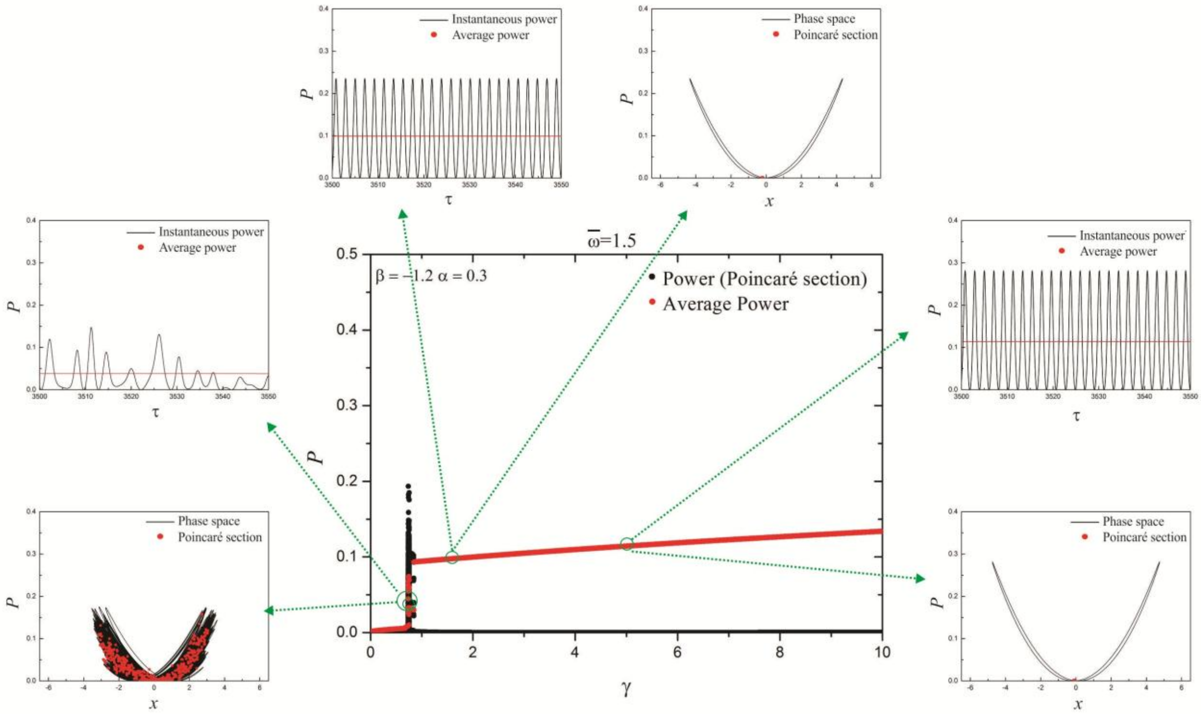

Figure 9. Power energy (P) and average power energy for different values of forcing amplitude $\xi_1 = \xi_2 = 0$ using different initial conditions for each parameter (bistable system).

Figure 10 presents details of the system dynamics for some set of parameters with $\bar{\omega} = 1.5$ and different kinds of solutions. Note that periodic solutions that oscillate around one equilibrium point, around three equilibrium points (around the two-well) and chaos are all highlighted.

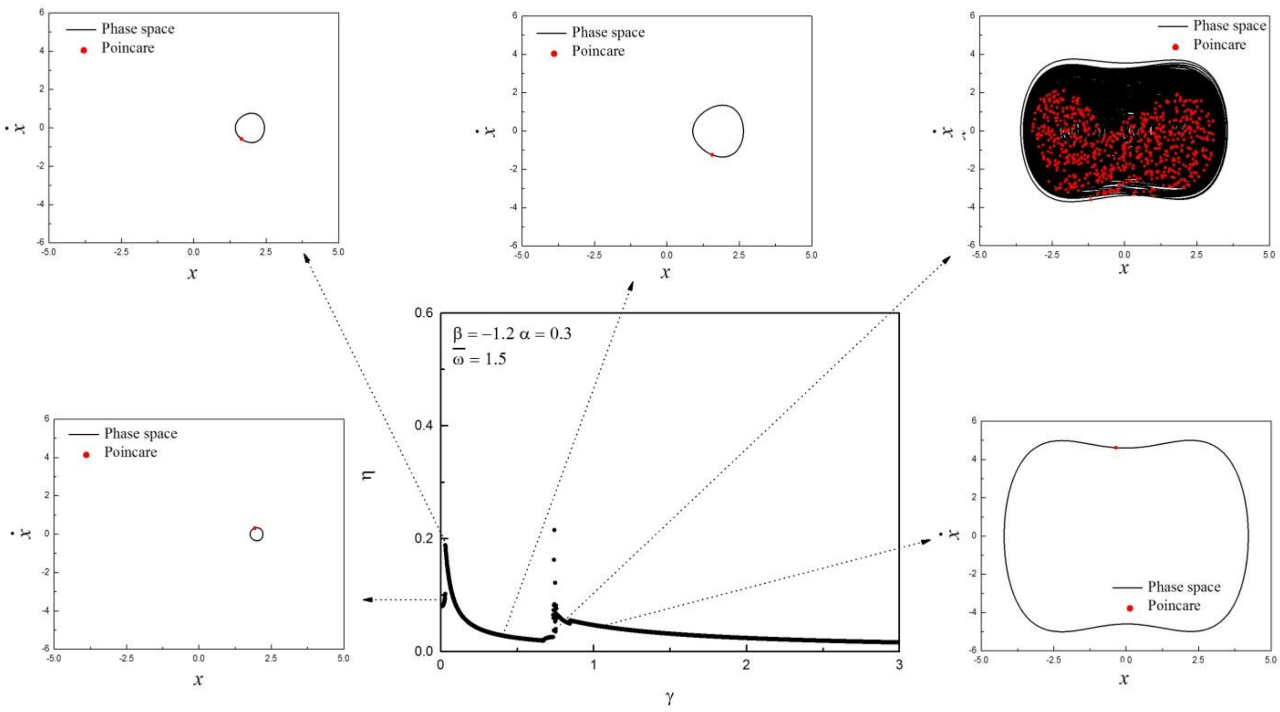

Figure 10. Details of bistable system dynamics (β = -1.2; α = 0.4; $\bar{\omega} = 1.5$; $\xi_1 = \xi_2 =$ 0).

Figure 11 presents bifurcation efficiency diagrams for comparing linear ($\xi_1 = \xi_2 = 0$) and nonlinear ($\xi_1 \neq 0$ and $\xi_2 \neq 0$) piezoelectric couplings. Under these conditions, it is possible to observe chaotic regions close to dynamical jumps. These regions are associated with good efficiency when compared with others.

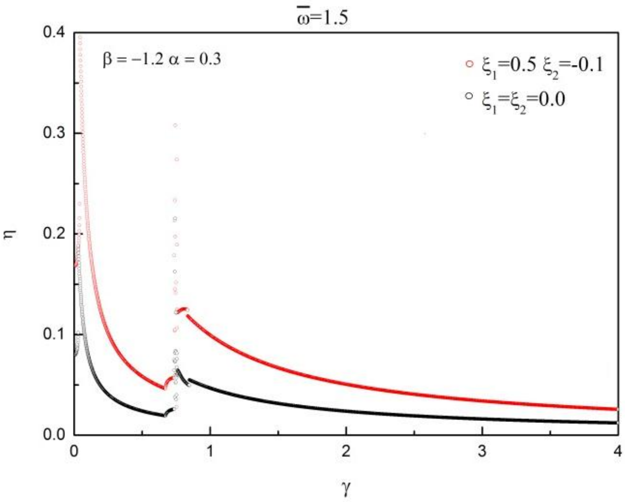

Figure 11. Efficiency for different values of forcing amplitude for linear ($\xi_1 = \xi_2 = 0$) and nonlinear ($\xi_1 \neq 0$ and $\xi_2 \neq 0$) bistable model.

Figure 12 presents the difference between the average power of monostable and bistable system with the same sets of parameters. It is observed that the bistable system generates greater average power values when the system is around three equilibrium points.

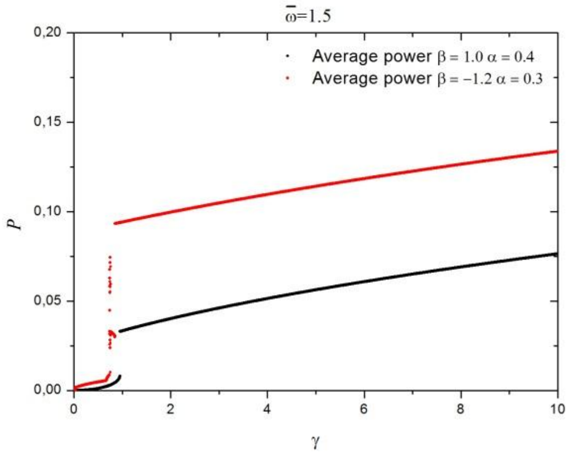

Figure 12. Average power energy for different values of forcing amplitude $\xi_1 = \xi_2 = 0$ using different initial conditions for each parameter (monostable and biestable system).

4 CONCLUSIONS

This article investigates nonlinear effects regarding vibration-based energy harvesting system. Mechanical nonlinearity is treated considering a Duffing-type oscillator analyzing either monostable or bistable systems. Piezoelectric electro-mechanical nonlinearity is treated considering a quadratic constitutive equation. Rich response is observed including chaotic and periodic solutions. Monostable system presents better performance under resonant conditions. On the other hand, bistable system presents better results. Situations where the system oscillates around three equilibrium points tend to be better due to their greater amplitudes. In general, chaotic response has an interesting amount of harvested power but bistable system presents an efficient generation in other situatios. Results show that piezoelectric nonlinearity has a significant influence on the system performance in terms of the harvested power and can enhance the power harvesting performance.

5 ACKNOWLEDGEMENTS

The authors would like to acknowledge the support of the Brazilian Research Agencies CNPq, CAPES and FAPERJ and through the INCT-EIE (National Institute of Science and Technology - Smart Structures in Engineering) the CNPq and FAPEMIG. The Air Force Office of Scientific Research (AFOSR) is also acknowledged.

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