AN HYBRID STOCHASTIC-DETERMINISTIC OPTIMIZATION ALGORITHM FOR STRUCTURAL DAMAGE IDENTIFICATION

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Abstract. *This paper presents a hybrid stochastic/deterministic optimization algorithm to solve the target optimization problem of vibration-based damage detection. The use of a numerical solution of the representation formula to locate the region of the global solution, i.e., to provide a starting point for the local optimizer, which is chosen to be the Nelder-Mead algorithm (NMA), is proposed. A series of numerical examples with different damage scenarios and noise levels was performed under impact and ambient vibrations. To test the accuracy and efficiency of the optimization algorithm, its results were compared to previous procedures available in the literature, which employed different solutions such as the genetic algorithm (GA) and the harmony search algorithm (HS). The performance of the proposed optimization scheme was more accurate and required a lower computational cost than the GA and HS algorithms, emphasizing the capacity of the proposed methodology for its use in damage diagnosis and assessment.*

Keywords: Structural health monitoring, Hybrid stochastic/deterministic (P-NMA) optimization method, Nelder-Mead algorithm, representation formula of Pincus.

INTRODUCTION

It is recognized that efficient methods to detect and quantify structural damage generate a wide interest in the civil, mechanical and aerospace fields. Thus, the application of robust procedures in programs to restore the reliability of engineering structures to initial design levels is highly desired. One method that may fulfil those requirements is dynamic non-destructive testing, which consists of monitoring the modal properties (e.g., natural frequencies, vibration modes and damping) during the lifetime of a structure.

In recent developments of these procedures, the main focus has been placed on taking advantage of ambient vibrations, such as traffic or wind- or pedestrian-induced vibrations, to determine the spectral properties at any time without operational interference or the use of special equipment for the excitation. Therefore, because it is more convenient to extract the modal parameters for output-only measurement conditions (Miguel et al., 2009), stochastic system identification techniques become very attractive. Reliable time-domain techniques, such as the Stochastic Subspace Identification (SSI) technique (Van Overschee and De Moor, 1993) or the Eigensystem Realization Algorithm (ERA) method (Juang and Pappa, 1985) coupled with the Natural Excitation Technique (NExT), have been successfully applied to in-situ measurements of ambient vibrations in long-term structural health monitoring (SHM).

Several strategies have been reported in the vibration-based damage detection field. A review of the state-of-the-art and developments of vibration-based structural damage detection were presented by Doebling et al. (1996) and Santos et al. (2008), to name just a few. Even considering the recent developments of vibration-based SHM techniques and the results of numerous studies with different degrees of success, this problem cannot be considered fully addressed and remains a challenging task. Special attention and additional work should be dedicated to develop robust and accurate techniques that are able to minimise experimental noise or numerical errors, thus smoothing false positive damage diagnoses.

Vibration-based damage detection may be treated as a bounded nonlinear optimization problem. The basic idea is to change the properties of the numerical model to fit the values provided by the experimental data, identifying damaged regions and the extent of damage on the structure. In other words, the optimization algorithm seeks the optimal parameter values, which are the reduction factors of element stiffnesses, to achieve a pre-defined performance in terms of the modal parameters defined by the experimental data. This procedure leads to a target performance optimization problem, which is usually very complex to solve because it generally leads to nonconvex and multimodal objective functions (see for instance, Gonçalvez and de Cursi (2001) and Lopez et al., 2011). Under these conditions, deterministic optimization algorithms such as gradient methods, Newton methods or sequential simplex methods may not converge to the global minimum of the problem due to their dependence on the quality of the starting point of the search. That is, if a given starting point is not on a basin of attraction of the global optimum, these methods will not converge to the global solution; the use of a global optimization algorithm is then required. In this framework, stochastic methods are often employed, including the following well-known examples: pure random search, GA, and simulated annealing (SA), among others. Recently, the problem was also solved using other more recent metaheuristics such as the bee algorithm (BA) (Moradi et al., 2011), the PSO algorithm (Kang et al., 2012), and the HS algorithm (Fadel Miguel et al., 2012).

However, these algorithms present some drawbacks, which include the following: (i) they require the tuning of many parameters by trial and error to maximise efficiency; (ii) the a priori estimation of their performance is an open mathematical problem; and (iii) an extremely large number of evaluations of the objective function are required to achieve global optimization, especially for continuous design variables. That is, they find in a reasonable time the region where the global solution is, yet they require a significant amount of computation to converge to the precise value of the global optimum, or sometimes they simply do not reach the exact solution. Hence, the use of these stochastic methods leads to a very high computational cost. Thus, to overcome this drawback, several classes of global optimization algorithms have been developed to increase the efficiency of the search. One such class consists of hybrid stochastic/deterministic methods in which a local optimiser, such as the deterministic methods cited above, is combined with a global optimiser, such as the stochastic methods previously mentioned.

Within this context, the main contribution of this paper is the application of a hybrid stochastic/deterministic (P-NMA) optimization scheme to solve the target optimization problem that identifies the structural damage. It is proposed the use of the numerical solution of the representation formula proposed by Pincus (1968) to locate the region of the global solution, i.e., to provide a starting point for the local optimiser, which is chosen to be the NMA (Nelder and Mead, 1965). Thus, once the starting point furnished by the representation formula is given to the NMA, it is expected that such a point is in a basin of attraction of the global optimum and that the NMA will be able to converge to the optimization problem's global solution, which identifies the damage scenario of the structure under analysis. The proposed optimization scheme is expected to find more accurate results while requiring much lower computational cost and time and, consequently, to overcome the drawbacks of the stochastic algorithms cited above. Initially, the damage detection process is formulated as a bounded nonlinear optimization problem in Section 2. Discussion on the proposed P-NMA optimization method is presented in Section 3. The efficiency and accuracy of the proposed method is highlighted in Sections 4 and 5 by comparing its results to the solutions of stochastic optimization methods available in the literature for the numerical analysis pursued. Finally, the main conclusions drawn from this work are summarised in Section 6.

1 THE PROBLEM FORMULATION

One classical approach to represent damage is to consider the reduction of the stiffness properties of the structure. Thus, it is useful to introduce the damage in the structure through the consideration of an elemental stiffness reduction factor (α_i), which enables the preservation of the original structural connectivity.

In this approach, the global stiffness matrix of the structure can be formulated as the assembly of damaged and undamaged element stiffness matrices in global coordinates, where the local element stiffness is multiplied by the reduction factor (α_i), such as

$$\left[\mathbf{k}_G(\alpha_i)\right]_i = \left[\mathbf{T}\right]_i^T \alpha_i \left[\mathbf{k}_\mathbf{e}\right]_i \left[\mathbf{T}\right]_i,\ i = 1, 2, 3, \ldots N, \tag{1}$$

$$[\mathbf{K}] = [\mathbf{K}(\boldsymbol{\alpha})], \tag{2}$$

In the above equations, N is the total number of elements of the structure, $\left[\mathbf{k}_G(\alpha_i)\right]_i$ is the element stiffness matrix in global coordinates, $\left[\mathbf{T}\right]_i$ is the transformation matrix of an element, $\left[\mathbf{k}_\mathbf{e}\right]_i$ is the local element stiffness matrix and $\left[\mathbf{K}\right]$ is the global stiffness matrix of the entire structure, which is assembled from $\left[\mathbf{k}_G(\alpha_i)\right]_i$. The reduction factor $\boldsymbol{\alpha} \in C \subset \Re^N$ can be defined as the ratio of the element stiffness reduction to the initial stiffness. The set C ranges from 0 to 1, where 1 signifies no damage in the element and 0 means that the element loses its stiffness completely. In damage detection techniques, through the solution of an optimization problem, structural damage is estimated from a model update process using damage-induced changes in the modal features. A numerical model is continuously updated until its difference from the experimental model is minimised. This process is formulated as the following optimization problem:

$$\boldsymbol{\alpha}^* = \arg\min\left(J(\boldsymbol{\alpha}) : \boldsymbol{\alpha} \in C\right), \tag{3}$$

where J is the objective function to be minimised. To solve this problem, J must be formulated in terms of the differences between the numerical and experimental values. A correct choice of this objective function is of paramount importance in the finite element model updating. Different objective functions have been applied, usually adopting frequency and mode shape residuals. Next, it is presented some strategies employed in the literature to construct J, which can be efficiently applied in the SHM context.

1.1 Frequency and Mode Shape Changes

Fractional changes in natural frequencies before and after damage can be used to construct J. For example,

$$J(\boldsymbol{\alpha}) = \sum_{i=1}^{NM}\left(\left(\frac{\delta\omega_i(\boldsymbol{\alpha})}{\omega_i}\right)^D - \left(\frac{\delta\omega_i}{\omega_i}\right)^E\right)^2, \tag{4}$$

in which NM is the number of modes analysed, the superscripts D and E represent numerical and experimental quantities, respectively, ω_i is the natural frequencies for the ith mode of the undamaged or healthy condition for both the experimental and analytical conditions, and, finally, $\delta\omega_i$ is a fractional change of the experimental and analytical natural frequencies for the ith mode of the structure. A finite element model should be used to represent the reference or healthy state of the target structure. Then, the stiffness reduction factor ($\boldsymbol{\alpha}$) of the finite element model should be updated until the differences of the numerical frequencies in the healthy and damaged states converge to the observed experimental frequencies in the pre- and post-damaged states. Because the natural frequencies can be accurately measured, this objective function is practical for real-time SHM under ambient vibrations. However, it is difficult to distinguish the damage in symmetric locations of a symmetric structure. In this situation, mode shapes may be introduced in the objective function as

$$J(\boldsymbol{\alpha}) = \sum_{i=1}^{NM}\left(\left(\frac{\delta\omega_i(\boldsymbol{\alpha})}{\omega_i}\right)^D - \left(\frac{\delta\omega_i}{\omega_i}\right)^E\right)^2 + \sum_{i=1}^{NM}\sum_{j=1}^{NP}\left(\left(\delta\phi_{ij}(\boldsymbol{\alpha})\right)^D - \left(\delta\phi_{ij}\right)^E\right)^2. \tag{5}$$

In practice, it is only possible to measure a few mode shapes and frequencies during vibration testing, even for free vibration tests. Thus, only those nodal displacements (denoted NP) that are really measured can be picked out of the numerical mode shapes. Then, any mode shape expansion procedure can be avoided (Fadel Miguel *et al.*, 2006).

1.2 Flexibility Matrix

The modal flexibility error residual may also be employed as an objective function. The modal flexibility matrix may be expressed as

$$[\mathbf{F}] = [\boldsymbol{\varphi}][\boldsymbol{\Lambda}]^{-1}[\boldsymbol{\varphi}]^T, \tag{6}$$

where $[\boldsymbol{\varphi}]$ is a mode shape matrix, and $[\boldsymbol{\Lambda}]$ represents a diagonal matrix containing the squares of the modal frequencies. The difference between the experimental model and the numerical model can then be employed as the objective function for damage quantification:

$$J(\boldsymbol{\alpha}) = \left\| [\mathbf{F}]_E - [\mathbf{F}(\boldsymbol{\alpha})]_D \right\|_{Fro}^2, \tag{7}$$

in which $\boldsymbol{\alpha}$ is the stiffness reduction factor, $\| \cdot \|_{Fro}^2$ represents the Frobenius norm of the residual matrix, $[\mathbf{F}]_E$ indicates the modal flexibility matrix from the experimental results, and $[\mathbf{F}(\boldsymbol{\alpha})]_D$ is the modal flexibility matrix calculated from the numerical model with the stiffness reduction factor.

It is not possible in practice to construct the flexibility matrix for all the degrees of freedom (DOFs) because only a limited number of measurements are available. Thus, the flexibility matrix may be obtained from only a few low-frequency modes in accordance with the measured DOFs.

2 THE HYBRID P-NMA OPTIMIZATION ALGORITHM

A hybrid stochastic/deterministic (P-NMA) algorithm is employed for the solution of Eq. (3). The stochastic part is given by the numerical solution of the representation formula first proposed by Pincus (1968) and reformulated by de Cursi (2006). As detailed below, the main goal of the stochastic part is to provide a starting point close to the global solution of Eq. (3) for the deterministic counterpart of the proposed hybrid optimization algorithm so that the local optimiser converges to the global solution. In this paper, it is employed the NMA to pursue the local search. Both the stochastic and deterministic parts of the algorithm are detailed in the following.

Consider that $\boldsymbol{\alpha}^*$ is the global solution of the optimization problem given in Eq. (3). As demonstrated by de Cursi (2006), the solution of such an optimization problem may be represented by the following relation:

$$\boldsymbol{\alpha}^* = \lim_{\lambda \to +\infty} \frac{E[\mathbf{A}\ g(\lambda, J(\mathbf{A}))]}{E[g(\lambda, J(\mathbf{A}))]}, \tag{8}$$

in which g is a continuous and strictly decreasing function and $\mathbf{A}$ is a convenient random variable, such as a random variable uniformly distributed on the domain C. The function g may be chosen as suggested by Pincus (1968): $g(\lambda, J(\mathbf{A})) = \exp(-\lambda J(\mathbf{A}))$. The general properties of **A** and g are detailed, for instance in Sonza de Cursi et al. (2006). It means that if it is possible to evaluate the limit and the expectation operators of Eq. (8), the global optimum $\boldsymbol{\alpha}^*$ of Eq. (3) can be obtained, even in the case where the objective function J is nonconvex. However, the analytical evaluation of these quantities is not possible for real engineering problems. Hence, the application of the representation of the optimal solution $\boldsymbol{\alpha}^*$ established above requires the numerical approximation of Eq. (8). Such an approximation may be based on the generation of finite samples of the random variables involved in the expressions and an approximation of the limit. For example, as described in de Cursi (2006), it must be chosen λ large enough and generated a sample $\hat{A} = \{\mathbf{A}_1,...,\mathbf{A}_{nr}\}$ comprising *nr* variates of $\mathbf{A}$. Thus,

$$\boldsymbol{\alpha}^* \approx \hat{\boldsymbol{\alpha}}^* = \frac{\sum_{i=1}^{nr} \mathbf{A}_i \; g(\lambda, J(\mathbf{A}_i))}{\sum_{i=1}^{nr} g(\lambda, J(\mathbf{A}_i))}, \tag{9}$$

which corresponds to the approximations

$$E[\mathbf{A} \; g(\lambda, J(\mathbf{A}))] \approx \frac{1}{nr}\sum_{i=1}^{nr} \mathbf{A}_i \; g(\lambda, J(\mathbf{A}_i)) \text{ and } E[g(\lambda, J(\mathbf{A}))] \approx \frac{1}{nr}\sum_{i=1}^{nr} g(\lambda, J(\mathbf{A}_i)). \tag{10}$$

As already noted, this approximation is able to locate a region on the domain close to the global solution. Thus, the approximation given by Eq. (9) may be employed as the starting point for a local optimiser, such as any gradient-based algorithm or the NMA, to obtain a more refined solution. This approach has been tested in some engineering problems. For example, Lopez et al. (2011) employed this approximation to supply the starting point for the random perturbation of the gradient algorithm in a stochastic programming problem. Gonçalves and de Cursi (2001) applied this strategy in the calibration of a transportation system. The local optimizer employed in this paper is the NMA, which is one of the most standard direct search methods for unconstrained minimisation problems. The NMA is initialized at $\boldsymbol{\alpha}_0$ and it is expected to converge to a local minimum. It is based on the comparison of function values at the *n+1* vertices $\boldsymbol{\alpha}_i$ of a simplex. In this paper, the full description of the NMA is not provided since it is widely available in optimization textbook, for instance, the reader is referred to Haftka and Gürdal (1992).

It should be observed here that the point of initialisation of the simplex $\boldsymbol{\alpha}_0$ is furnished by the numerical approximation of the representation formula of Eq. (8) given by Eq. (9). Because the computational code of the NMA is widely available, the implementation of the presented hybrid P-NMA optimization algorithm reduces, for instance, to the implementation of the code used to evaluate Eq. (9) and its coupling to an existing NMA code, which is quite simple. This fact shows the ease in using and implementing the proposed scheme. Moreover, any other local optimiser may be employed instead of the NMA; *e.g.,* if it is easy to obtain the gradient information about the objective function under analysis, the user of the proposed algorithm could employ a gradient-based algorithm such as the interior point or sequential quadratic programming methods. Finally, it is the main goal of the next section to show that despite the fact that the proposed algorithm is easy to implement and use, it provides more accurate results requiring much lower computational cost and/or time to solve the problem at hand than well-known global optimization algorithms such as the GA and the HS.

3 NUMERICAL ANALYSIS

Standard test problems recently reported in the literature are used to validate the P-NMA optimization algorithm. This section starts with a noise-free portal plane frame previously solved by Gomes and Silva (2008) using the GA. Next, a cantilever beam used by Miguel et al. (2012) is investigated to assess the influence of measurement noise.

3.1 Portal plane frame

To assess the accuracy of the P-NMA algorithm, the same portal frame model that was recently studied using the GA by Gomes and Silva (2008) was considered. The structure has a rectangular cross-sectional area with height h = 0.24 m, width b = 0.14 m and lengths of L = 2.4 m and H = 1.6 m. The material has a Young's Modulus of $E = 2.5x10^{10}$ N/m^2 and a material density of $\rho = 2.5 \times 10^3$ kg/m^3. In the structural model, 56 plane frame elements were used, as shown in Fig. 1.

This is an interesting initial example to verify the accuracy of the proposed optimization approach, as no general conclusions could be reached in the previous study (Gomes and Silva, 2008) despite the fact that it was tested with noise-free data and two distinct damage detection methodologies were applied. In the paper by Gomes and Silva (2008), the location for single damage scenarios was successfully found for both approaches using only the first five natural frequencies. However, some spurious damaged elements also appeared in the procedure. In addition, it was not possible to differentiate the damage in the symmetric sites, as the authors used an objective function based on changes in natural frequencies. Finally, the procedures failed to locate multiple damage scenarios and quantification for single and multiple damage cases.

Thus, this example attempts to reproduce two damage scenarios in order to assess the accuracy of the P-NMA algorithm: (1) the static moment of inertia about the z-axis of element 20 is reduced by 10% and, (2) the static moments of inertia about the z-axis of elements 10, 28 and 52 are reduced by 10%. The objective function employed in this example is described by Eq. (7). The eigenproblem is solved by an in-house finite element code to obtain the frequencies and mode shapes that are necessary to employ Eq. (7). To represent the truncated mode shapes at the sensor locations, which are close to a real condition, only the first 5 natural frequencies and 17 nodal displacements were adopted in the mode shapes, which follow: node 6, 10, 13, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 45, 48 and 52.

Thus, there are 56 updating parameters (*i.e.*, N = 56), which represent each element of the portal frame. As seen in Figs. 2 and 3, the damage locations may be accurately identified for both scenarios. The results of the proposed optimization scheme are compared to the results presented by Gomes and Silva (2008) in Figs. 2 and 3.

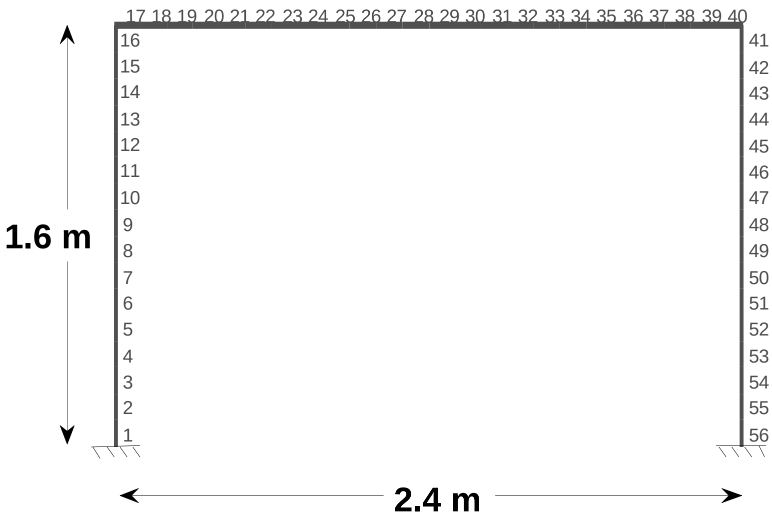


Figure 1. Portal plane frame modeled with 56 finite elements

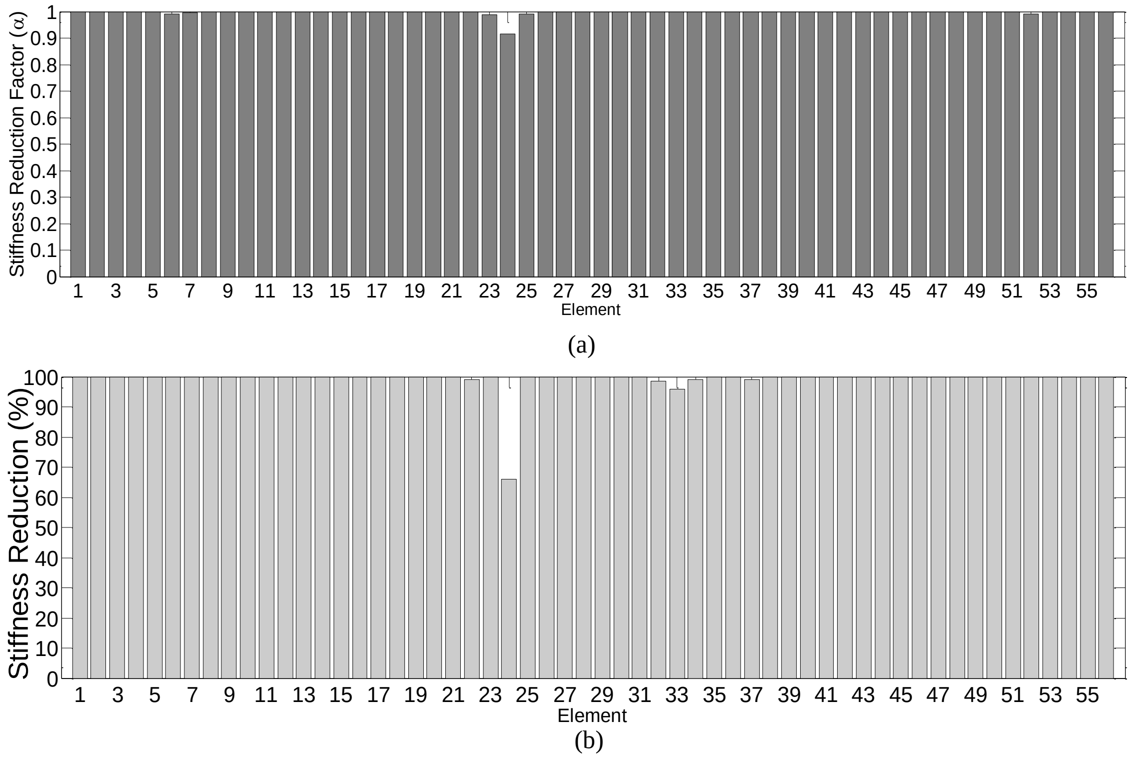


Figure 2. Damage Scenario 1 - 10% in element 24: (a) P-NMA, (b) Gomes and Silva (2008)

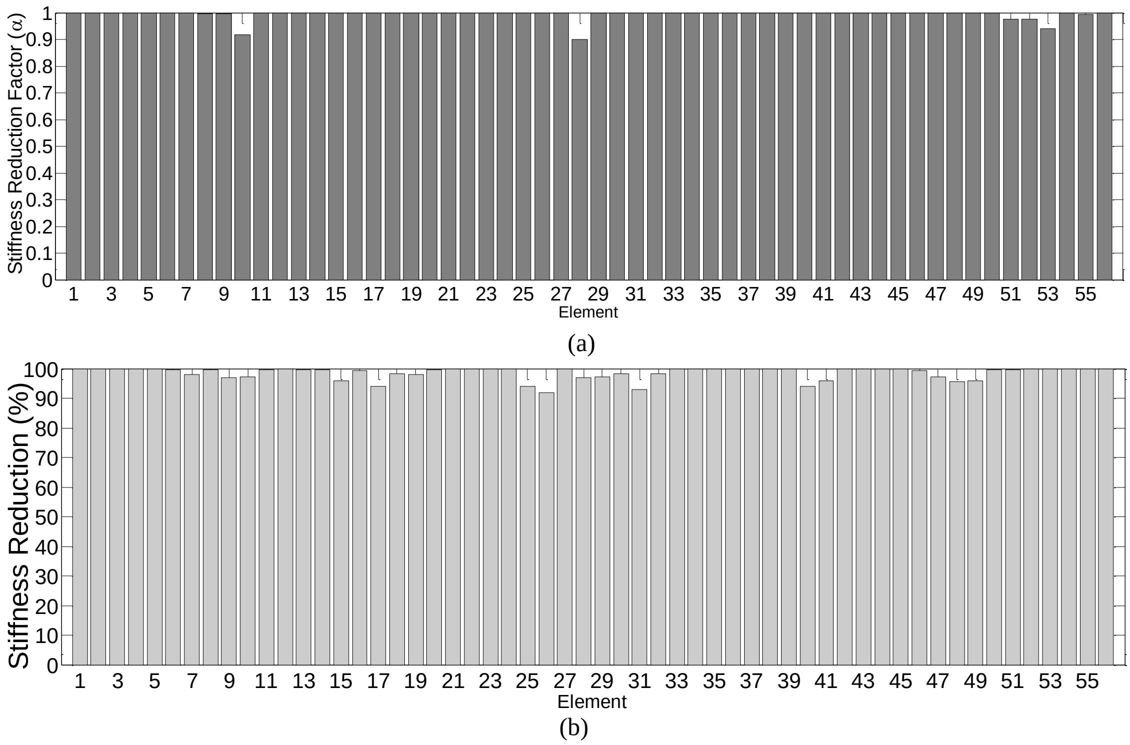


Figure 3. Damage Scenario 2 - 10% in element 10, 28 and 52: (a) P-NMA, (b) Gomes and Silva (2008)

For the single damage scenario (Fig. 2), the proposed P-NMA optimization method accurately identified both the damaged element and the extent of the damage. On the other hand, the GA presented by Gomes and Silva (2008) was not able to identify the damage extent and presented some spurious values around element 33. Using the proposed optimization scheme in the multiple damage case, some negligible values of damage appeared around member 52, which did not disturb the diagnosis as they are in the neighbourhood of the true damage element. However, the GA (Gomes and Silva, 2008) could not even identify the damaged element. Hence, the P-NMA algorithm was more accurate than the GA as proposed by Gomes and Silva (2008).

It is also important to note the computational cost of each approach: Gomes and Silva (2008) carried out a total of 1,000,000 objective function evaluations (OFE) to obtain each of their results, but in the present study, 5000 OFE were employed to approximate numerically the representation formula (Eq. ()) and 11,200 OFE to run the NMA, totalling 16,200 OFE for each final damage scenario.

In this example, it was shown that the optimization scheme employed in this paper presents an outstanding performance, achieving more accurate damage scenarios with a much lower computational cost than the recent literature, although in such literature only natural frequencies were used in the objective function, which in part may diminish a little the value of P-NMA algorithm in this specific context.

3.2 Cantilever beam

To assess the influence of noise, a numerically simulated cantilever beam with several assumed damage elements is considered. The structure adopted here is 750 mm long, and it has a square box cross section with external dimensions of 25.4 mm and a wall thickness of 1 mm. It was modelled with 25 Timoshenko beam elements, as shown in Fig. 4. The specific weight, elastic modulus of the material, Poisson's coefficient and Timoshenko's shear factor of the beam are 28 kN/m^3, 68.6 GPa, 0.3 and 0.5, respectively. Also, a concentrated mass of 18.2 g is also included in all DOFs of the numerical model in order to correctly represent the presence of the accelerometers.

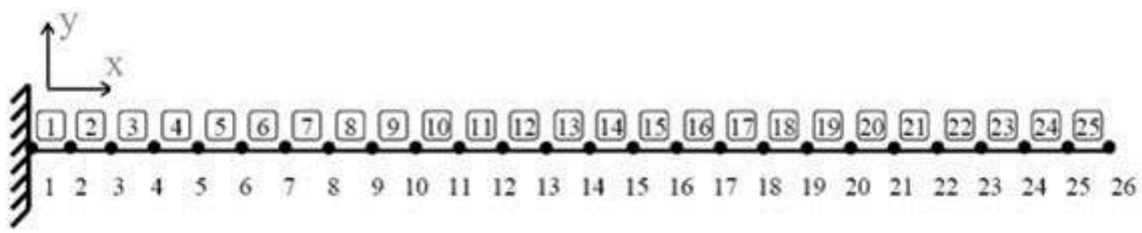


Figure 4. Beam modeled with 25 finite elements

To test the sensitivity to uncertainties in the measurements, numerically simulated dynamic tests in the presence of noise were performed by considering impulsive and ambient excitations. The latter was modeled by 75 uncorrelated Gaussian white noise signals, with zero mean and standard deviation equal to one, applied at all the generalized coordinates of the structure. The impulsive loading is represented by the application of an impact at node 8 in the vertical direction.

Three different damage scenarios are considered for both excitations to assess the ability to locate and quantify damage of the proposed SHM approach. Scenario 1 was represented by reducing the stiffness of element 20 to 80% of the initial value. Scenario 2 also used member 20 but reduced its stiffness to 30% of the initial value. Finally, Scenario 3 was represented by reducing the original stiffness of element 8 to 70% of the initial value.

The dynamic problem is solved by numerical integration of the equations of motion using the Newmark method, (in-house finite element code) with an integration time step equal to 10^{-5} s for the free vibration case and 10^{-6} s for ambient vibration. Rayleigh damping is assumed with proportionality constants determined for yield damping ratios in the 1st mode equal to ξ = 1%. The output data are filtered with an eight-order Chebyshev type I lowpass filter with a cutoff frequency of 1600Hz and the data is re-sampled at a rate of 4000Hz, in order to reduce the number of data points and to make the procedure more accurate in the range of frequency of interest.

In order to determine the variation of structural modal parameters due to noise effects and to evaluate the robustness of the damage detection procedure in situations closer to field conditions, noise levels were simulated through the addition of white noise signals with RMS amplitude of 3% and 5% of the mean measured response for free vibration and 3% for ambient vibration. In real dynamic testing, this is consistent with the assumption of uncorrelation between the primarily electronic noises with the actual measurement signal. The damage Scenario 3 is not considered for 5% noise level.

After getting the output for each excitation, damage scenario and noise level, the output-only system identification is carried out through the SSI method. It is assumed that only the first 5 modes are available and that measurements are only obtained from the translational DOFs of the model. The SSI algorithm presents the main advantage of avoiding any preprocessing to obtain spectra or covariances, identifying models directly from time signals. As earlier described by Fadel Miguel *et al*. (2007), the performance of the two different algorithms SSI-DATA (DATA-driven Stochastic Subspace Identification) and SSI-COV (COVariance-driven Stochastic Subspace Identification) and three different variants CVA (Canonical Variate Analysis), PC (Principal Component) and UPC (Unweighted Principal Component Algorithm) is quite similar, thus a combination of the second algorithm (SSI-COV) together with the variant PC was chosen to carry out the identification approach for the two damage scenarios. The identified frequencies for both damage scenarios, both excitations and two noise levels are shown in Tabs. 1 and 2.

Table 1: Identified frequencies for free vibration

Modes	Noise 3% (Hz)				Noise 5% (Hz)		
	Healthy	Scenario 1	Scenario 2	Scenario 3	Healthy	Scenario 1	Scenario 2
1st	26.563	26.552	26.521	26.250	26.552	26.542	26.518
2nd	164.458	164.007	160.285	163.892	164.459	164.007	160.286
3rd	451.723	447.801	418.614	445.937	451.723	447.800	418.611
4th	861.458	853.360	803.738	856.940	861.453	853.362	803.733
5th	1376.73	1370.11	1330.49	1370.87	1376.71	1370.11	1330.492

Table 2: Identified frequencies for ambient vibration

Modes	Noise 3% (Hz)			
	Healthy	Scenario 1	Scenario 2	Scenario 3
1st	26.563	26.558	26.527	26.250
2nd	164.505	164.050	160.303	163.920
3rd	451.969	447.976	418.836	445.999
4th	862.400	853.593	803.749	857.463
5th	1378.17	1369.87	1332.30	1371.33

The objective function employed in this example is described by Eq. (4). The identified damage locations after updating for both excitations and noise levels are shown in Figs. 5, 6, 7 and 8. To compare the accuracy of the P-NMA algorithm, the results for free vibration and 3% noise, obtained through the HS algorithm by Fadel Miguel *et al*. (2012), are shown in Figs. 5c and 5d.

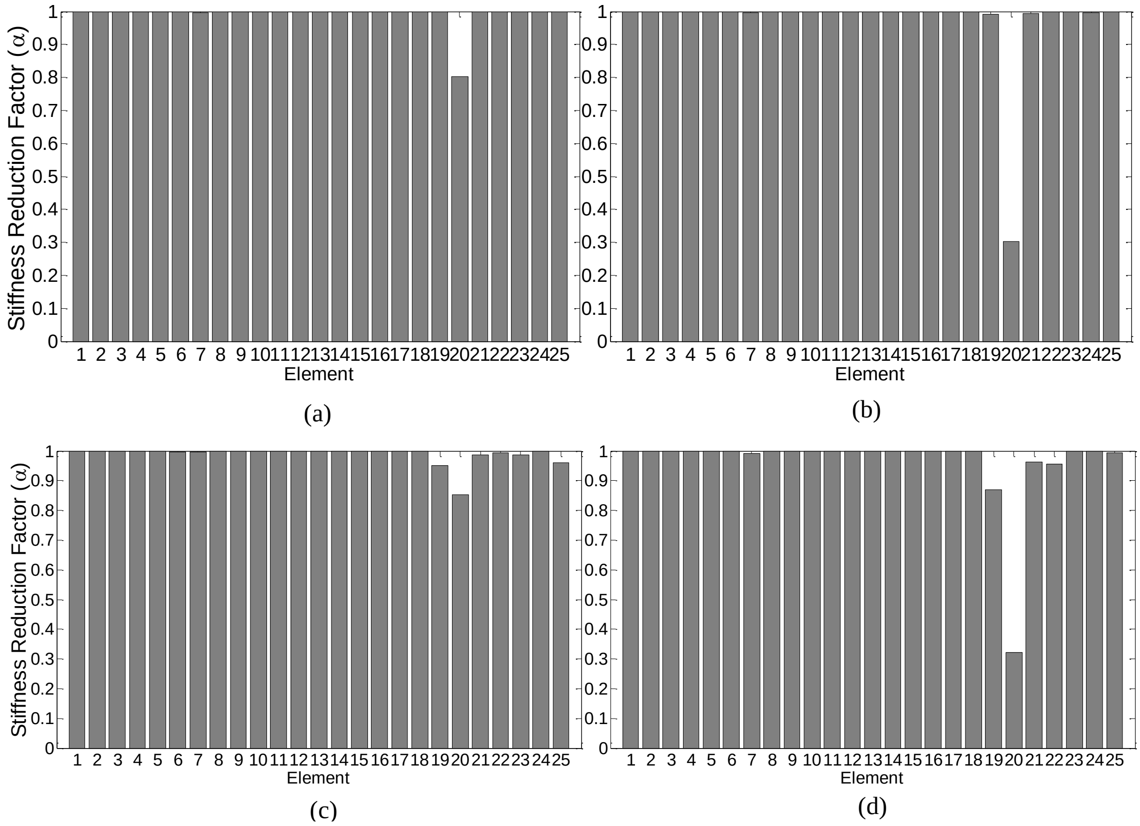
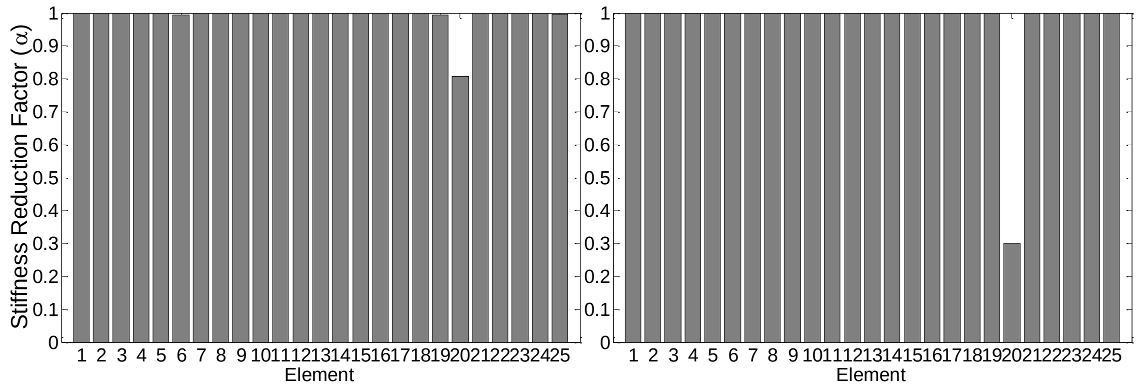
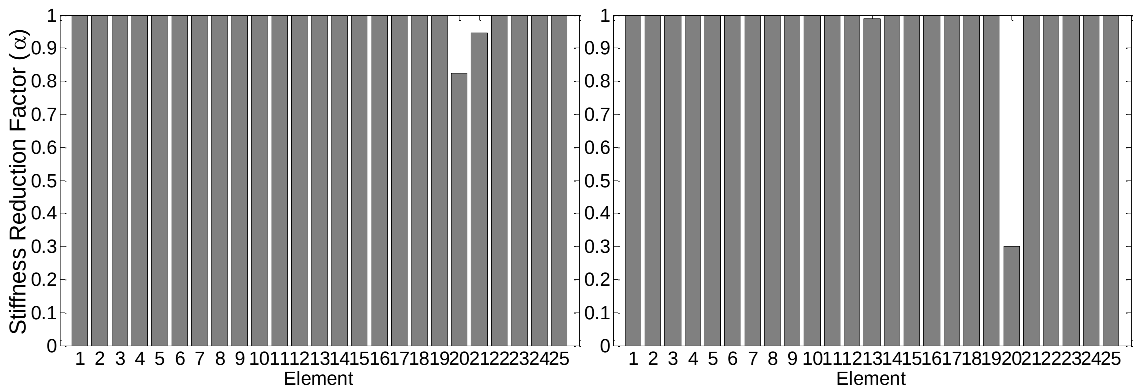


Figure 5. Free vibration and 3% noise: (a) Damage Scenario 1 through the P-NMA, (b) Damage Scenario 2 through the P-NMA, (c) Damage Scenario 1 with HS (Fadel Miguel et al., 2012), (d) Damage Scenario 2 with HS (Fadel Miguel et al., 2012)



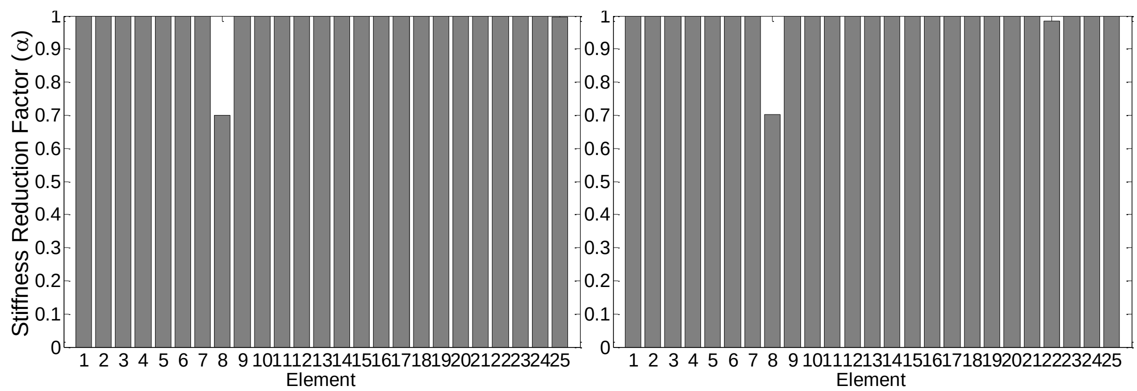
(a) (b)

Figure 6: Free vibration and 5% noise through the P-NMA: (a) Damage Scenario 1, (b) Damage Scenario 2.



(a) (b)

Figure 7: Ambient vibration and 3% noise through the P-NMA: (a) Damage Scenario 1, (b) Damage Scenario 2.



(a) (b)

Figure 8: Damage Scenario 3 and 3% noise through the P-NMA: (a) Free vibration, (b) Ambient vibration.

Note that the location and extent of damage may be accurately identified for both excitations and noise levels regardless of the position of damage in the beam. It is important to notice that the optimization matches the real damage condition almost exactly without any spurious values in other elements for both excitations and noise levels, slightly improving the diagnosis achieved in Fadel Miguel *et al*. (2012). The results of the damage detection for the cantilever beam are summarised in Tab. 3.

It is important to note that while Fadel Miguel *et al*. (2012) carried out a total of 200,000 OFE to achieve each damage scenario, in the present study only 1000 OFE were performed to numerically approximate the representation formula given by Eq. (9) and 5000 OFE by the NMA, resulting in only 6000 OFE to obtain each damage scenario.

In this example, it was shown that the proposed optimization scheme is more accurate and requires a much lower computational cost compared to the HS algorithm (Fadel Miguel *et al*., 2012).

Table 3: Summary of damage detection results for the numerical cantilever beam analysis.

Damage case	Exact damage		Predicted damage		Error	
	Element number	Stiffness reduction (%)	Element number	Stiffness reduction (%)	Localizati on (%)	Extent (%)
Free vibration (Noise 3%)	20	20	20	19.66	0.0	-1.70
	20	70	20	69.81	0.0	-0.27
	8	30	8	29.93	0.0	-0.23
Free vibration (Noise 5%)	20	20	20	19.21	0.0	-3.95
	20	70	20	69.99	0.0	-0.01
Ambient vibration (Noise 3%)	20	20	20	17.48	0.0	-12.60
	20	70	20	70.00	0.0	0.00
	8	30	8	29.90	0.0	-0.33

4 CONCLUDING REMARKS

In this paper, a hybrid stochastic/deterministic optimization algorithm (P-NMA) for vibration-based damage detection was proposed. The P-NMA approach was verified with a series of numerical examples to compare its performance to algorithms previously reported in the literature.

The comparisons started with a noise-free portal plane frame previously solved by Gomes and Silva (2008) through the GA. The P-NMA procedure showed a better performance than Gomes and Silva (2008), being able to correctly identify the damage location and its extent for both single and multiple damage scenarios with a considerably lower number of OFE. In

Gomes and Silva (2008) the extent of the damage could not be determined for the single damage scenario, and even the damage locations were not correctly determined for the multiple damage scenario. Next, a typical cantilever beam was used to assess the influence of measurement noise when considering different damage scenarios and noise levels under impact and ambient vibrations. Once again, the P-NMA results were more accurate than those reported by Fadel Miguel et al. (2012) and demanded a considerably lower computational cost. Two different objective functions were employed in this paper: for the first numerical example, because the structure is symmetric the objective function based on flexibility matrix was chosen and for the second numerical example, where the structure is unsymmetrical the objective function based on natural frequency was adopted. The achieved results demonstrated better efficiency and accuracy of the proposed P-NMA optimization scheme compared to the heuristic or the hybrid algorithms. It is important to note, however, that the stochastic part of P-NMA algorithm (Eq. (8)) is influenced by the magnitude of parameter λ and the sample $\mathbf{A}$, and so, it is objective of future work to simulate several times the proposed algoritm as well as the use of real data for the experiments.

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