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NUMERICAL EVALUATION OF THE EFFECT OF UNCERTAINTIES IN ROTATING MACHINERY USING REDUCED MODEL

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Abstract. *Rotating machines are extensively used in industrial applications considering the flexibility of the equipment, capable of being operated at extreme speeds, the study of uncertainties are required, due the influence in the dynamic behavior. The use of stochastic techniques have played an important function in engineering problems, the Monte Carlo method (MC) and your variant called Latin Hypercube (LHS) are widely used to model uncertain parameters. In this context, the present work is devoted to analysis of the uncertainties in the parameters of a flexible rotor discretized by finite element (FE). Aiming a considerable saving of time, models reduced by Iterative Improved Reduction System (IIRS) method were used in the numerical analysis process. The analysis procedure is limited to the frequency domain. Solution envelopes are obtained by LHS method that allow us to describe the system behavior considering some parameters as random.*

Keywords: *Rotating machines, model reduced, Latin Hypercube method, uncertainty analysis.*

1 INTRODUCTION

Rotating machines require attention, because the efficiency of this equipment is related to its operating speed, fact that enables the use of increasingly flexible rotors (Koroishi, 2013). Problems coming from the vibration phenomenon may result from small noise to the catastrophic failure of the equipment, discomfort to users and in major economic loss.

The increased flexibility of the rotors involves the increased level of vibration when the rotor is operate near one of its critical speeds or when the system goes through a transitional regime. Excessive vibration levels increase the mechanical stress on the rotor and bearings which affects the performance, service life and reliability of the machine (Koroishi, 2013; Vance et al, 2010; Simoes et al., 2007).

The active vibration control can reduce the vibration level by adding energy to the system. The most common devices used in the active control of vibration in rotating machines are hydraulic actuators, piezoelectric actuators and active magnetic bearings (MMAs). Due to its characteristics, the use of active magnetic bearings as actuator active control systems is a major highlight object and interest in the military, aerospace, biomedical, high-speed machining, among others.

Studies focused on the use of magnetic bearings are increasing. This type of bearing has as main feature the absence of friction and lubrication, fact that allows the operation of rotating machines at high speeds. The ability to actively act and change their own parameters during operation also enables unique capabilities to MMAs as control the vibration level and act as an intelligent capable of self diagnosis system (Ujihara, 2011).

In this context, the computational numerical simulation of rotating machines is a step indispensable for researchers and engineers. The numerical simulation allows a comprehensive understanding of the dynamic behavior of the system and the various state variables involved, besides the prediction of undesired mechanical behavior of the device. Therefore a mathematical model of greater reliability to be obtained considering various subsystems, as follows: first, the subsystems that can be defined by its geometry, such as the shaft (usually modeled via Finite Element Method), and the coupling discs; subsequently, the subsystems are dependent on the frequency and / or conditions, such as bearings and; finally, the gyroscopic effect (Cavalini Jr. et al., 2012, Meggiolaro, 1996).

In consonance, industrial applications require mechanical systems that work with optimum performance under certain operating conditions requiring high reliability, robustness to environmental conditions and low operational requirements. Thus, the need to develop numerical models that adequately consider the uncertainties in parameters and system inputs to evaluate the effect of the variability of these parameters in the dynamic behavior of the generating units is of great value in the design problems (Lara-Molina et al, 2014; Cavalini Jr. et al, 2012).

According to Möller and Beer (2004), uncertainties in modeling dynamic systems are considered mainly using three different approaches. In the first approach, the uncertainty is represented using a formula based on the theory of probability; the uncertainties are represented by probability distribution. In the second, the uncertainties are modeled by pre-defined intervals. In the third, the uncertainties are represented using the theory of possibilities.

In accordance with Lara-Molina et al. (2014), the parametric uncertainties of numerical models of dynamic systems have been represented using a probabilistic approach that concentrated parameters are modeled by random variables with an arbitrary probability density function and the fields or random processes are modeled by expanding Karhunen-Loève (Spanos and Ghanem, 1991). In most cases, the numerical simulation of the stochastic models is carried out with methods that show the probability density function, highlighting the Monte Carlo simulation and Latin Hypercube. In simulations with few random variables and large dispersion is used methods that do not sample the probability distribution function, as is the case of Chaos Polynomial (Xiu, 2010).

The study of uncertainties in rotating machines are performed by stochastic approaches mainly based on the Finite Element Method Stochastic. Didier et al. (2012) quantified the effects of uncertainties in the response of flexible rotors by the theory of chaos polynomial expansion. Koroishi et al. (2012) represented the uncertainties in the parameters of a rotor as homogeneous Gaussian stochastic fields discretized by expanding Karhunen-Loève. The system dynamic response was characterized by Latin Hypercube Sampling technique and Monte Carlo simulations. Lara-Molina et al. (2014) used the Finite Element Method Stochastic Fuzzy to quantify the effects of high order uncertainties in the response of rotating machinery, among others.

The uncertainty analysis is a great strategy for implementing the evaluation of variability of dynamic systems response. However, the use of this strategy is an onerous task and of high computational cost due the use of finite element models containing high number of degrees of freedom and the need to calculate a large number of response samples to achieve the required statistical significance. Thus, computational tools have been constantly developed to decrease this problem. Emphasis is given to the use models reduced methods (De Lima et al, 2010; Guedri et al., 2006).

In accordance with what was presented, this work aims to a preliminary study which analyzes the uncertainties in parameters and system inputs that affect the dynamic behavior of a flexible rotor model supported by active magnetic bearings, discretized by finite element. For this purpose we used the Latin Hypercube method. To reduce the computational cost time, the original model is reduced by models reduction method Iterative Improved Reduction System (IIRS) that provided good results in the simulations and with great saved time.

2 MODELING OF FLEXIBLE ROTORS

In the study of rotating machines, modeling tools are required. Emphasis is given to the use of Finite Element Method (FEM) and the Transfer Matrix of Method (MMT) (Steffen Jr., 1981; Lallement et al, 1982; Berthier, et al., 1983).

In the context of this work, flexible rotors are dynamic systems whose models consist of elements such as drives, flexible shafts and bearings. This theoretical model is constituted by a set of differential equations describing the system movement and are obtained using Newton's laws, voltage relationships - deformation and Lagrange equations (Craig, Jr. and Roy, 1981). It is considered that the elements coupled to the shaft are disks, the axis is an elastic system and the bearings as support elements, the formulation is based on the potential energy, kinetic, virtual work and strain energy (Koroishi, 2013; Ujihara 2011; Lallane and Ferraris, 1998).

Layane and Ferraris (1998) present the general steps to be followed to determine a rotor equations of motion. These steps are shown below:

- The kinetic energy, the strain energy and the virtual work of the external forces are calculated for system components;
- Applies a numerical method (FEM);
- Applies the Lagrange equations as follows:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = Fq_i \tag{1}$$

where q_i are independent generalized coordinates, F_{qi} are the generalized forces and, T and U are the kinetics and deformation of the components energies, respectively.

A finite element axis is shown in Fig. 1, with the following features: the element has 2 nodes, each node has 4 degrees of freedom, i.e., two translations (u, w) and two rotations (θ, ψ). It is assumed the cross section as linearly variables. Assume the effects of rotational inertia of the cross section (Rayleigh) and the shear effect of the cross-section (Timoshenko).

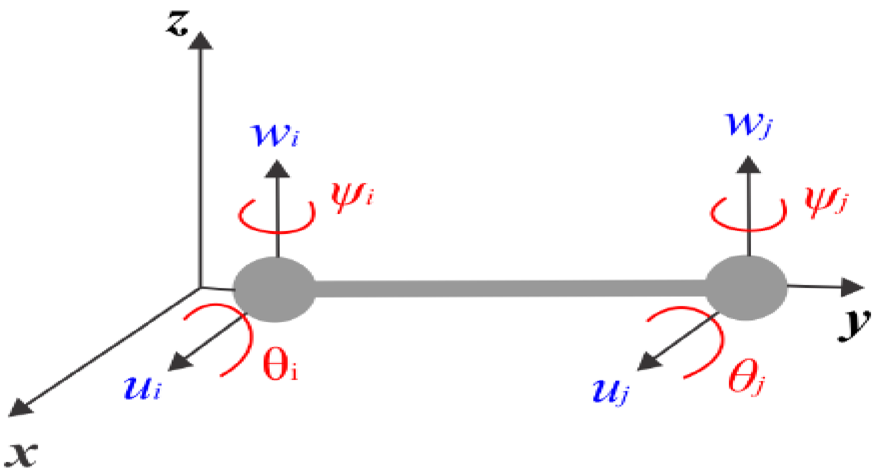

Figure 1 - Degrees of freedom about an axis finite element.

Using the FEM equation of motion of a flexible rotor is determined and described in a matrix form by Eq. (2),

$$[M]\{\ddot{x}(t)\} + [[C] + \dot{\Omega}[C_G]]\{\dot{x}(t)\} + [[K] + \ddot{\Omega}[K_{ST}]]\{x(t)\} = \{F(t)\} \tag{2}$$

where $[M]$ is the mass matrix of system, $[C]$ is the proporcional damping matrix, $[C_G]$ is the gyroscopic effects matrix, $[K]$ is the stiffness matrix can this be added to the matrix $[K_{ST}]$ which represents the stiffness of the system when in transient regime. All these matrices are associated with rotating machine parts, such as disks, the coupling and the shaft. The generalized displacement vector is represented by $\{x(t)\}$ and the speed of rotation is given by $\dot{\Omega}$. The vector forces is given by $\{F(t)\}$, can be, in many applications, added to the force weight vector of the unbalancing forces and the force vector produced by the bearings.

3 MODELS REDUCTION METHODOLOGY

In dynamic systems, the generalized movement equations are described as a set of linear differential equations of second order:

$$[M]\{\ddot{x}(t)\}+[C]\{\dot{x}(t)\}+[K]\{x(t)\}=\{F(t)\}. \tag{3}$$

where $[M]$, $[C]$, $[K]$, order $n \times n$, are the respective mass, damping and stiffness matrices; $\{x(t)\}$, $\{F(t)\}$, order $n \times 1$ are the displacement and forces vectors; n indicates the dof of the complete system.

According Koutsovasilis and Beitelschmidt (2008), many types of reduction procedures involve obtaining a coordinate transformation matrix. Generally, determining a subspace-called coordinate transformation matrix $[T]$, order $m \times n$, with $m << n$, such that:

$$\{x(t)\} \approx [T]\{x_R(t)\}, \tag{4}$$

where $\{x_R(t)\}$ is the reduced displacement vector.

Introducing the Eq. (4) in the Eq. (3) and then multiplying the transposed portions by the transformation matrix, a new reduced system is obtained:

$$[\overline{M}]\{\ddot{x}_R(t)\}+[\overline{C}]\{\dot{x}_R(t)\}+[\overline{K}]\{x_R(t)\}=\{\overline{F}(t)\}, \tag{5}$$

where $[\overline{M}]$, $[\overline{C}]$, $[\overline{K}]$, order $m \times m$, are respective mass, damping and stiffenss matrices of reduced order; $\{\overline{F}(t)\}$, order $m \times 1$ is the reduced forces vector; m indicates the number dof retained in the reduced model. These matrices are defined as:

$$[\overline{M}]=[T]^T[M][T], \quad [\overline{C}]=[T]^T[C][T], \quad [\overline{K}]=[T]^T[K][T], \quad \{\overline{F}\}=[T]^T\{F\}. \tag{6}$$

Although the size of the reduced model is much smaller than the full model, given the coordinate transformation matrix that is particular in each reduction method, the dynamic characteristics of the complete model can be preserved.

3.1 Iterative improved reduction system (IIRS)

Over the years, various formulations for iterative scheme IIRS are suggested. According Friswell et al. (1995), to obtain the transformation matrix it is part of the generalized eigenvalue problem of a system undamped in substructure way in terms of dof masters (m) and slave (s),

$$\begin{bmatrix} [k_{mm}] & [k_{ms}] \\ [k_{sm}] & [k_{ss}] \end{bmatrix} \begin{bmatrix} \Phi_{mm} \\ \Phi_{sm} \end{bmatrix} = \begin{bmatrix} [m_{mm}] & [m_{ms}] \\ [m_{sm}] & [m_{ss}] \end{bmatrix} \begin{bmatrix} \Phi_{mm} \\ \Phi_{sm} \end{bmatrix} [\Lambda_{mm}], \tag{7}$$

where Φ are the normalized eigenvectors; Λ is the diagonal matrix containing the corresponding eigenvalues arranged in ascending order. From the second set of equations of Eq. (7), we have:

$$[k_{sm}][\Phi_{mm}] + [k_{ss}][\Phi_{sm}] = [m_{sm}][\Phi_{mm}][\Lambda_{mm}] + [m_{ss}][\Phi_{sm}][\Lambda_{mm}], \tag{8}$$

where $[\Phi_{sm}]$ can be expressed by,

$$[\Phi_{sm}] = -[k_{ss}]^{-1}[k_{sm}][\Phi_{mm}] + [k_{ss}]^{-1}\left([m_{sm}][\Phi_{mm}][\Lambda_{mm}] + [m_{ss}][\Phi_{sm}][\Lambda_{mm}]\right). \tag{9}$$

Makes,

$$[\Phi_{sm}] = [T_0][\Phi_{mm}], \tag{10}$$

where $[T_0]$ is the transformation matrix, written in the form

$$[T_0] = [T_{Guyan}] + [k_{ss}]^{-1}\big([m_{sm}] + [m_{ss}][T_0]\big)[\Phi_{mm}][\Lambda_{mm}][\Phi_{mm}]^{-1}. \tag{11}$$

Then, the processing degrees of freedom between the masters and the degrees of freedom of the complete model is written as,

$$[\Phi] = \begin{bmatrix} \Phi_{mm} \\ \Phi_{sm} \end{bmatrix} = \begin{bmatrix} [I_{m,m}] \\ [T_0] \end{bmatrix} [\Phi_{mm}] = [T]\ [\Phi_{mm}], \tag{12}$$

Substituting Eq. (12) in Eq. (7) and multiplied by the transpose of the transformation matrix obtained above, there is obtained a reduced eigenvalue problem

$$[\overline{K}][\Phi_{mm}] = [\overline{M}][\Phi_{mm}][\Lambda_{mm}]. \tag{13}$$

From Eq. (13),

$$[\Phi_{mm}][\Lambda_{mm}][\Phi_{mm}]^{-1} = [\overline{M}]^{-1}[\overline{K}]. \tag{14}$$

Substituting Eq. (14) to (11), Obtain:

$$[T_0] = [T_{Guyan}] + [k_{ss}]^{-1}\big([m_{sm}] + [m_{ss}][T_0]\big)[\overline{M}]^{-1}[\overline{K}]. \tag{15}$$

Note that $[T_0]$ is implicit in Eq. (15) and can not be set directly, so we propose an iterative scheme (Friswell et al., 1995) given by:

$$[T]^{(k)} = [T_{Guyan}] + [k_{ss}]^{-1}\Big([m_{sm}] + [m_{ss}][T]^{(k-1)}\Big)\Big([\overline{M}]^{(k-1)}\Big)^{-1}[\overline{K}]^{(k-1)}, \tag{16}$$

$$[T]^{(k)} = \begin{bmatrix} [I_{m,m}] \\ [T]^{(k)} \end{bmatrix}, \tag{17}$$

$$[\overline{K}]^{(k)} = [T^{(k)}]^T [K][T]^{(k)}, \tag{18a}$$

$$[\overline{M}]^{(k)} = [T^{(k)}]^T [M][T]^{(k)}, \tag{18b}$$

Where the index k denotes k - th iteration ($k \geq 2$).

For the iterative scheme above, if $k = 1$, $[T]^{(1)} = [T_{Guyan}]$, is the static condensation method (Guyan, 1965); when $k = 2$, is equivalent IRS technique (Friswell et al., 1995). Eigenvectors and eigenvalues are estimated by solving the generalized eigenvalue of the reduced system problem:

$$[\overline{K}]^{(k)}[\Phi_m]^{(k)} = [\overline{M}]^{(k)}[\Phi_m]^{(k)}[\Lambda_m]^{(k)}. \tag{19}$$

In the iterative approach, the coordinate transformation matrix is updated repeatedly until the required accuracy. However the process of convergence of this method may require high computational time. Some improvements have been made by Xia et al., (2004). In this approach, iterative formula for transformation matrix is modified such as to improve convergence speed. The formula for the new iterative scheme is:

$$[T]^{(k)} = [T_{Guyan}] + [k_{ss}]^{-1}\left([m_{sm}] + [m_{ss}][T]^{(k-1)}\right)[M_d^{(k-1)}]^{-1}[\overline{K}]_{Guyan}, \tag{20}$$

$$[T]^{(k)} = \begin{bmatrix} [I_{m,m}] \\ [T]^{(k)} \end{bmatrix}, \tag{21}$$

$$[M_d^{(k-1)}] = \left([m_{mm}] + [m_{ms}][T]^{(k-1)}\right) + [T_{Guyan}]^T\left([m_{sm}] + [m_{ss}][T]^{(k-1)}\right) \tag{22}$$

For this approach, Eq. (17) and (18) are valid for estimating of eigenvectors and associated eigenvalues.

4 NUMERICAL SIMULATIONS

In this paper, for the various cases of uncertainty analysis in the parameters of rotor using Latin Hypercube Sampling (LHS) it is considered a flexible rotor supported by active magnetic bearing whose parameters for the beam and disk elements are shown in Table 1.

Table 1. Rotor Parameters

Beam elements			Disk elements				
NV	*L*	*D*	*ND*	*NOD*	*L*	D_i	D_e
1	32.6x10^{-3}	17 x10^{-3}	1	2	30.4x10^{-3}	39.35x10^{-3}	74.25 x10^{-3}
2	30.4 x10^{-3}	39.35x10^{-3}	2	3	8.8x10^{-3}	39.35x10^{-3}	74.25 x10^{-3}
3	8.8x10^{-3}	39.35x10^{-3}	3	4	8.8x10^{-3}	27.56x10^{-3}	52 x10^{-3}
4	8.8 x10^{-3}	27.56x10^{-3}	4	6	7x10^{-3}	55x10^{-3}	70 x10^{-3}
5	28.9x10^{-3}	21.45x10^{-3}	5	7	37.75x10^{-3}	39.35x10^{-3}	74.25x10^{-3}
6	7x10^{-3}	21.45x10^{-3}	6	9	37.75x10^{-3}	39.35x10^{-3}	74.25x10^{-3}
7	37.75x10^{-3}	39.35x10^{-3}	7	10	9.9x10^{-3}	55x10^{-3}	70x10^{-3}
8	37.75x10^{-3}	39.35x10^{-3}	8	12	8.8x10^{-3}	27.56x10^{-3}	52x10^{-3}
9	9.9x10^{-3}	21.45x10^{-3}	9	13	8.8x10^{-3}	39.35x10^{-3}	74.25x10^{-3}
10	22.05x10^{-3}	21.45x10^{-3}	10	14	30.4x10^{-3}	39.35x10^{-3}	74.25x10^{-3}
11	8.8x10^{-3}	27.56x10^{-3}					
12	8.8x10^{-3}	39.35x10^{-3}					
13	30.4x10^{-3}	39.35x10^{-3}					
14	71.95x10^{-3}	17x10^{-3}					

In this table, NV is the number of beam element, L is length, ρ is material density (kg/m^3), where the beam element is 7850 and of disk is 6770, modulus of elasticity of steel or young's modulus is 210 GPa. ND is number of disk, NOD is node where the disk is applied, D_i internal diameter (m) and D_e external diameter (m). Also, bearings are located in node 3 (first bearing) and node 13 (second bearing). This distribution can be observed in Fig. 2.

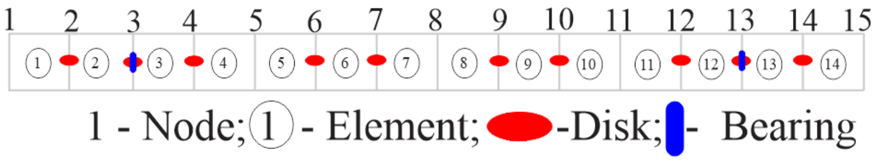

Figure 2. Rotor representation.

Through the resolution of an eigenvalue problem the natural frequencies were obtained. The Figure 3 illustrates the frequency response function (FRF) estimated in a range of 0 to 800 Hz for the rotor model shown in Fig. 2.

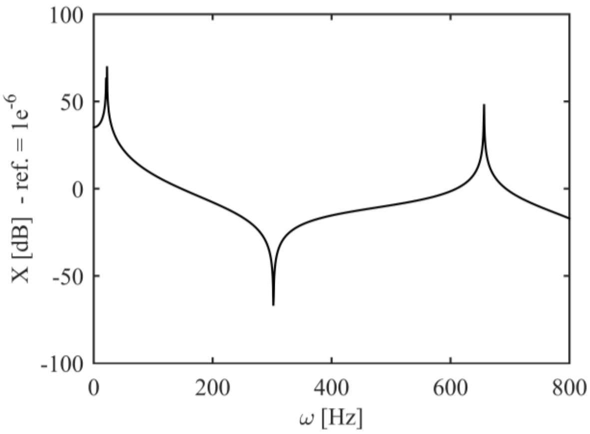

Figure 3. Frequency response function of the rotor.

4.1 Study of the effect of uncertainty in bearing of the rotor

The bearing has the function of supporting the rotors, absorb or attenuate the vibratory energy and to restrict the degree of freedom for the drive shaft. It is considered some random parameters using the method of sampling LHS, where 1000 samples are generated. In this case, two bearings are applied to nodes 3 and 13 and have the following rigidity values $49x10^3$ N/m and $60x10^3$ N/m.

Considering the effect of 30% obtained the Fig. 4. In these figures, it is observed that the full model and the reduced were obtained similar curves, i.e., range of values are close in both cases.

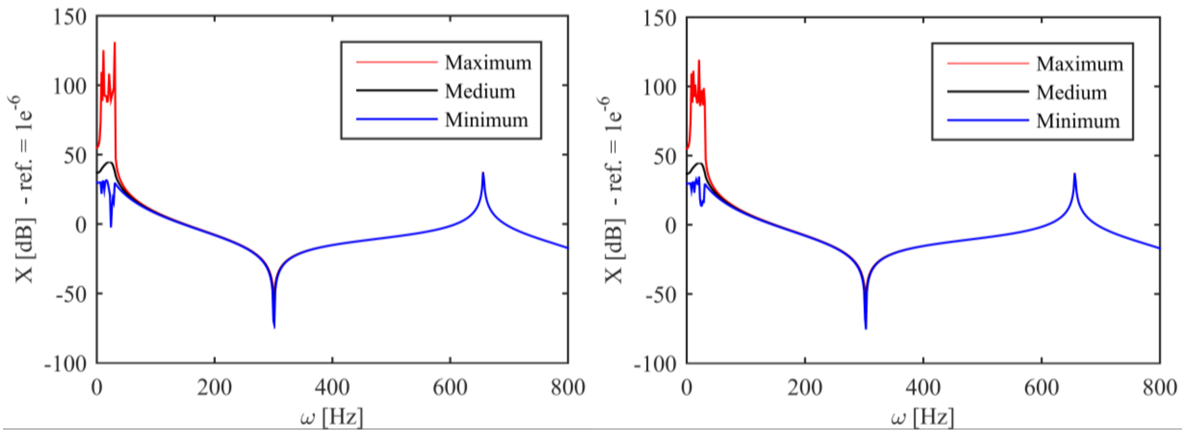

Figure 4. Effect of 30% uncertainty in the stiffness of the bearing to (a) complete model and (b) reduced model.

In Table 2 are listed the values of computational time.

Table 2. Values of computational time and standard deviation for the complete model and reduced.

Figure	Time (s)	Percentage (%)
(a)	144660	100
(b)	72410	50

With this simulation, it was found that the bearings are not sensitive, in this case, the consideration of uncertainty, since, even considering a high value of uncertainty (30%) range of values showed up close.

Furthermore, it was obtained a satisfactory time when used the *IIRS* reduction method. In Table 2 shows savings of 50%. These simulations showed that the reducing method was of great value in reducing the computational time, i.e., this will be used in the next simulations.

4.2 Study of uncertainty effect in the diameter of beam element

Once confirmed the time savings obtained by IIRS reduction method, so in the next simulations will be used this method.

First, it is considered the diameter of the second element of the beam as a random variable. After, is performed again simulations with the LHS method where there is the consideration of the uncertainty in the diameter of the third, subsequently, has two random variables such as, in this case, the diameters of the third and ninth and finally, is considerate the seventh and thirteenth simultaneously.

It is noteworthy that in the tests the bearings will be used applied to us [3 13], with the values of $49x10^3$ N/m and $60x10^3$ N/m.

Therefore, it has the effect of 10% uncertainty in the diameter of the second beam element, thus, by LHS method, the envelopes of solution are obtained.

The figure 6 is shown the effect of 10% of the uncertainty in the diameter of the second beam element, where it was observed that the value ranges are distant, that is, a sensitive parameter the consideration of uncertainty.

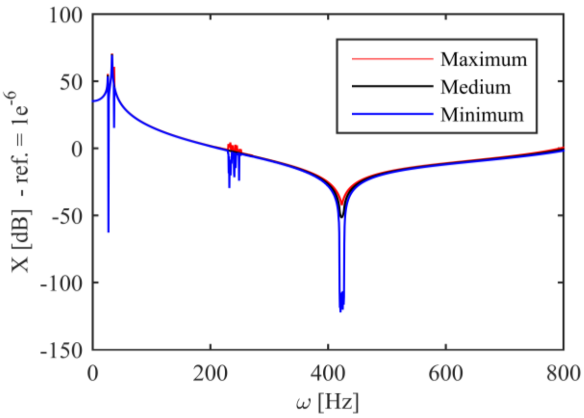

Figure 6. Effect of 10% of the uncertainty in the diameter of the 2° beam element.

In Table 4 illustrates the value of the standard deviation. It is observed that the ranges of values are far since the value of the standard deviation obtained is high.

Table 4. Value of the standard deviation.

Standard deviation
8.01

In this case, the simulation is carried out considering the diameter of the third beam element. Therefore, in Fig. 7 is illustrated the envelope of solution with 10% of uncertainty, it is concluded that the range of values (maximum, medium and minimum) with a lower sensitivity than in the case described in Fig. 6.

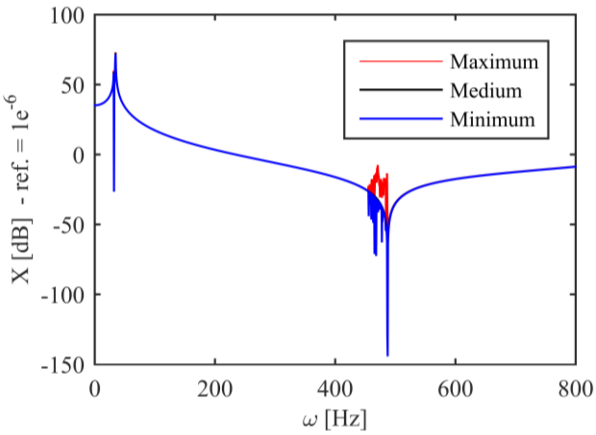

Figure 7. Effect of 10% of the uncertainty in the diameter of the 3° beam element.

Given the value in Table 5, it can be concluded that the range of values are closer compared to the value obtained where it was considered the uncertainty in the diameter of second beam element shown in Table 4.

Table 5. Value of the standard deviation.

Standard deviation
5.41

Once, it was perceived a small sensitivity of the parameters corresponding the diameters of second and third beam element will be now considered two cases of two random variables, namely, first is diameter corresponding the third and ninth beam element and in the last case, considers the diameter the seventh and thirteenth simultaneously.

In Figure 8 is considered the effect of 10% uncertainty in the diameters corresponding to the third and ninth beam elements. It is observed by envelopes of solution, the sensitivity parameters the uncertainty, as is visible in the distance between ranges of values.

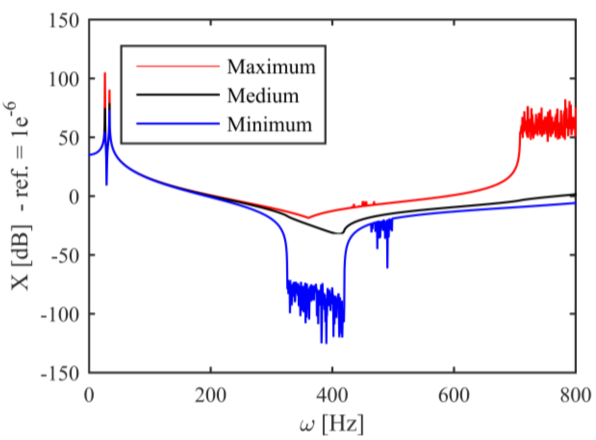

Figure 8. Effect of 10% of the uncertainty in the diameters of the 3° and 9º beam element.

The Table 6 shows the value of the standard deviation found for this case. Can be confirmed with the value of the standard deviation emphasizing that the envelope of solution illustrated in Figure 8 are distant, that is, there is a case that sensitivity to external factors (uncertainty).

Table 6. Value of the standard deviation.

Standard deviation
27.29

In addition, in this simulation has the effect of 10% uncertainty in the diameters of the corresponding beam element to the seventh and thirteenth. In Figure 9 (a) has the envelope of solution where it is observed that the ranges of values (maximum, medium and minimum) show their distance, i.e., are parameters that are sensitive to uncertainty.

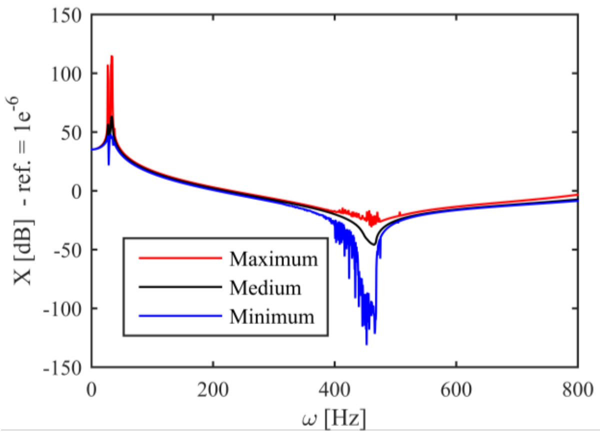

Figure 9. Effect of 10% of the uncertainty in the diameters of the 7° and 13º beam element.

In Table 7 describes the value of the standard deviation relating to the envelope of solution Fig. 9. As in the previous case, it is evident a large distance between the maximum value of the range as the minimum.

Table 7. Value of the standard deviation.

Standard deviation
14.86

4 CONCLUSION

This paper presented a preliminary study which analyzes the uncertainties in the parameters of a flexible rotor supported by assets of magnetic bearings was discretized by finite elements. We used a stochastic method called Latin Hypercube Sampling. Through numerical simulations it was characterized the response in the frequency domain in the presence of uncertainties. Furthermore, the use of reduced models provides good representability and low computational cost, and hence, great savings of time.

In particular, modeling by finite elements of a flexible rotor supported by active magnetic bearings have been proposed and implemented. The reduction of the model was obtained using the IIRS method. Thus, the uncertainties in the design variables that characterize the flexible rotor were inserted to different case studies that show that the envelopes of the solution lead to important information regarding the effect of the variability of these parameters in the dynamic behavior of the rotor in question.

In general, it is concluded that in the consideration for uncertainty in the rigidity of the bearings is seen that this is not a sensible variable in the consideration of uncertainty, since the ranges of values were close. As for the uncertainty of consideration in diameter of the beam element, was obtained for the cases considered ranges that are distant, mainly when it was considered more of a parameter as uncertain where, we can see a great sensitivity to external factors.

Therefore, with the numerical results, it is that the proposed strategy shows the importance of introducing uncertainties in the design variables from the perspective of the design of rotating machines, constituting itself as an important tool for analysis and design.

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