EXPERIMENTAL INVESTIGATION ON THE VARIABILITY OF THE DYNAMIC RESPONSE OF ASSEMBLIES OF NOMINALLY IDENTICAL COMPONENTS

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Abstract. *Usually, structures contain inherent variability in geometric and material properties due to the complexity of manufacturing process. This variability, combined with components and subcomponents assemble uncertainties, provide relevant changes in the structure dynamic behavior with respect to the nominal design. Therefore, including those uncertainties in the dynamic analysis provides a wider range of response predictions improving structure reliability and hence reducing costs of design. A stochastic modelling is required to add these variabilities on the solution and probabilistic approaches are commonly used with Finite Element Analysis (FEA) to represent those uncertainties in dynamics analysis, named Stochastic FEA (SFEA). In this work, nominally identical structural components, built-up from beams and plates, are characterized from frequency response function measurements and natural frequencies. Some of their statistics, like histograms and percentiles, are then calculated. Then, some of these nominally identical structures are assembled and the variability of the dynamic response is investigated under the different possible permutations. Results are compared towards the existence of possible permutations leading to decreased or increased variability on the response of the assembly.*

Keywords: *Ensemble variability, Parameters uncertainties, Stochastic analysis, Structural dynamics.*

1 INTRODUCTION

The use of structural elements such as trusses or frames is very common in the engineering practice. Their mechanical behavior depends on material properties, types of joints and loads, geometry, etc. Their manufacturing cost increases for increasing accuracy of dimensions and material quality. Therefore, an analysis of parameters that influence the dynamic behavior of such structures becomes relevant. The geometry and material properties variability of mass-produced components result in rising safety factor (William D. Callister, Jr. 2006). The computational prediction for better safety factors has been the main goal of all structure developers because it affects directly the cost, then it should not be ignored.

The inherent variability of manufacturing process cannot be avoided, so it must be treated by theories that feature this random behavior, such as the probability theory (Athanasios Papoulis and S. Unnikrishna Pillai 2002). By including this randomness on dynamic analysis, most times using Finite Element Analysis (FEA), other dimension arises to the problem (Stefanou, 2009, Sudret, and Der Kiuereghian, 2000), as a probability set, called Stochastic Finite Element Analysis (Ghanem, and Spanos, 2012, Matthies, et al., 1997, Ostoja-Starzewski, 2007, Shang, and Yun, 2013). Many studies related with those uncertainties theories (Babuška, and Motamed, 2016, Moens, and Vandepitte, 2005, Möller, et al., 2000, Muscolino, et al., 2016, Sniady, et al., 2013, Wang, et al., 2014) combined with FEA have been done on several areas like, composites (Babuška, and Motamed, 2016, Murray, et al., 2015), kinematics of multi-body systems (Wasfy, and Noor, 1998), dynamic analysis with geometry and material variability (Chang, 2014, Murray, et al., 2015, Noh, 2005, Shang, and Yun, 2013, Stefanou, and Papadrakakis, 2004). During the last decades, this randomness has been considered by the researchers and the industry sectors. However, there are only a few experimental studies concerning variabilities and coupling combinations of structures, for instance (Murray, et al., 2015, Wang, et al., 2014). The addition of those non-deterministic properties on the design can improve the estimate of reliability.

Most engineering structures are composed of assemblies of components, generally joined by welds or screws. Usually, the effect of the uncertainties from each component on the dynamic behavior can be insignificant when compared with the whole assembly (W D’Ambrogio and A Fregolent 2009). This effect of the assembly uncertainty becomes even greater when considering joints variability (Hinke, et al., 2009, Octavio de Alba Alvarez, 2012). The assembly behavior prediction can be more computationally expensive when dimensional variabilities are included on finite element model, due to the remeshing necessary for every sample on a Monte Carlo sampling scheme (Reuven Y. Rubinstein and Dirk P. Kroese 2007).

Frame structures are build-up from beams, plates connected by joints. The beams are responsible for the flexure and torsional modes as well as the plates for the plate modes. The joint is responsible for the stiffness of the coupling between the plates and beams and are often different from each other. This difference can have a great influence on the dynamics of the structure. The dimension of each component of the frame also have an inherent variability due to manufacturing process that can only be reduced. In addition, every component created has different mechanical properties, including stiffness and density. The welded joints properties are also very difficult to be controlled, because the fusion/solidification process never create the same crystalline set, influencing on the stiffness of each joint (Robert W. Messler, Jr. 1999). These variabilities, when combined, changes dynamic behavior of each frame, increasing the resonances range and affecting safety factors for engineering applications.

Usually, the uncertainties on experimental analysis are composed by a random and a systematic part. This last one is associated to the part that remains constant on a sample of repeated measurements. It can be obtained by past experience, means, design specifications, or other information. The random part is associated to the part that cannot be predicted on repeated experiments obtained by statistical analysis of a sample of measurements. Possible sources of error that influences on FRF behavior are experimental setup errors, for example, different excitation points, clamping mechanisms and joints not providing the constraint condition, etc.

In this paper, an experimental investigation on the variability of the dynamic response of nominally identical frames and assemblies of such frames joined by screws is presented. The investigation aims to compare the statistics of the ensemble of components with the ensemble of assemblies towards the existence of possible permutations leading to decreased or increased variability on the response of the assembly. Frequency response measurements are obtained from a modal hammer and accelerometer. Section 2 presents the experimental procedure. Section 3 shows some preliminary experimental results obtained and a discussion. In Section 4, some concluding remarks are drawn as well as the further steps of the work.

2 EXPERIMENTAL PROCEDURE

In this section, the experimental setup and procedure are presented. Ten nominally identical frames, i.e., manufactured under the same specification, are analyzed. Each individual structure is built-up from 4 beams and 2 plates spot welded and with 4 holes on each plate for further assembly. The beams and plates are made of steel, except the welded spots. The assembly is done by steel screws, using a torquemeter to normalize the grip level and guarantee that the joints are clamped, necessary condition to transfer the injected energy through the joint. Each frames is numbered from 1 to 10, with 5 measuring and 1 excitation positions on the top plate of each frame, as illustrate at Fig. 1.

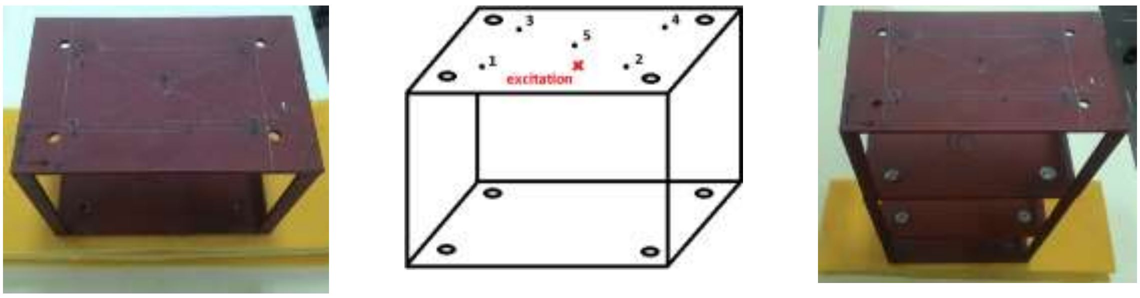

Figure 1. Picture of single frame test setup (left), schematic frame draft (middle) and picture of one assembly (right).

The test procedure was divided in two main steps. The first one aims the characterization of dynamic behavior variability of the ensemble of the nominally identical structures. Then, each one of the 10 single frames is measured via impact hammer/accelerometer in the 5 positions and FRF were calculated using a H_2 estimator and the Welch's method (Periodogram) (Kihong Shin and Joseph K. Hammond 2008) approach with 10 samples. The measurements are presented as $H_i^{(j)}$, where i is the measured position and j is the component number. In addition, the natural frequencies are given by $\omega_k^{(j)}$, for the k^{th} frequency from the j^{th} component. The component number 10 was arbitrarily chosen for comparison.

The second step is the characterization of the variabilities in the dynamic behavior of the assemblies. For that, 3 frames were chosen and assembled under the 6 possible combinations, in order to analyze the variability imposed by the assembly position. The measurements were made on the top plate of the top frame of the assembly, using the same positions as shown in Fig 1. The combinations were assembled using the components 7, 9 and 10, as shown in Table 1. They were chosen because they were the more physically similar each other.

Table 1. Assembly combinations using the numbered frames 7, 9 and 10.

Cj	Order
C_1	7-9-10
C_2	7-10-9
C_3	9-7-10
C_4	9-10-7
C_5	10-7-9
C_6	10-9-7

The measurements were performed through an acquisition data board (NI cDAQ-9174, frame 4 slots USB, 15 W, 9 – 30 V, 5 – 500 Hz), transduced by a quartz accelerometer (PCB 353B03, 10 mV/g, 1 – 7000 Hz) and impact hammer (PCB 086C03), acquired and processed on LABView.

Two kinds of support were tested to simulate the free boundary condition. Supporting the structures on sheets of foam material and hanging up the structure on thin strings. Figure 2 shows a typical FRF using both supports. It can be seen that they present a very good agreement between 80 Hz to 500 Hz. The hammer spectrum remains flat up to 500 Hz, where after the results diverge with low coherence. For convenience, due to the ease of handling, the foam support was selected to carry out all of the experiments.

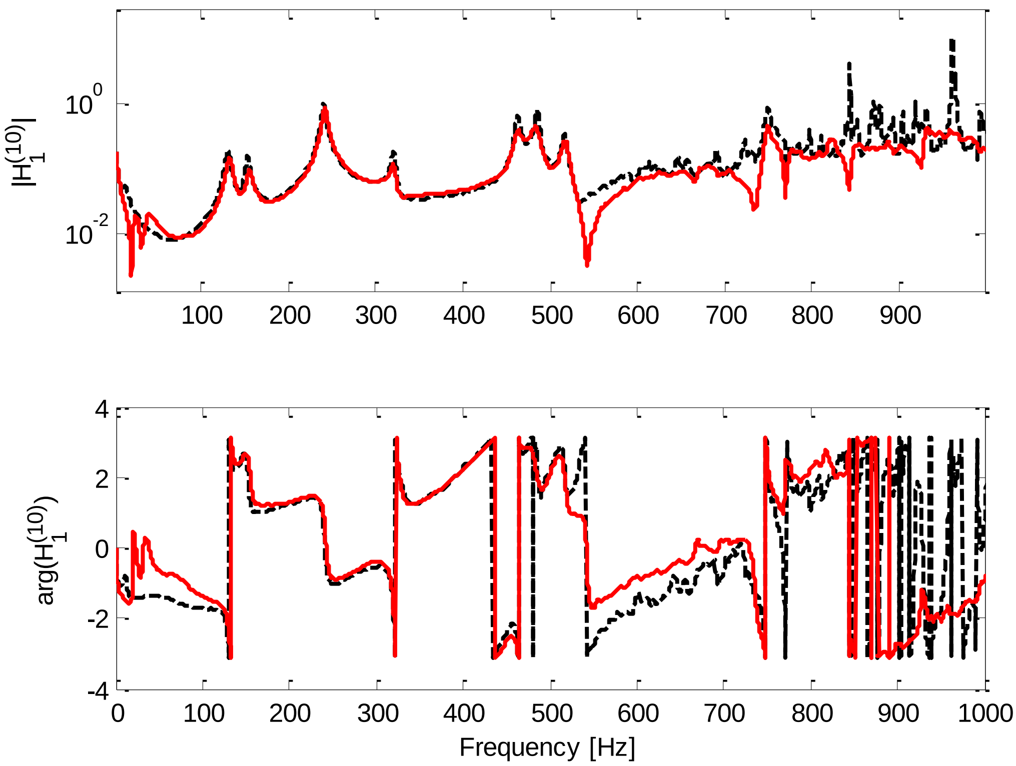

Figure 2. Typical amplitude and phase of the FRFs from a frame tested on foam (red) and hang by strings (black).

3 RESULTS AND DISCUSSION

In this section, measures and results obtained by statistical treatment are presented. All of the components are made of steel, small damping factor. It is assumped that randomness in the shape, geometry and material properties are responsible for changes on the mass and stiffness parameters, affecting the ensemble FRFs. Material e geometrical properties are assumed to homogeneous over the plate and beams. The uncertainty due to the assembly using screws is assumed to be negligible due to the normalization of the grip level by the torquimeter. The frame welded joints are assumed to be the similar. Ten samples were made of each test to ignore the systematic error. Component 10 was arbitrary chosen for represent the ensemble behavior. Table 2 shows the mean, standard deviation and coefficient of variation, defined by $\text{COV} = (\text{standard deviation})/(\text{mean value})$, of the geometrical parameters from measuring all of the 10 frames.

Table 2. Mean and deviation of dimensional parameters

	Parameter	Mean [mm]	Standard Deviation [mm]	COV
Plate	**Thickness**	6.56	0.21	0.034
	Length	301.28	4.73	0.016
	Width	200.69	3.94	0.020
Beam	**Thickness**	6.49	0.17	0.026
	Length	199.21	4.15	0.021
	Width	20.14	0.76	0.038

Figure 3 shows modulus, phase and coherence of $H_1^{(10)}$. At the very low frequencies (0 – 50 Hz), an oscillation behavior can be seen on Fig. 2 due to foam material support. The first three resonance peaks are seen on 100 – 250 Hz and are presented on Fig. 4. The frequency band selected for analysis was between 100 – 250 Hz where 3 natural frequencies because they have the best coherences. The natural frequencies from each of the 10 components, estimated assuming light damping and a peak picking approach, are shown on Table 3.

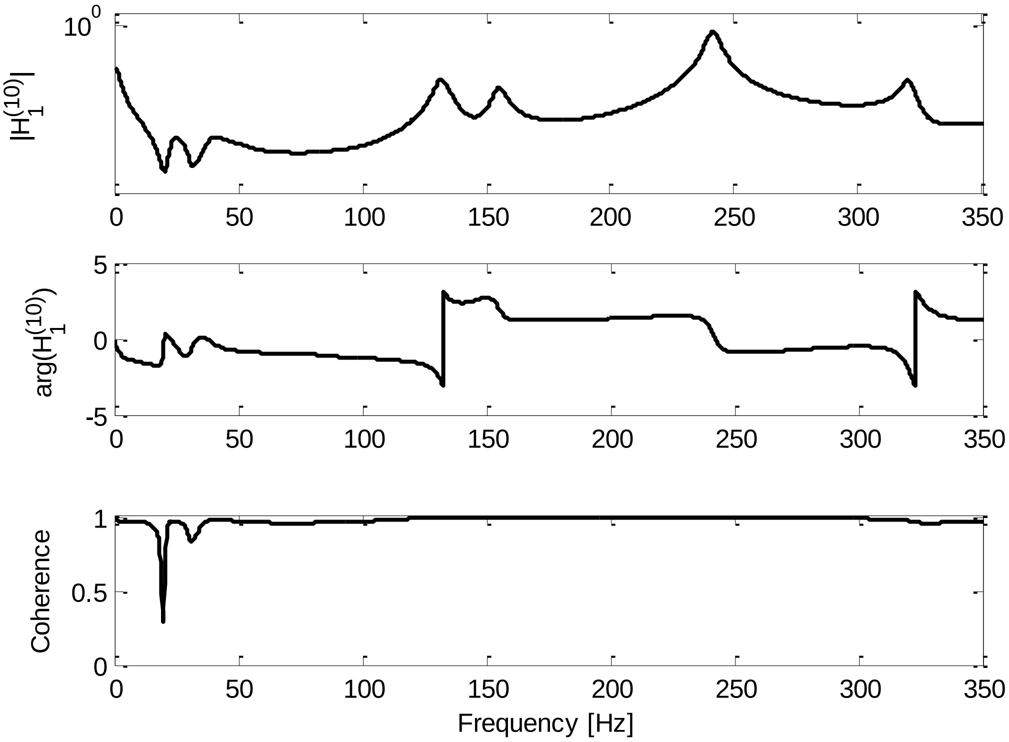

Figure 3. FRF modulus, phase and coherence of component #10 measured at point 1

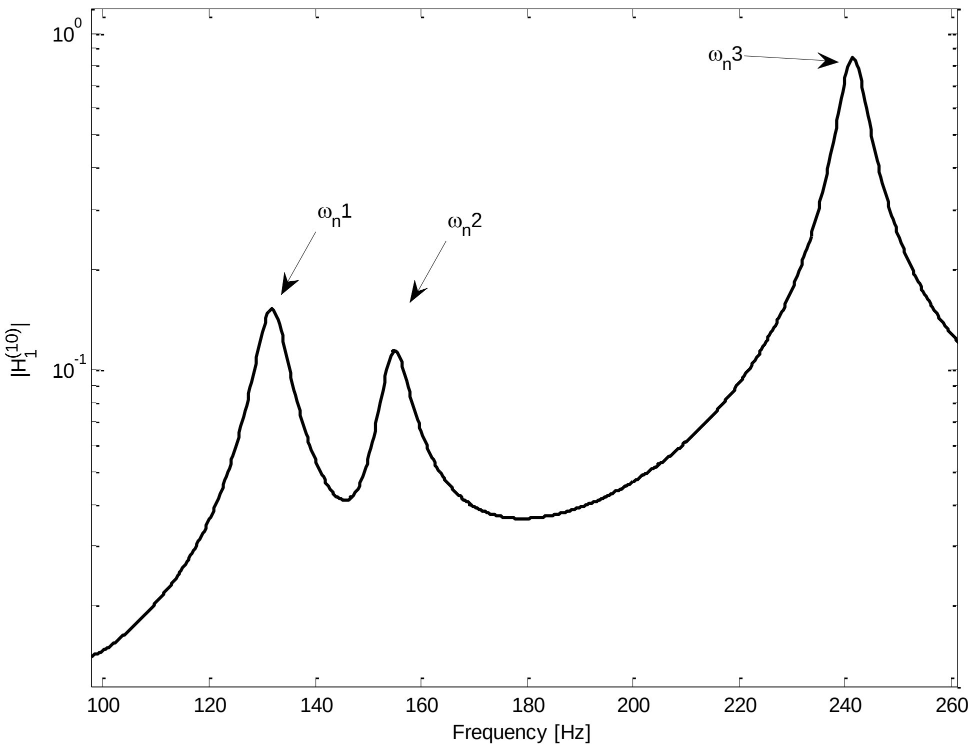

Figure 4. FRF of component #10 with analyzed frequencies pointed

Table 3. Natural frequencies estimated from the *j-th* frame within the frequency band under analysis.

j	$\omega_1^{(j)}$	$\omega_2^{(j)}$	$\omega_3^{(j)}$
1	118.76	149.76	236.02
2	130.01	154.01	240.27
3	127.01	152.01	237.02
4	123.76	149.75	234.77
5	114.76	149.76	234.77
6	127.51	150.51	235.77
7	134.26	158.26	244.52
8	127.01	151.26	237.52
9	134.76	153.51	239.27
10	131.76	155.01	241.52

Figure 5 shows the measured FRF for each component and the mean value. It can be noticed that the randomness arising from differences on the dimensional/material properties significantly affect frequency response in the range in frequency of the third resonance frequency. This typical behavior is seen throughout the frequency band under analysis.

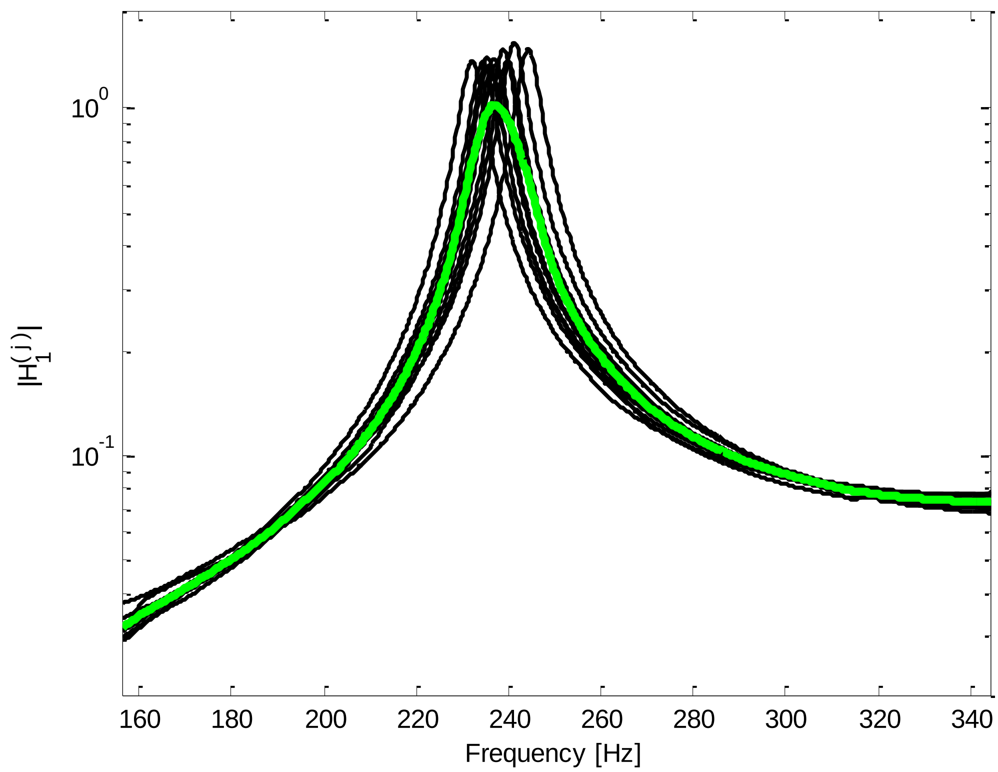

Figure 5. FRF of component *j* (black) and mean FRF (green) pointing the third natural frequency

Table 4 shows the mean value, standard deviation and COV calculated for each natural frequency within the frequency band under analysis.

Table 4. Mean, standard deviation and Coefficient of Variation (COV) of the natural frequencies estimated from the component ensemble.

Frequency	Mean value ± standard deviation[Hz]	COV
$\omega_n 1$	126.91 ± 6.46	0.0509
$\omega_n 2$	152.33 ± 2.68	0.0175
$\omega_n 3$	238.08 ± 3.18	0.0134

Figure 6 shows the 95% percentile of FRFs determined on the 5 positions, for the ensemble. All positions mean FRFs and components FRFs are in this range and it can be seen on Fig. 7 which shows the histogram of natural frequency estimated from of all of the components.

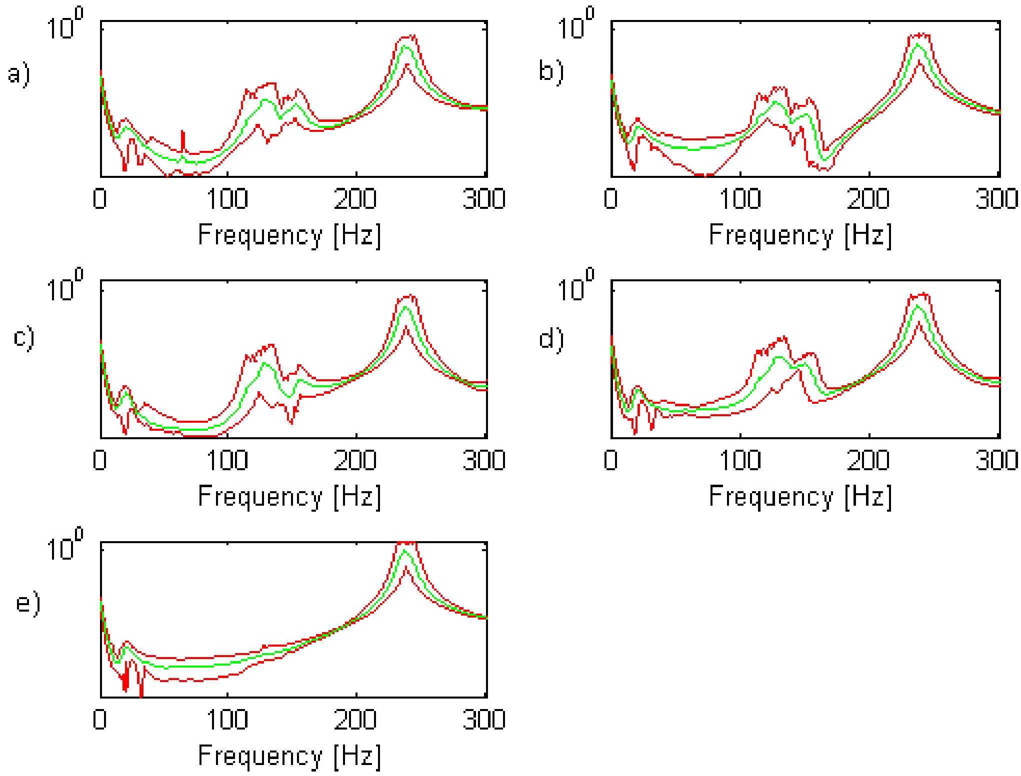

Figure 6. Mean value and upper and lower 95% percentile from the measured FRFs on positions (a) 1, (b) 2, (c) 3, (d) 4 and (e) 5.

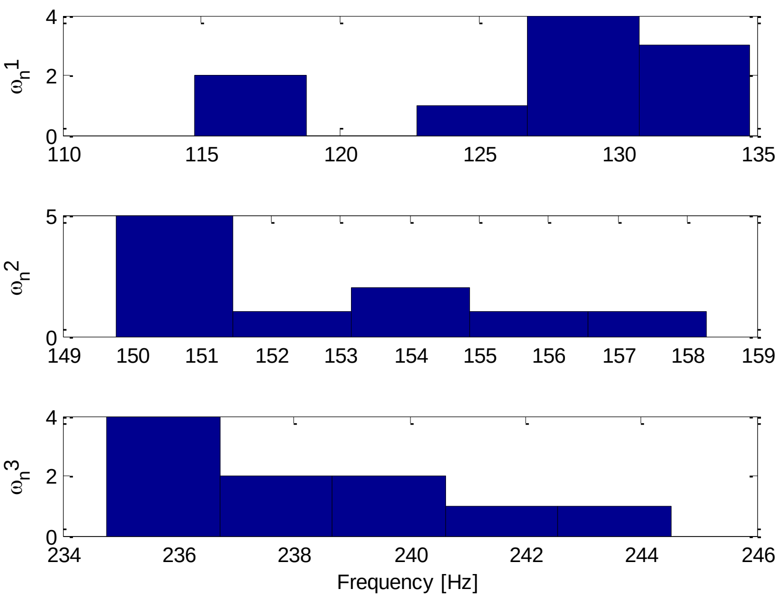

Figure 7. Experimentally obtained histogram of the natural frequencies from the ensemble of components.

Table 5 shows the estimated natural frequencies from the six possible combination of assemblies, Tab. 1, and Tab. 6 presents the mean value, standard deviation and COV for three natural frequencies estimated within the frequency band under analysis. Figure 8 shows histogram of measures for each natural frequency of the predetermined assemblies with a percentile analysis with 95% range of confidence.

Table 5. Measurements of resonance frequencies of j-th ensemble and these respective coherences

C_j	$\omega_1^{(j)}$	$\omega_2^{(j)}$	$\omega_3^{(j)}$
C_1	123.26	148.26	260.52
C_2	123.76	146.01	258.52
C_3	121.51	150.76	235.27
C_4	121.01	137.51	224.76
C_5	121.76	145.51	240.02
C_6	121.26	139.51	252.77

Table 6. Mean, standard deviation estimation and covariance factor of ensembles resonance frequencies

	Frequency ± Standard Deviation [Hz]	COV
$\omega_n 1$	122.09 ± 1.14	0.0093
$\omega_n 2$	144.59 ± 5.11	0.0353
$\omega_n 3$	245.31 ± 14.23	0.0580

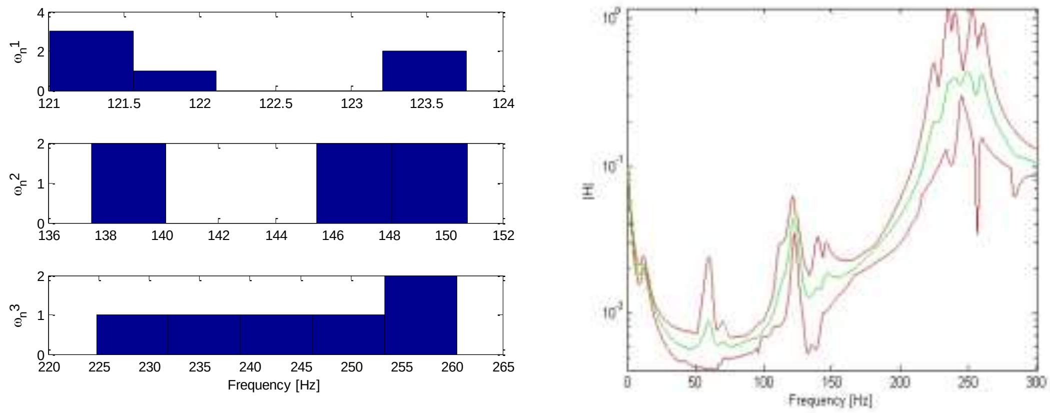

Figure 8. Experimentally obtained histogram from the natural frequencies (left) and 95% upper and lower percentile of the frequency response (right) from the assemblies.

Figure 9 presents the upper and lower 95% percentile of the FRF obtained at position 1 from the ensemble of components and from the ensemble of assemblies. Overall, it can be noticed that most of the dynamical behavior observed in the components ensemble has changed when compared to the assemblies, as expected, due to the structural changes. However, it can also be noticed, by analyzing only the percentiles, i.e. only the variability, that the assembly variability is comparable to the component ensemble variability, even though the same three components have been used in the assemblies, only by changing their positions. This result suggests that there might be best choices to reduce the variability in the frequency response when selecting components build the given assembly. Moreover, the assembly variability affects differently some the frequency bands. This is likely due to the vibration modes sensitivity with respect to the kind of assembly, in this by connecting the upper and lower plate by screws. Therefore, it can be inferred that the modes sensitivity are differently influenced by the structure changes. The same kind of variability change can be noticed when comparing the natural frequency statistics from Tabs. 4 and 6.

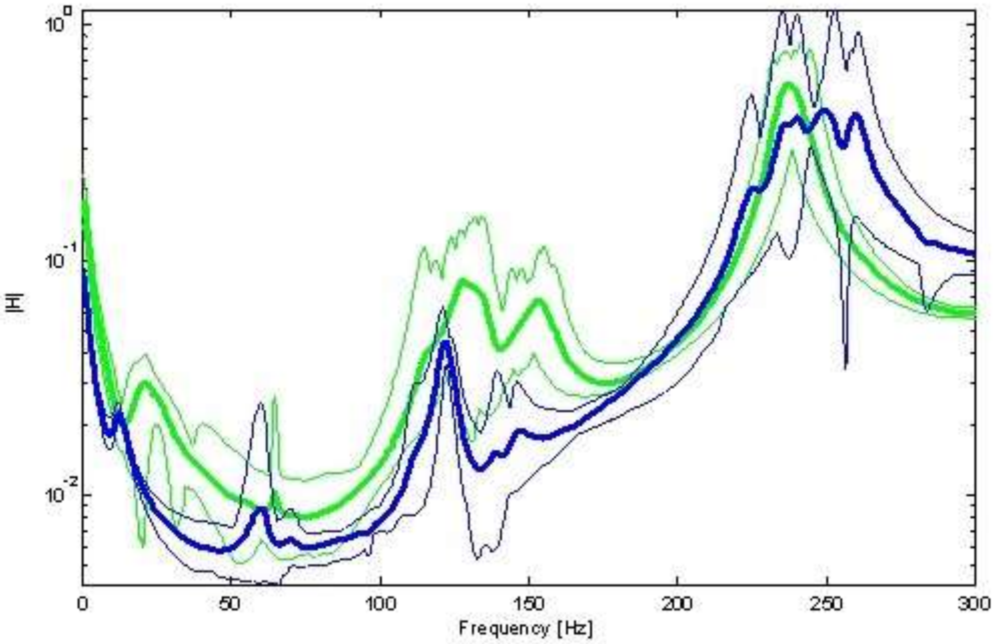

Figure 9. Mean FRFs (thick) and respective percentiles (thin) of component (green) and ensemble (blue)

4 CONCLUSIONS

An experimental investigation on the variability of the dynamic response of nominally identical frames and assemblies of such frames joined by screws was presented. The characterization of dynamic behavior variability of the ensemble of components and ensemble of assemblies was done and compared with statistical treatment. Over this analysis, it can be concluded that the assembling of components with uncertainties parameters included, need to be considered. The variabilities arises from parameters uncertainties and also from different combinations of assemblies even considering the same components on different positions. Furthermore, the assembly variabilities of a chosen set of components can be much bigger than ensemble variabilities considering all components.

Variabilities can be powered depending on the others parameters influence and band analyzed, leading to different sensitivities on different modes. On frames assembled case, the structural changes affected the sensitivity on low frequency band decreasing the variabilities on natural frequencies. The structural changes influenced directly on changing the dynamic behavior almost on all bands. The sensitivity caused by assembling components on different positions using the same components should be considered. On low working frequencies of the assembled structure this variabilities can be reasonable instead high ones. Previous sensitivity analysis of uncertain parameters and assembly variabilities can reduce costs of production aiming the tolerance allocation on parts that are more influent on dynamic behavior.

For further steps, a stochastic finite element analysis will be done to compare results from the assembly and ensemble variabilities, in order to fit a model that consider the joint stiffness and material properties with measurements, towards some criteria for selective assembly.

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