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EXPERIMENTAL STUDY OF THE DYNAMIC BEHAVIOR OF A BEAM WITH SPATIALLY CORRELATED MASSES FOR DIFFERENT RANDOM FIELD MODELS

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Abstract: *There is no such thing as absence of variability in manufacturing processes. Therefore, there is a need for improved prediction models which take this variability into account. Experimental studies of these variabilities often are complex and hardly found in the literature mainly because the characterization of a great number of samples has a high cost associated and the variability model can only be defined after the samples are produced. In this work, an experimental setup capable of imposing a spatially correlated variability model a priori is used. The mass density distribution of a straight beam is approximated by small and evenly distributed masses whose values are given by a random field model. The masses are small magnets attached to the beam, allowing for easily changing the distribution and providing a great number of samples. Different configurations of masses are generated from different discrete random field models, i.e. with different PDFs, correlation functions and lengths. For each configuration, an ensemble of dynamic responses is measured from an impact hammer test and effects of the different random field configurations on the statistic of the bending natural frequencies of the beam are investigated.*

Keywords: *random fields, variability, natural frequencies, experimental characterization.*

1 INTRODUCTION

There always exists variability in productive processes, regardless of the scale of the processes. Material and/or geometrical properties such as Young's and shear modulus, density, thickness, moments of inertia and area, will always present some level of variability with respect to the nominal design (Fabro, 2015; NIST/SEMATECH, 2012). These differences can and often will have implications on the structure´s performance, mainly in its dynamic behaviour. Therefore, it is imperative that a model that includes the effects of variability should be taken into account.

It is usually very difficult to produce experimental results taking into account effects of such variabilities, mainly because it is necessary to characterize a great number of samples to provide adequate levels of statistical significance, with a high cost associated. For that reason, such studies are scarcely found in the literature. Moreover, it is not trivial to find a model that describes this variability due to the fabrication/production processes, because this model can only be identified *a posteriori,* i.e. it depends on intrinsic characteristics of the manufacturing process, which is a restrictive factor once we try to study the effects of the uncertainties in the dynamic response of a number of structures.

The objective of this paper is twofold. One, it is to propose a simple experimental setup aiming to introduce spatially correlated random variability on the material proprieties of a structure, derived from a *a priori* stochastic model, following a former experimental procedure (Fabro et al. 2015). Second, to make available an experimental database for further researches on the dynamic response of simple structures of random spatially correlated material properties. In this work, a stochastic model from the random fields theory (Vanmarcke, 2010) is chosen and a number of samples are generated accordingly. Such approach allows us to have control over the experimental results, once the parameters are known.

The experiment consists in introducing spatial variability in the beam's mass density with small magnets attached to different points equally spaced across the beam. The number of magnets attached to each point, i.e. the value of mass added to each point is dictated by a random field model. Then, the natural bending frequencies of the beam are measured with an impact hammer test, for each different configuration.

For that, some statistical moments are analysed, such as average, standard deviation and coefficient of variation (COV) of the natural bending frequencies of a beam with density defined by a random field model. The parameters of the random field will be varied for different probability density functions (PDFs), correlation functions and lengths. The results of this work will be gathered for the creation of a data base that can be used by other research groups further researches on stochastic structural dynamics.

This paper is organized as follows. In Section 1 a brief introduction and aims are described. Section 2 discusses the manufacturing and characterization process of the beam used during all experiments, section 3 discusses how the distribution of the random masses along the beam is generated. Section 4 presents the experimental setup and section 5 the results and some discussion. In section 5, some concluding remarks and further steps are drawn. The appendix presents each configuration of masses used in this work.

2 DESIGN AND EXPERIMENTAL CHARACTERIZARION OF THE BASELINE BEAM

The baseline beam is made of steel and with rectangular cross section was made at the mechanical engineering workshop of the University of Brasilia. The beam had its dimensions and mass measured in the metrology laboratory using a ruler for the length measurement, a micrometer for the width and thickness measurements and a scale with precision ± 0.001 kg, as shown in “Table 1”. These values were obtained by measuring each dimension of the beam 7 times and taking the average.

Table 1. Baseline beam geometrical and material properties.

Length [mm]	Width [mm]	Thickness [mm]	Volume [mm³]	Mass [kg]	Young’s modulus [GPa]
500.5	25.48	4.90	62488.426	0.466	181.81

With the value of the volume and the mass, it is then possible to estimate the beam’s density as $\rho = 7452.8$ kg/m³. After obtaining the geometrical proprieties and density, the Young’s modulus was estimated by measuring the natural frequencies of the baseline beam, hung by thin wires, i.e. free-free boundary conditions, and the Elasticity modulus could be estimated using the following expression obtained from the available analytical solution, using Euler-Bernoulli beam theory (Inman, 2001),

$$E = \frac{(2\pi w_n)^2 \rho A}{\beta_n^4 I}, \quad (1)$$

in which w_n is the n-th natural frequency, and $\beta_1 = 4.73004074/L$, $\beta_2 = 7.85320462/L$, $\beta_3 = 10.9956078/L$ and $\beta_4 = 14.1371655/L$. These values for β are obtained by solving the transcendental equation

$$\cos(\beta_n L)\cosh(\beta_n L) = 1. \quad (2)$$

Two independent experiments were performed, and the four first bending natural frequencies were estimated by a peak picking procedure. The dimensions of the beam are such that the first torsional and longitudinal natural frequencies are always higher than the fourth flexural mode, even when the masses are attached. This is important so that only one measurement point is needed with no mode shape estimation. The experimental setup is the same used for measuring FRF with the attached masses and it is shown in section 4. Results are summarized in “Table 2”.

Table 2. Bending natural frequencies obtained experimentally from the baseline beam.

Experiment	1st mode [Hz]	2nd mode [Hz]	3rd mode [Hz]	4th mode [Hz]
1	101.5	280.0	548.4	905.8
2	101.7	280.0	548.6	906.1

The results of the first experiment yields a Young's modulus value of 189.71 GPa, and the second experiment yields 189.91 GPa. The average value is then used as the baseline, "Table 3". The estimated material and geometrical properties are then used to calculate the first four bending natural frequencies of the baseline beam, using Euler-Bernoulli beam theory, as shown in "Table 3".

Table 3. First 4 bending natural frequencies of the beam obtained with estimated material and geometrical parameters.

1st mode [Hz]	2nd mode [Hz]	3rd mode [Hz]	4th mode [Hz]
101.5	279.9	548.7	907.0

3 DISTRIBUTION OF RANDOM MASSES

Masses with random values are attached in equally spaced points across the otherwise bare beam according to a random field model, with different correlation lengths. The masses are small magnets with 0.733 grams each. Such approach allows for changing the mass locally and approximate the discrete distribution to a continuous mass density random field model for the mode shapes corresponding to the first natural frequencies. The points in which the magnets are attached are kept the same, but the quantity of magnets, i.e. the value of mass added, are dictated by the random field model. Moreover, it allows to generate a large number of configurations easily, i.e., a large number of samples, which is necessary for the statistical significance of the results.

A MATLAB routine was made to generate the number of magnets attached to each point of the beam. This routine first creates a vector of 10 elements, and then correlate this vector according to a correlation matrix. The correlation matrix is 10 by 10 and each element of the matrix specify how each point of the beam will correlate to each other. This will depend on the correlation function used, and also the correlation length.

The random vector $\boldsymbol{\xi}(\theta)$ approximates the continuous random field $H(x,\theta)$. Let $\boldsymbol{\zeta}(\theta)$ be a vector of uncorrelated Gaussian random zero mean and unit variance variables and with $\mathbf{C} = \langle \boldsymbol{\xi}\boldsymbol{\xi}^T \rangle$ the correlation matrix, where $\langle\cdot\rangle$ represents the mathematical expectation, and the superscript T represent transpose. This matrix is symmetric and positive-definite, so a Cholesky decomposition of the kind $\mathbf{C} = \boldsymbol{\Sigma}\boldsymbol{\Sigma}^T$, where $\boldsymbol{\Sigma}$ is a lower triangular matrix and $\boldsymbol{\Sigma}^T$ is its transpose, is possible. Then a realization of the random field can be given by a realization of as

$$\xi(\theta) = \Sigma\zeta(\theta). \tag{3}$$

In this work, the following continuous correlation is used

$$C(\tau) = e^{-\tau/b}, \tag{4}$$

in which τ is the distance between two point and b is the correlation length and three different correlation lengths were used, as shown in "Table 4".

Table 4. Correlation lengths used in the random field model for the masses distribution normalized by the beam length L.

b_1/L	b_2/L	b_3/L
$1/5$	3/5	1

For each correlation length, ten different mass density samples were generated, and for each configuration a frequency response measurement was made in order to obtain the first four bending natural frequencies of the beam.

4 EXPERIMENTAL SETUP

The FRF measurements were made using a modal hammer PCB 086C01 and a Laser Doppler Vibrometer (LDV) Polytec PDV 100, shown in “Figures 1 and 2” along with the acquisition system. The LDV is used so that there is no added mass from an accelerometer, for instance. A H_2 estimator was used for the FRFs with 10 averages and frequency discretization $\Delta f = 0.15625$. The same experimental setup was used to estimate the material properties of the baseline beam.

Figure 1. LDV Polytec PVD 100 (left) and PCB 086C01 modal hammer and acquisition board used (right).

The baseline beam was hung by two nylon wires, such that free-free boundary condition can be assumed, as shown in “Figure 2”. A reflexive tape was fixed at one end of the beam and the hammer excitation were done on the other end. The magnets were equally spaced at ten different positions with $l = L/10$ apart, as shown schematically in “Figure 3”. The four first bending natural frequencies were estimated by a peak picking procedure.

Figure 2. Experimental setup including modal hammer, LDV, acquisition system, and the hung beam(left) and a details of a sample configuration of the attached magnets and measuring point (right).

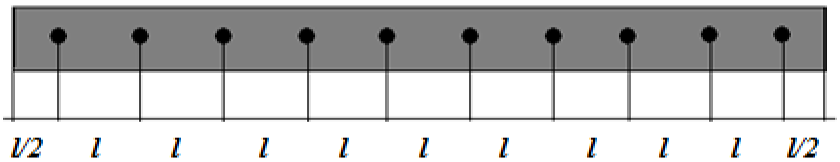

Figure 3. Schematic representation of the masses distribution location along the beam.

5 RESULTS AND DISCUSSION

Ten independent random field samples were generated using the Gaussian random field with exponentially decaying correlation function model, with three different correlation length, as presented in section 3. All of the values of the added masses are presented in the appendix. First four bending natural frequencies for each sample are presented in "Tables 5 to 7" for $b = L/5$, $b = 3L/5$ and $b = L$, respectively. "Table 8" presents the average added mass for each correlation length case.

Table 5. Bending natural frequencies obtained experimentally for each mode, as well as total mass added for each configuration for the correlation length $b = L/5$.

Sample	Added mass [g]	1st mode [Hz]	2nd mode [Hz]	3rd mode [Hz]	4th mode [Hz]
1	29.320	98.4	272.2	532.8	882.1
2	23.456	99.4	273.8	537.0	886.3
3	37.383	98.1	270.0	530.1	873.8
4	25.655	99.4	273.1	536.1	882.2
5	22.723	99.7	273.6	537.0	887.0
6	37.383	98.3	270.0	530.0	874.5
7	36.650	97.7	270.8	530.0	874.7
8	39.582	98.1	270.3	527.7	873.0
9	36.650	98.0	270.2	530.3	875.3
10	22.723	99.4	273.6	537.8	886.7

Table 6. Bending natural frequencies obtained experimentally for each mode, as well as total mass added for each configuration for the correlation length $b = 3L/5$

Sample	Added mass [g]	1st mode [Hz]	2nd mode [Hz]	3rd mode [Hz]	4th mode [Hz]
1	16.859	100.0	275.5	540.3	891.6
2	33.718	98.4	271.3	530.9	878.4
3	29.320	98.6	272.2	533.4	881.7
4	11.728	100.3	276.9	543.6	897.3
5	21.990	99.5	274.4	537.0	887.5
6	31.519	98.4	271.4	532.7	878.6
7	30.786	98.6	271.7	533.0	879.4
8	24.922	99.4	273.4	535.8	883.9
9	24.922	99.2	273.1	535.8	884.7
10	20.524	99.4	274.2	538.4	888.4

Table 7. Bending natural frequencies obtained experimentally for each mode, as well as total mass added for each configuration for the correlation length $b = L$.

Sample	Added mass [g]	1st mode [Hz]	2nd mode [Hz]	3rd mode [Hz]	4th mode [Hz]
1	31.519	98.4	271.6	531.9	879.7
2	21.257	99.4	274.4	537.3	888.1
3	27.854	98.6	272.5	533.9	882.3
4	24.189	99.4	273.6	535.5	885.6
5	43.247	97.7	268.8	527.0	869.7
6	26.388	99.2	273.1	532.5	882.2
7	32.985	98.4	271.1	531.6	878.6
8	17.592	100.2	275.3	539.4	890.8
9	47.645	97.3	267.7	524.8	867.0
10	22.723	99.2	273.9	536.7	887.0

Table 8. Average added mass for the correlation length.

$b = L/5$	$b = 3L/5$	$b = L$
31.153	24.629	29.540

The sample mean value of μ and standard-deviation σ of the experimentally obtained natural frequencies are then used to calculate the coefficient of variation (COV)

$$COV = \sigma/\mu, \tag{5}$$

for each mode and correlation length, and results are shown in "Figure 4". It can be noticed that the COV as a function of the normalized correlation length is approximately the same for each mode, suggesting that the random variability affects equally the first bending modes.

It can be noticed that the COV tends to increase for increasing correlation length, which is expected (Fabro et al, 2015). For the case $b = L/5$, the correlation length is two times larger than the spacing between masses, so the poor spatial resolution can misrepresent the correlation in the random field. In this case, the first mode presented a higher COV than for the $b = 3L/5$. For correlation length much larger than the beam length, i.e. $b < L$, the spatial distribution tends to be homogeneous.

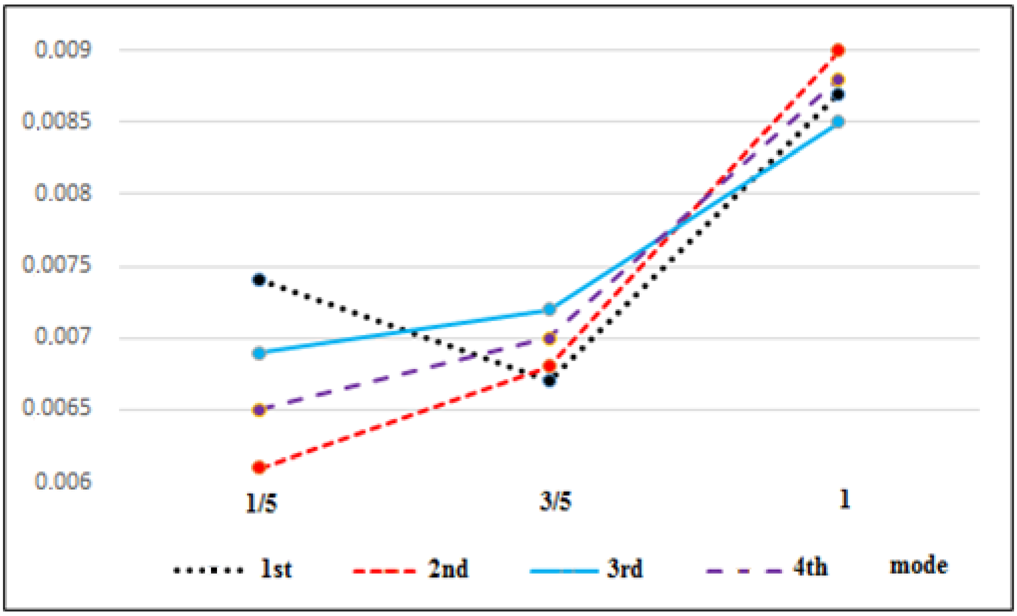

Figure 4. Coefficient of variation as a function of the ratio between the correlation length and beam length for each mode.

6 CONCLUDING REMARKS

In this work, a simple experimental procedure was presented to introduce spatially correlated random variability on the material proprieties of a structure, aiming to characterize its influence on the statistics of the dynamic response of a simple structure. It allows to generated a set of masses evenly distributed over a bare baseline beam, according to a random field model defined *a priori*. This setup allows for easily generate a great number of samples, which is need for statistical significance of the results.

The baseline beam is such that the first four bending natural frequencies are far from the first torsional and longitudinal frequencies. It allows measurement from on single point, with no need for mode shape estimation. Moreover, the generated set of masses and corresponding measured bending natural frequencies are made readily available, so it can be used by other research groups further researches on stochastic structural dynamics

It was shown that the correlation length directly influences the statistics of the natural frequencies of a structure. The increasing correlation length increases the variability of the natural frequencies, until this value reaches a maximum for correlation lengths larger than the beam length, because the distribution becomes homogenous over the beam. This results shows that it is very important to properly include spatial correlation in any stochastic dynamic model, because it can significantly affect the variability it dynamic behaviour.

Further works includes extending this investigation for different correlation functions and non-Gaussian random fields.

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8 APPENDIX

In this appendix, “Table 9 to 10” present all of the configurations of masses along the beam used for each sample for each correlation length case, i.e. the number of magnets in each position, for all the three correlation lengths. The mass of each magnet is 0.733 grams.

Table 9. Number of magnets attached to each position of the beam, for each sample, for the correlation length $b = L/5$.

Sample	Pos. 1	Pos. 2	Pos. 3	Pos. 4	Pos. 5	Pos. 6	Pos.7	Pos.8	Pos. 9	Pos. 10
1	3	3	4	5	6	6	3	3	2	5
2	3	4	4	5	4	2	1	3	3	3
3	6	5	5	6	5	5	6	5	5	3
4	2	3	4	7	5	5	4	2	2	1
5	1	1	4	3	4	4	6	3	3	2
6	8	7	7	5	4	4	5	4	5	2
7	5	6	3	4	5	3	3	4	8	9
8	2	6	6	5	6	5	4	6	7	7
9	5	3	4	6	6	6	6	5	5	4
10	5	3	3	2	0	3	4	3	5	3

Table 10. Number of magnets attached to each position of the beam, for each sample, for the correlation length $b = 3L/5$.

Sample	Pos. 1	Pos. 2	Pos. 3	Pos. 4	Pos. 5	Pos. 6	Pos.7	Pos.8	Pos. 9	Pos. 10
1	4	4	3	3	2	2	2	1	1	1
2	5	5	5	4	5	4	4	5	5	4
3	4	3	3	4	5	5	4	4	4	4
4	2	1	0	1	1	2	2	2	2	3
5	2	3	2	2	3	3	3	4	4	4
6	4	4	3	4	4	4	5	5	5	5
7	4	4	4	4	4	5	5	4	4	4
8	3	4	4	4	3	3	3	3	4	3
9	3	4	4	4	3	3	3	3	3	4
10	5	4	3	3	3	3	3	1	1	2

Table 11. Number of magnets attached to each position of the beam, for each sample, for the correlation length b = L.

Sample	Pos. 1	Pos. 2	Pos. 3	Pos. 4	Pos. 5	Pos. 6	Pos.7	Pos.8	Pos. 9	Pos. 10
1	4	4	4	4	4	4	4	4	5	6
2	3	3	2	2	2	4	3	3	3	4
3	3	3	3	4	5	5	4	3	3	5
4	3	4	4	4	4	3	2	3	3	3
5	6	6	6	7	6	6	6	6	6	4
6	2	3	3	5	5	5	5	3	3	2
7	5	4	4	4	4	4	4	5	6	5
8	1	1	2	2	3	3	4	3	3	2
9	8	8	8	7	6	6	6	6	6	4
10	4	3	2	3	3	3	2	3	4	4