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RESPONSE VARIABILITY WITH RANDOM UNCERTAINTY IN A TUNED LIQUID COLUMN DAMPER

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Abstract. *Passive energy dissipation systems encompass a range of materials and devices for enhancing damping. They can be used both for natural hazard mitigation and for rehabilitation of aging or deficient structures. Among the current passive energy dissipation systems, tuned liquid column damper (TLCD), a class of passive control that utilizes liquid in a "U" shape reservoir to control structural vibration of the primary system, has been widely researched. Uncertainties can arise from simplifications in the model and from nonlinearities. To quantify uncertainties, random variables need to be associated with the systems parameters, such as stiffness and damping ratio, along with their probability density function. The Maximum Entropy Principle is used to construct the probability density function since it avoids using misinformation in the construction of model. In this paper the frequency response function of a system with a TLCD is investigated considering two cases of parameter uncertainty: the first considering uncertainties only in one parameter, the absorber damping ratio and the second considering uncertainties in two parameters, the absorber damping ratio and the structure stiffness. The results showed that, for the first case, the uncertainty is only predominant near the resonance and anti-resonance region and can indeed interfere in the optimum condition of the absorber. For the second case, the uncertainties are presented in all frequencies.*

Keywords: *Tuned Liquid Column Damper,Vibration Absorber, Model Uncertainty, Stochastic Analysis*

1 INTRODUCTION

Remarkable progress in the technology used in wind turbines has been made over the past years, advances in the field of structural and dynamic analysis allow the creation of larger and more efficient wind turbines. However, higher and slender structures poses challenges concerning its integrity in relation to the dynamic loads from wind, ocean waves or earthquakes. Serious efforts have been undertaken to develop the concept of vibration control of wind turbines.

The main goal of the project is reduce vibration in wind turbines using a tuned liquid column damper (TLCD). As shown in Fig. 1, the TLCD operates based on the movement of the liquid column. The column may have different shapes, particularly in this paper, the TLCD has a "U" shape. The TLCD requires no extra mechanism such as springs or joints, besides that, its geometry may vary according to design needs, making them very versatile devices. While the apparent simplicity of the system, the damping is dependent on the amplitude of the liquid, and therefore the dynamics of TLCD is nonlinear, which brings some mathematical complications to the model.

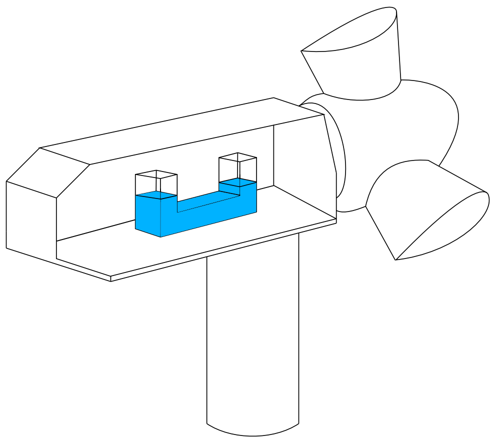

Figure 1: Possible tuned liquid column damper scheme applied in wind turbine.

In this paper, the aim is to simulate the structure response to a variability in the system parameters. It is common to find, in the literature, the excitation load modeled as a stochastic process even though the assumption that uncertainties in structures have negligible response can be unacceptable in real situations. For instance, the uncertainties can arise from many reasons such as inadequate modeling of boundary conditions, fabrication process, effect of nonstructural elements, degradation due to aging and temperature, fluctuations in structural mass, member capacities, yield strength, inertial moment, elasticity module, etc (Marano, Greco and Sgobba, 2010). Another major limitation of the deterministic approach is that uncertainties in the performance-related cannot be included in the damper parameter optimization since the damper efficiency can drastically reduce if the parameters are off-tuned to the vibration mode (Chakraborty and Roy, 2011). For that reason, the probabilistic approach offers a rational basis of accounting for both load and structural uncertainties in the design process.

To increase the credibility of the model, these uncertainties need to be modeled appropriately. The study of randomness associated with mechanical systems was introduced in the early 20th century. However, only the external loading was considered random leaving uncertainties related to the model unconsidered (Newland, 2012).

To quantify uncertainties in dynamic structures, random variables need to be associated with the system parameters along with their probability density function. Building the probability density function that best represents the physical problem is not trivial and requires experimental data to assist in its construction (Soize, 2001). One way around this problem is to associate the random variable a Gaussian probability density function. However this procedure is not always advisable as it may lead to physically incoherent results.

The Maximum Entropy Principle can be used to construct the probability density function of the random variable uncertainties of the model. The principle consist of using only the information available to build possible probability density functions (pdf) and from there, search for the function with maximum entropy (or uncertainty). This method avoids using misinformation in the construction of model ranging from the physics of the problem (Sampaio and Ritto, 2008). After defining the proper pdf's, a Monte Carlo simulation is made to describe the implications of this variability in the system.

The outline of the paper is as follows. After a brief review of the equation of motion of the deterministic system (Section 2), the outline of the proposed uncertainty model is presented in Section 3. Section 4 describes how the structural uncertainties are modeled using Monte Carlo simulation. Section 5 presents the case study and the results of the application of the proposed procedure. Finally, some concluding remarks are given in Section 6.

2 DETERMINISTIC MODEL

Consider the TLCD model mounted structure as sketched in Fig. 2. The idealization for the structure is acceptable because the support has negligible mass, thus, it is possible to approach the shear-frame system as a one degree of freedom model with stiffness and equivalent damping.

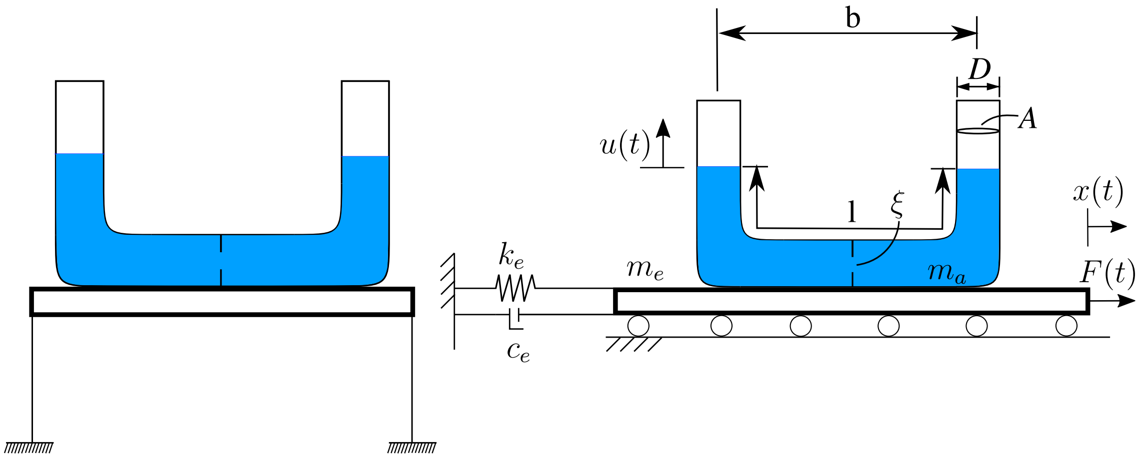

Figure 2: Schematic model of the system.

The equation describing the motion of the fluid in the TLCD is given by

$$\rho A l \ddot{u}(t) + \frac{1}{2}\rho A \xi |\dot{u}(t)| \dot{u}(t) + 2\rho A g u(t) = -\rho A b \ddot{x}(t), \tag{1}$$

where $u(t)$ is the displacement of fluid function, $x(t)$ is the displacement of the primary system function, ρ is the fluid density, ξ is the head loss coefficient, A is the cross section area of the column, b and l are the horizontal and total length of the column respectively and g is the gravity constant. It can be observed that the TLCD mass is given by $m_a = \rho A l$, the TLCD damping is $c_a = \frac{1}{2}\rho A \xi |\dot{u}(t)|$ and the TLCD stiffness is given by $k_a = \rho A g$. The natural frequency of oscillation in the column can be obtained by $\omega_a = \sqrt{2g/l}$.

The equation of motion of the primary structure is given by

$$(m_e + m_a)\ddot{x}(t) + \rho A b \ddot{u}(t) + c_e \dot{x}(t) + k_e x(t) = F(t), \tag{2}$$

where the parameter m_e is the structure mass, k_e the structure stiffness, c_e the structure damping and $F(t)$ the external force. Thus, combining Eq. (1) and Eq. (2). The equation of motion in matrix form can be written as

$$\begin{bmatrix} m_e + m_a & \alpha m_a \\ \alpha m_a & m_a \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{u} \end{Bmatrix} + \begin{bmatrix} c_e & 0 \\ 0 & c_a \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{u} \end{Bmatrix} + \begin{bmatrix} k_e & 0 \\ 0 & k_a \end{bmatrix} \begin{Bmatrix} x \\ u \end{Bmatrix} = \begin{Bmatrix} F(t) \\ 0 \end{Bmatrix} \qquad |u| \leqslant \frac{l-b}{2}, \tag{3}$$

where $\alpha = b/l$ is the dimensionless length ratio. The condition presented by Eq. (3) is needed to ensure that the liquid in the column do not spill water and consequently change its damping characteristic. Eq. (3) can also be written with the mass matrix in its dimensionless form, given by

$$\begin{bmatrix} 1+\mu & \alpha\mu \\ \alpha & 1 \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{u} \end{Bmatrix} + \begin{bmatrix} 2\omega_e\zeta_e & 0 \\ 0 & \frac{\xi|\dot{u}|}{2l} \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{u} \end{Bmatrix} + \begin{bmatrix} \omega_e^2 & 0 \\ 0 & \omega_a^2 \end{bmatrix} \begin{Bmatrix} x \\ u \end{Bmatrix} = \begin{Bmatrix} \frac{F(t)}{m_e} \\ 0 \end{Bmatrix}, \tag{4}$$

where ζ_e and ω_e are the damping ratio and natural frequency of the structure, respectively. The dimensionless parameters mass ratio μ and tuning ratio γ are defined as

$$\mu = \frac{m_a}{m_e}; \qquad \gamma = \frac{\omega_a}{\omega_e}. \tag{5}$$

The nonlinear nature of the damping requires the determination of a equivalent value to the damping coefficient. Roberts and Spanos (2003) proposed a procedure to estimate the optimum value of the damping coefficient utilizing the statistical linearization method. It is possible to express the error between the nonlinear system with the equivalent linear system as $\epsilon = (1/2)\rho A \xi |\dot{u}|\dot{u} - c_{eq}\dot{u}$, where the value of the equivalent damping c_{eq} can be obtained by minimizing the standard deviation of the error value, namely $E\{\epsilon^2\}$. Assuming that the liquid velocity has Gaussian form due to wind random excitation, the expression for the equivalent damping is given as (Roberts and Spanos, 2003)

$$c_{eq} = \sqrt{\frac{2}{\pi}}\rho A \xi \sigma_{\dot{u}} = 2\omega_a\zeta_a, \tag{6}$$

where $\sigma_{\dot{u}}$ is the standard deviation of the fluid velocity. Therefore, the equivalent damping approached by statistical linearization c_{eq} can replace the nonlinear value c_a in Eq. (3). This method needs an iterative procedure since the value of $\sigma_{\dot{u}}$ is not known. Furthermore, minimizing the mean square response does not necessarily correspond to the optimal design in terms of reliability.

Another possible strategy to find an equivalent damping is by expressing the damping as a function of the absorber damping ratio ζ_a and then apply a optimization method as described in Yalla and Kareem (2000). This method is preferred over the mean square response since it does not rely in iterative method and produces good results.

The equation of motion can be written in the Fourier domain and one can obtain the the frequency response function (FRF) for the two degree of freedom as follows,

$$\hat{H}(i\omega) = \frac{\hat{x}}{\hat{F}}; \qquad \hat{G}(i\omega) = \frac{\hat{u}}{\hat{F}}, \tag{7}$$

where ω stands for the driving frequency, $\hat{x}$ denotes the Fourier transform of x, $\hat{u}$ denotes the Fourier transform of u and $\hat{F}$ denotes the Fourier transform of F, then it follows

$$\hat{H}(i\omega) = \frac{-\Delta\mu\alpha(i\omega)^2 + (i\omega)^2 + \zeta_a\omega_a(i\omega) + \omega_a^2}{((i\omega)^2(1+\mu) + 2\zeta_e\omega_e(i\omega) + \omega_e^2)((i\omega)^2 + 2\zeta_a\omega_a(i\omega) + \omega_a)^2 - (i\omega)^4\alpha^2\mu}, \tag{8}$$

$$\hat{G}(i\omega) = \frac{-\alpha(i\omega)^2 + \Delta}{((i\omega)^2(1+\mu) + 2\zeta_e\omega_e(i\omega) + \omega_e^2)((i\omega)^2 + 2\zeta_a\omega_a(i\omega) + \omega_a)^2) - (i\omega)^4\alpha^2\mu}, \tag{9}$$

where Δ indicates the reference in the analysis of the system, when $\Delta = 1$, one has base excitation and x is a relative displacement. When $\Delta = 0$ one has excitation in the primary system and x is the absolute displacement. Eq. (8) is a function of frequency and it depends on the parameters of the system such as the absorber damping ratio ζ_a and the structure natural frequency, hence the structure stiffness k_e. In the next section, we will concentrate in study the uncertainties in these two parameters.

3 UNCERTAINTY MODEL

Uncertainty analysis is important in order to describe how the system parameters may impact the device performance and improve design reliability considering the optimum damping and its variability. The probabilistic parameters are assumed to be the viscous damping coefficient ζ_a and the stiffness of the structure k_e. The masses are assumed to be deterministic.

First, the probability distribution function (pdf) will be constructed using the Maximum Entropy principle (Soize, 2001). By relying only on the information available, it is possible to obtain the optimum probabilistic model using the one with maximum entropy (uncertainty).

The parameters considered as uncertain are the TLCD damping ratio ζ_a and the structure stiffness k_e. The random variable Z is associated to the damping ratio and K for the structure stiffness. A underline bar is used to represent the mean value of these parameters. The procedure to find the pdf is similar for both parameters, for that reason, the following analysis will show only the pdf construction of the Z parameter.

The basic available information are the mean reduced model, the positive-definiteness of the random variable and the existence of second-order moments, in other words: (1) the support of the probability density function is $]0, +\infty[$, (2) the mean value is assumed to be known, $E\{Z\} = \underline{Z}$ and (3) the condition $E\{\ln(Z)\} < +\infty$, which implies that zero is a repulsive value (Cataldo, Bellizzi and Sampaio, 2010). The probability density function p_Z has to verify the following constraint equations (Kapur and Kesavan, 1992)

$$\int_{-\infty}^{+\infty} p_Z(z)dz = 1, \tag{10}$$

$$\int_{-\infty}^{+\infty} zp_Z(z)dz = \underline{Z}, \tag{11}$$

$$\int_{-\infty}^{+\infty} \ln(Z)p_Z(z)dz < +\infty, \tag{12}$$

applying the Maximum Entropy Principle yields the following probability density function (Cataldo, Bellizzi and Sampaio, 2010)

$$p_Z(z) = \mathbf{1}_{]0,+\infty[}(z)\frac{1}{\underline{Z}}\left(\frac{1}{\delta_Z^2}\right)^{\frac{1}{\delta_Z^2}}\frac{1}{\Gamma(1/\delta_Z^2)}\left(\frac{z}{\underline{Z}}\right)^{\frac{1}{\delta_Z^2}-1} e^{\frac{-z}{\delta_Z^2\underline{Z}}}, \tag{13}$$

where $\delta_Z = \sigma_Z/\underline{Z}$ is the coefficient of dispersion of the random variable Z and σ_Z is the standard deviation of Z such that $0 \leq \delta_Z \leq 1/\sqrt{2}$. It can be verified that Z is a second-order random variable and that $E\{1/Z^2\} < +\infty$ (Soize, 2001). The Gamma function is defined as

$$\Gamma(1/\delta_Z^2) = \int_0^{+\infty} t^{1/\delta_Z^2-1}e^{-t}dt, \qquad 1/\delta_Z^2 > 0. \tag{14}$$

The pdf for the structural stiffness follow the same procedure and is given by

$$p_K(k) = \mathbf{1}_{]0,+\infty[}(k)\frac{1}{\underline{K}}\left(\frac{1}{\delta_K^2}\right)^{\frac{1}{\delta_K^2}}\frac{1}{\Gamma(1/\delta_K^2)}\left(\frac{k}{\underline{K}}\right)^{\frac{1}{\delta_K^2}-1} e^{\frac{-k}{\delta_K^2\underline{K}}}, \tag{15}$$

since the values of coefficient of dispersion are not known for both parameters, the following sections will show results for a variation of this parameter. From the constructed pdf's, we can now perform a Monte Carlo simulation.

4 MONTE CARLO SAMPLING

The Monte Carlo method is a class of computational techniques based on synthetic generation of random variables in order to deduce the implications for the probability distribution. In probabilistic simulations, we must ensure that the probability density function of the random variable has significant physical meaning.

Simulation convergence criterion is given by (Sampaio and Ritto, 2008)

$$conv(n_s) = \frac{1}{n_s}\sum_{j=1}^{n_s}\int_B \|H_j(\theta,\omega) - \underline{\hat{H}}(\omega)\|^2 d\omega, \tag{16}$$

where $\|H_j(\theta,\omega)\|$ is the stochastic system response in the frequency domain calculated for the θ realization, $\|\underline{\hat{H}}(\omega)\|$ is the mean stochastic system response.

The deterministic model is obtained by using the mean value of damping ratio $\underline{Z}$. The value of $\underline{Z}$ is determined by a optimization method developed by Yalla and Kareem (2000) which, for a white-noise excitation and considering undamped primary system, it can be expressed as

$$\underline{Z} = \frac{\alpha}{2}\sqrt{\frac{2\mu\left(\alpha^2\frac{\mu}{4} - \mu - 1\right)}{(\alpha^2\mu^2 + \alpha^2\mu - 4\mu - 2\mu^2 - 2)}}, \tag{17}$$

where for $\alpha = 0.9$ and $\mu = 0.05$ it follows that $\underline{Z} = 0.0965$.

The mean values of the structure stiffness $\underline{K}$ is obtained by a simplified model of cantilever beam (Murtagh, Basu and Broderick, 2004)

$$\underline{K} = \frac{\pi^4}{32L}EI \tag{18}$$

where L is the beam length and EI is the flexural stiffness. Using $L = 60\ m$, $3\ m$ width and $0.015\ m$ thickness, $E = 2.1$ x $10^{11}\ N/m^2$, density of the steel $\rho = 7,850\ kg/m^3$ one can find $\underline{K} = 463,671\ N/m$. The rotor mass is $M = 19,876\ kg$. Using the dimensionless parameter length ratio $\alpha = 0.9$ and $\nu = 0.1$, it follows that $\omega_e = 3.6450\ rad/s$ and $\zeta_e = 0.0018$ (Avila et al., 2009).

5 RESULTS

Two cases are studied in this section, in the first case, uncertainties are considered only in the damping ratio parameter. In the second case, uncertainties are also included in the structural stiffness parameter. In both cases, we are interested in construct the frequency response function of structure $H(\omega)$ from Eq. (8) for different coefficients of dispersion using Monte Carlo (MC) simulation.

Figure 3 shows, for different values of δ_Z, the mean model, the mean response of the stochastic model and the boundary lines representing the confidence region of 95%, which means that the response is inside the envelope with probability 95%. The statistics of the response were calculated using 3,000 MC samples.

It can be noticed from Fig. 3 that the mean value of all realizations does not coincide with the deterministic value except for the first case, $\delta_Z = .2$, in Fig. 3 (a), in which they are very similar. The uncertainty is only predominant near the resonance and anti-resonance region. As the value of δ_Z increases, the uncertainty also increases in the two peaks and in the region in between the peaks. This shows that the uncertainty in the damping parameter interfere in the amplitude of displacement of the primary structure not changing the resonance frequencies, as expected, significantly affecting the performance of the damper, by changing the amplitude values at design frequency.

The convergence rate for all dispersion coefficients is shown in Fig. 4, occurring way bellow the 3,000 Monte Carlo samples.

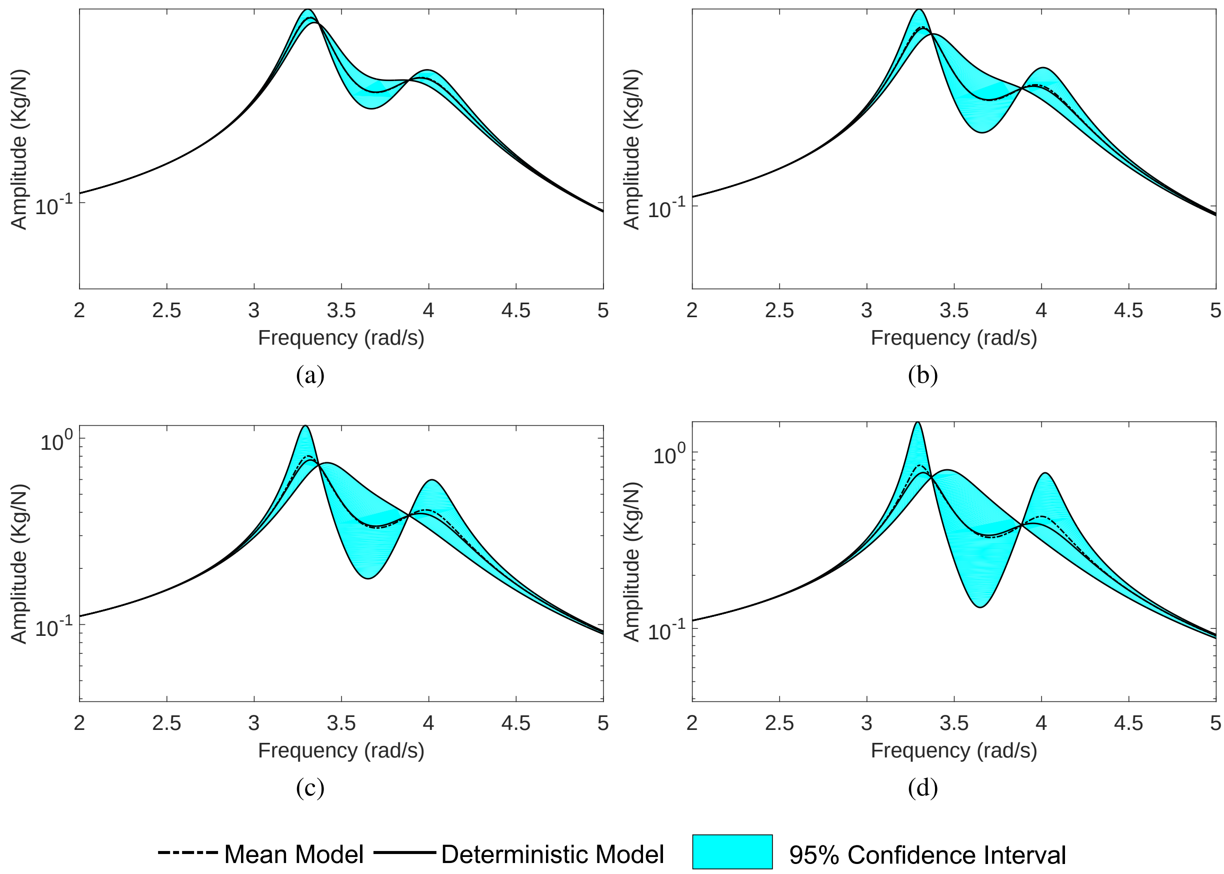

Figure 3: FRFs with damping ratio uncertainties of the deterministic model, mean response of the stochastic model, and 95% confidence region for different values of δ_Z's: (a) .2 (b) .4 (c) .6 (d) .7.

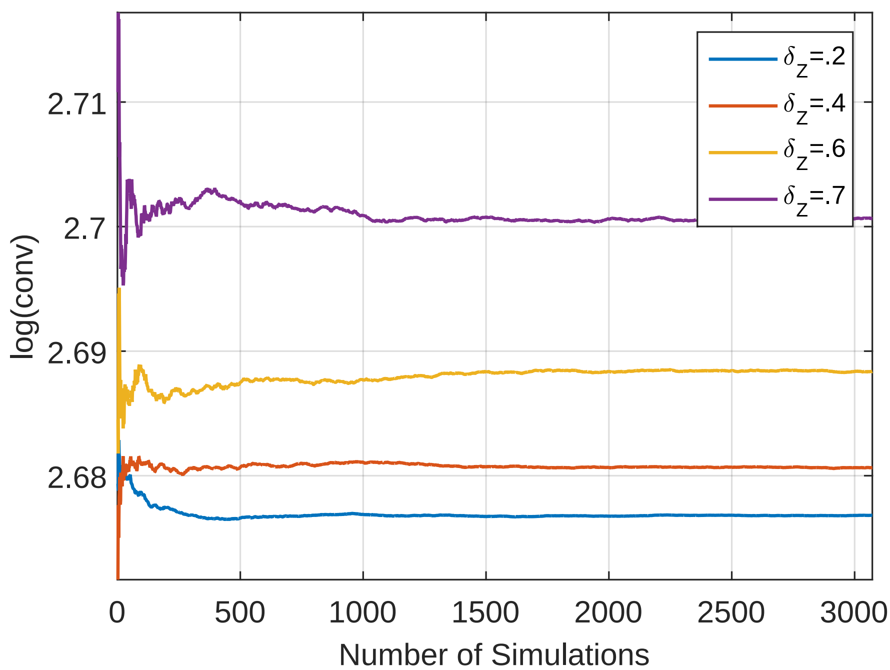

Figure 4: Mean square convergence for different values of δ_Z.

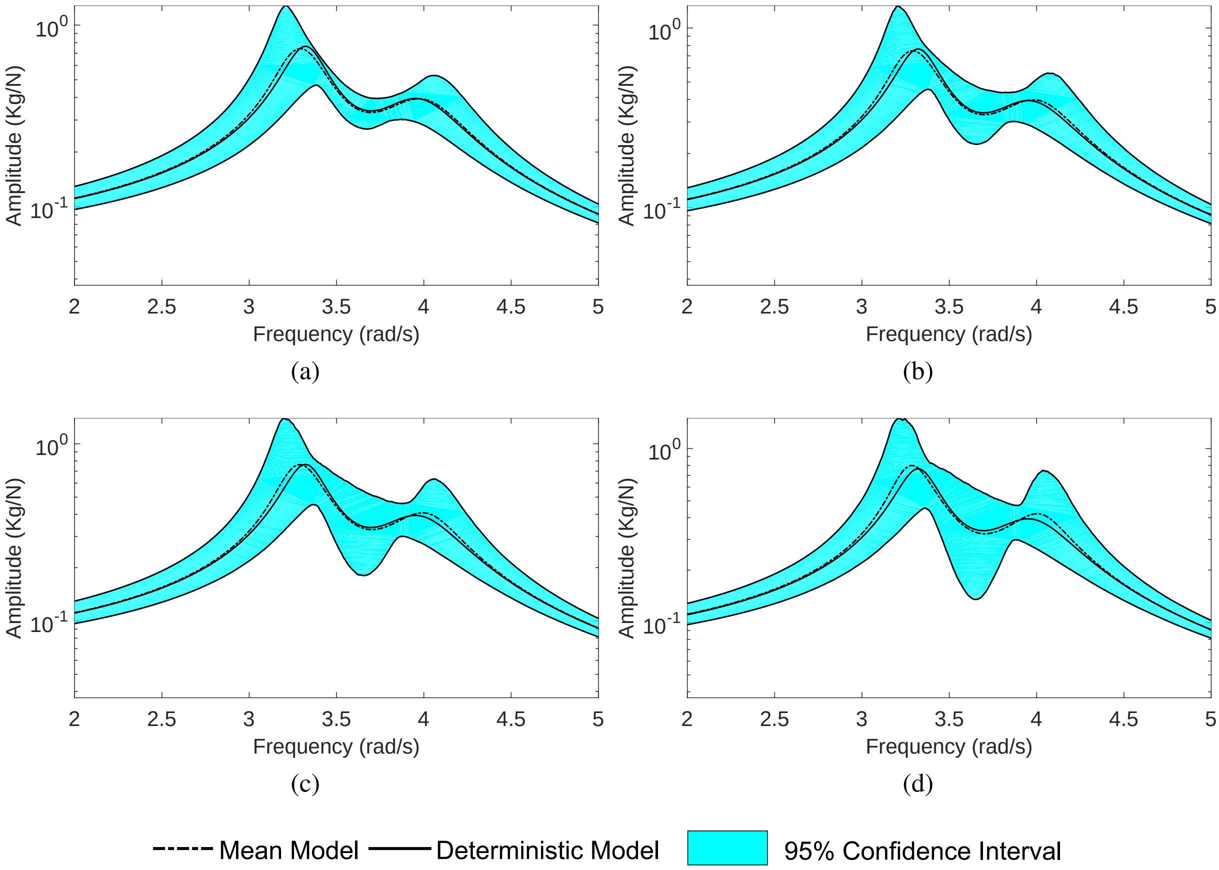

Figure 5: FRFs with damping ratio uncertainties of the deterministic model, mean response of the stochastic model, and 95% confidence region for different values of δ_Z and δ_K: (a) .2 & .05 (b) .4 & .15 (c) .6 & .25 (d) .7 & .35.

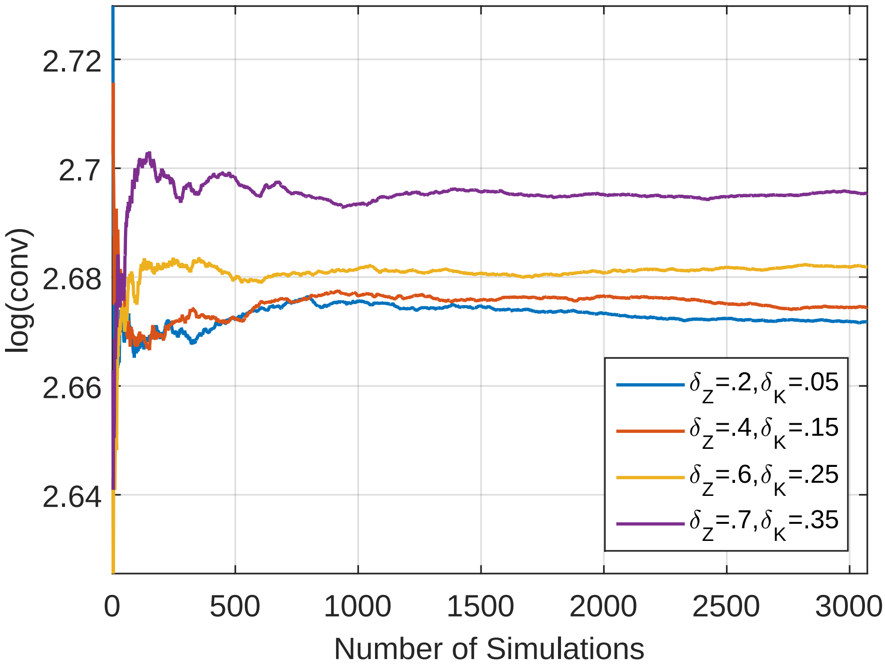

Figure 6: Mean square convergence for different values of δ.

For the second case, the uncertainty in the structure stiffness parameter is included in the model. The values of coefficients of dispersion have a small variation which means that the random variable associated to the structural stiffness has a small standard deviation since the mean value is fixed.

Figure 5 shows, for different values of δ_Z and δ_K, the mean model, the mean response of the stochastic model and the boundary lines representing the confidence region of 95%. The statistics of the response were calculated using 3,000 MC samples.

From Fig.5, it is clear that uncertainty in the primary-system stiffness is much more significant than uncertainty in the damping ratio parameter, since the damping ratio uncertainty only changes the FRF amplitudes and in this case we have uncertainties occurring in all frequencies. The uncertainties in the primary-system stiffness comes from, generally, the reduction in model to a 1 degree of freedom model. When the value of the coefficients of dispersion increases, the response limits becomes to wide to give any satisfactory insight in the dynamic of the system. Furthermore, since the magnitude of the structural stiffness is big, a large variation would result in unsatisfactory results and the simulation would not converge. For that reason, it is important to keep the dispersion of this parameter as small as possible.

The convergence rate for all dispersion coefficients is shown in Fig. 6, occurring way bellow the 3,000 Monte Carlo samples.

6 CONCLUDING REMARKS

In this paper, we investigated parameters uncertainties in a TLCD applied in wind turbines. The assumption that uncertainties in structures have negligible response can be unacceptable in real situations and beside that, the uncertainties in the performance-related cannot be included in the damper parameter optimization. For that reason, to increase the credibility of the model, these uncertainties were included to help describe the range of potential outputs of the system at some probability level and estimating the relative impacts of input variable uncertainties.

The method consisted of inserting uncertainties in the absorber damping ratio and the structural stiffness element, constructed the probabilistic model from the Maximum Entropy principle and then, performed a Monte Carlo simulation. Two cases were studied in this paper, the first only considering uncertainties in the absorber damping ratio and the second case considering both uncertainties in the absorber damping ratio and the structural stiffness. The results showed that the uncertainties can indeed interfere in the TLCD performance since it change the FRF amplitude considerably in both cases and that uncertainty in the primary-system stiffness is relatively more significant than uncertainty in the damping ratio parameter although the last one interferes in the design performance.

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