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STATIONARY DYNAMIC RESPONSE OF A CIRCULAR RIGID FOUNDATION PARTIALLY SUPPORTED BY A FLEXIBLE PILE AND INTERACTING WITH A TRANSVERSELY ISOTROPIC SOIL

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Abstract. *This paper investigates the vertical response of rigid circular foundation resting on the surface of a three-dimensional, transversely isotropic soil. The foundation interacts with the soil by two mechanisms. First, the foundation is partially supported by a flexible pile embedded in soil, in which case there are no stresses at the soil-foundation interface. In the second mechanism, the foundation is partially supported by the soil through soil-foundation interface stress. This article studies the change in the foundation dynamic response when supported by these two distinct mechanisms. The present analysis is based on two previously solved problems, namely the interaction of a rigid foundation with a transversely isotropic soil (Labaki, Mesquita and Rajapakse, 2014) and the response of a flexible pile embedded in the transversely isotropic soil (Labaki, Mesquita and Rajapakse, 2015). The solutions to these two problems are computationally very demanding. The numerical studies reported in this article investigate the role of the mass-to-pile inertia ratio as well as the relative stiffness of the soil and the pile. An external vertical force acting on the foundation is considered as the excitation mechanism. The results are expressed in terms of vertical displacements of the foundation for distinct sets of parameters.*

Keywords: *Dynamic Soil-structure Interaction, Foundations Dynamics, Pile Dynamics, Green's Function.*

1 INTRODUCTION

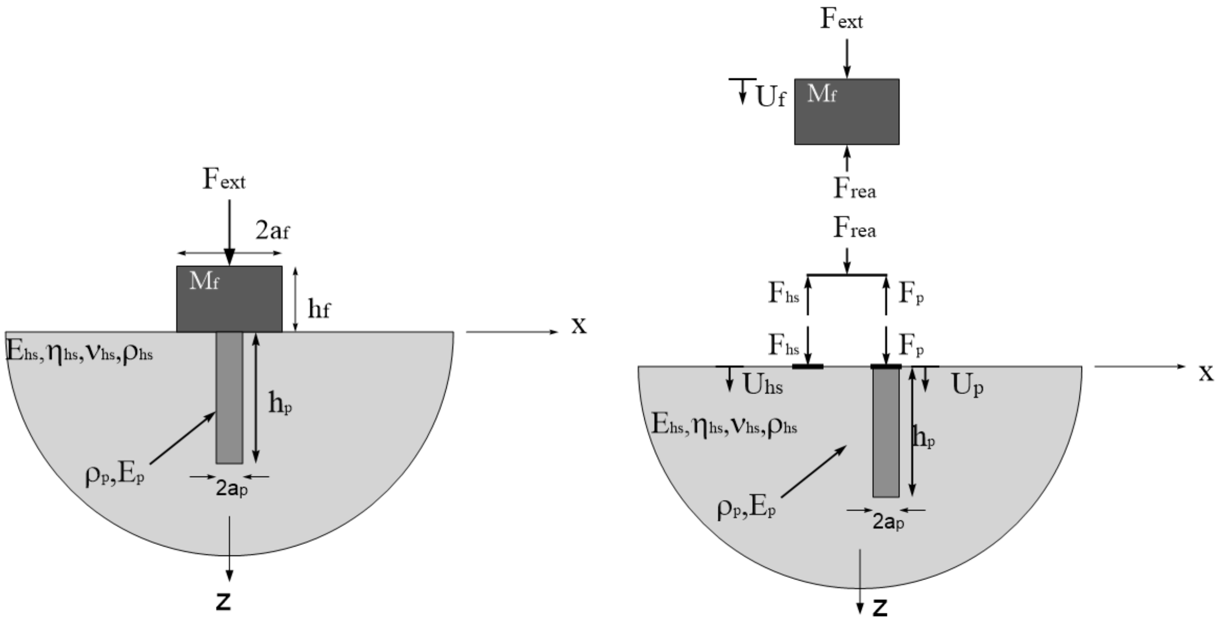
In recent years, a number of studies have dealt with the problem of the dynamic response of circular plates resting on the soil using different types of numerical methods. Some of these studies were motivated by practical applications such as vibration control of nano-facilities and nuclear powerplants. The case of a plate resting on the surface of a transversely isotropic soil is of special interest to the analysis of vibration-sensitive foundations such as synchrotron light sources (Labaki, Mesquita and Rajapakse, 2014).

If the bearing capacity of the soil is inadequate for the structural load of the foundation, the introduction of a pile or a group of piles may increase the soil bearing capacity. The idea of using piles to support foundations and others structures on the soil is to transfer the load to the bearing ground with greater bearing capacity (Nazir and Azzam, 2010). One issue to be investigated is whether this increase in bearing capacity through the use of piles is followed by a corresponding increase in the suppression of vibration levels. It is known that the introduction of piles in soil stratum makes the soil system stiffer, and consequently, the soil vibration response is considerably affected (Rangwala et al., 2012).

The present paper introduces a method to study the coupled response of a surface foundation partially supported by the pile and partially supported by the soil. The method of coupling requires the determination of displacements and forces acting at the interface between the plate and the pile due to an external excitation. The displacement and force responses of the plate and pile are determineded and must satisfy the criteria of kinematic compatibility and equilibrium of forces at the interface. The paper investigates the response of the foundation supported only by the soil as well as the response of the foundation supported only by the pile. A strategy is introduced, which allows intermediate cases to be studied, in which the foundation is partially supported by the soil and partially supported by the pile. The section of numerical results considers different inertia of foundation, different geometry and relative stiffness of the pile as well as different constitutive properties of the soil.

2 STATEMENT OF PROBLEM

Consider the problem of dynamic soil-foundation-pile interaction under vertical load, F_{ext}, shown in Fig. 1. The index ‘hs’ is related to the soil, modeled here as a homogeneous, transversely isotropic half-space, with Young’s modulus E_{hs}, Poisson’s ratio ν_{hs}, mass density ρ_{hs} and material damping coefficient η_{hs}. The index ‘p’ is related to the pile, with Young’s modulus E_p, density ρ_p, radius a_p, and length h_p. The index ‘f’ is related to the foundation, with properties radius a_f, length h_f, Poisson’s ratio ν_f and mass density ρ_f. The coordinate system is placed so that the x-y plane is aligned with the surface of the soil, and the pile is aligned along the z-axis. The center of the plate coincides with the origin of the coordinate system (Fig. 1a).



(a) **(b)**

Figure 1. (a) Foundation interacting with soil and pile and (b) equilibrium of forces.

The foundation interacts with the underlying soil and the pile and it is only studied in the vertical (z) direction. In others words, this is a one-dimensional problem. The foundation mass is considered to be concentrated at the center of the soil-pile-foundation interface. The foundation is subjected to an external force.

In one particular case, the foundation is considered to be perfectly bonded to the soil surface. In another case, the foundation is only supported by the pile. In the second case, all the load transfer from the external force to the soil occurs throught the pile. This paper aims to investigate how the continuous change from one support mechanism to the other influences the foundation dynamic response. In each configuration, different amounts of the external force are directly transferred to the soil, through the soil-foundation interface, or from the foundation throught the pile to the soil.

Figure 1b shows the forces acting on the foundation structure. The reaction force ($\vec{F}_{rea}$), as shown in Eq. (1), is partially caused by the pile resistance and partially generated by the forces at the foundation-soil interface:

$$\vec{F}_{rea} = \vec{F}_{hs} + \vec{F}_{p} \quad (1)$$

The total reaction force is a composition that varies continuously the soil force F_{hs} to pile force F_p.

As the soil displacement U_{hs} and the pile displacement U_p has to be equal, it is possible to vary the soil force F_{hs} and the pile force F_p.

The total soil-pile reaction force acting on the foundation F_{rea}, is given by Eq. (2):

$$\vec{F}_{rea} = \alpha \vec{F}_{p} + (1-\alpha)\vec{F}_{hs} \quad (2)$$

in which

$$0 \leq \alpha \leq 1 \tag{3}$$

When α=1, then force acting on the foundation is solely due to the soil reaction, F_{hs}. This corresponds to the case in which the plate is completely supported by the soil. When α=0, the external force is balanced solely by the pile reaction. This corresponds to the case in which the plate is completely supported by the pile.

3 FORMULATION

The models of soil-foundation interaction in this paper require the derivation of Green's functions corresponding to vertical loads applied on the surface or within the half-space.

Consider an elastic, transversely isotropic, three-dimensional full-space. The problem is governed by the Cauchy-Navier differential equations, that are described in terms of the displacement components $u_i=u_i(r,\theta,z,\omega)$ (i=r,θ,z). Rajapakse and Wang (1993) proposed a solution for this coupled problem in terms of Hankel integral transforms and series expansion. The solution is written in terms of arbitrary functions, the values of which are determined from the boundary and continuity conditions of a given problem.

3.1 Soil-Foundation interaction

This section investigates the soil-foundation interaction system, in which the foundation is completely supported by the soil (Fig. 2a).

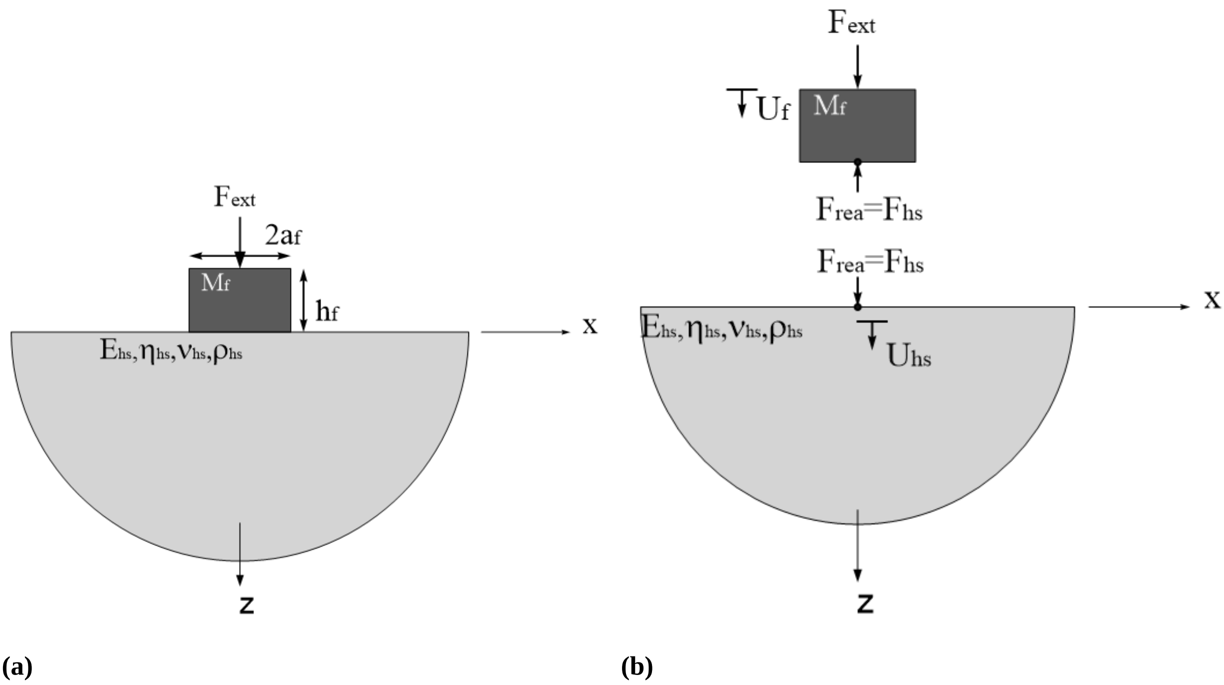


Figure 2. (a) Foundation interacting with soil and (b) equilibrium of forces.

The boundary and continuity conditions corresponding to the case of a rigid circular plate resting on the surface of the half-space were presented by Labaki, Mesquita and Rajapakse (2014). In that paper, the model of rigid plate is obtained by discretizing the plate in a number of concentric annular disc elements. It is then imposed that all disc elements are displaced vertically by the same amount. The contact tractions at the plate-soil interface are determined upon the solution of the flexibility equation arising from this kinematic constraint. In the present paper, the vertical displacement of the rigid plate resting on the surface of the half-space, due to a unit load, is the dynamic compliance S_{zzhs}.

Figure 3 shows an example of plate response obtained by Labaki, Mesquita and Rajapakse (2014). This case considers a plate with ν_f =0.3, a_f=1m, ρ_f =10kg/m³ and h_f=0.01m, and a half-space with E_{hs}=2.5Pa, ν_{hs}=0.25, ρ_{hs}=1kg/m³ and η_{hs}=0.01. The plate is under a uniformly distributed unit load. Normalized frequency a_0, is defined as $a_0=a_f\omega(\rho_{hs}/G_{hs})^{1/2}$.

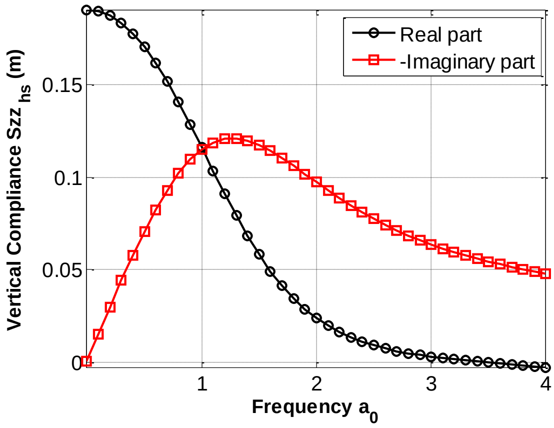


Figure 3. Response of a rigid foundation interacting with the soil

Figure 2b shows the forces acting on the structure. The equilibrium equation of the structure, as shown in Eq. (4), is:

$$\vec{F}_{ext} - \vec{F}_{hs} = -\omega^2 M_f U_f \tag{4}$$

The soil response to external excitation is determined through a flexibility matrix, S_{zzhs}, as shown in Eq. (5),

$$U_{hs} = S_{zzhs} \vec{F}_{hs} \tag{5}$$

In this case, F_{rea} is equal to F_{hs}.

The only force that acts on the surface is F_{hs}, which is responsible for the soil displacement U_{hs}.

Substituting Eq. (4) into equation Eq. (5), it is possible to find an equation that relates the displacement of soil and foundation, as shown in Eq. (6):

$$U_{hs} = \omega^2 M_f U_f S_{zzhs} + S_{zzhs} \vec{F}_{ext} \tag{6}$$

In order to satisfy the criteria of kinematic compatibility, the displacement of the soil must be equal to the displacement of the rigid foundation, as shown in Eq. (7):

$$U_{hs} = U_f \tag{7}$$

Applying Eq. (7) into Eq. (6) and isolating U_f yields:

$$U_f = \left(\frac{S_{zzhs} \vec{F}_{ext}}{1 - S_{zzhs} \omega^2 M_f} \right) \tag{8}$$

Equation (8) gives the displacement of the rigid foundation that is perfectly bonded with the soil excited by an external force F_{ext}.

3.2 Pile-foundation interaction

This section considers the case of soil-pile-foundation interaction. In this case, the external load is transferred to the soil through the pile. The foundation is not in contact with the soil (Fig. 4a).

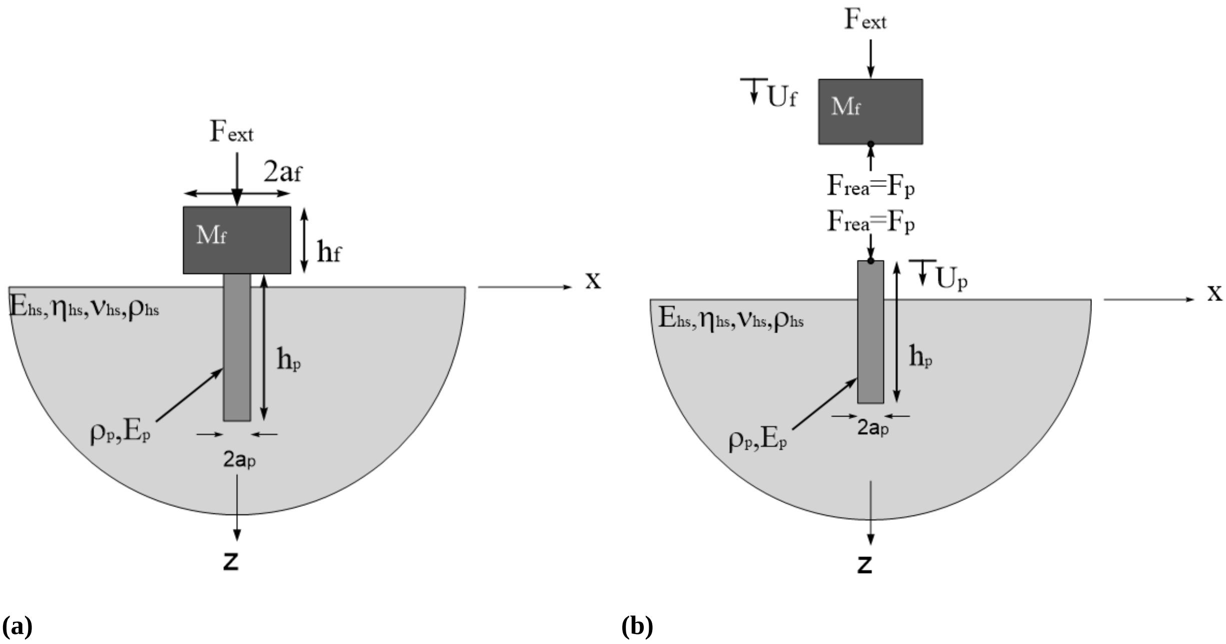


Figure 4. (a) Foundation interacting with pile and (b) equilibrium of forces

The boundary conditions for the modeling of the pile problem are defined as follows. Consider a half-space containing a buried vertical load of unit intensity. The load is uniformly distributed on a finite cylindrical shell within the half-space. A solution for the pile problem was presented by Labaki, Mesquita and Rajapakse (2015), in which the displacement of the pile is described by an Ansatz function in terms of generalized coordinates. The body forces in the half-space corresponding to each term of the Ansatz function are obtained numerically by solving a flexibility equations based on the Green's functions corresponding to the buried forces described above. The generalized coordinates in the Ansatz function are obtained from a Lagrange's equations of motion for the pile. In this paper, the displacement of the top end of the buried pile is the dynamic compliance S_{zzp}.

Figure 5 shows an example of the pile response obtained by Labaki, Mesquita and Rajapakse (2015). This case considers a half-space with E_{hs}=2.5Pa, ν_{hs}=0.25, ρ_{hs}=1kg/m³ and

η_{hs}=0.01, and E_p/E_{hs}=10, ρ_p/ρ_{hs}=1 and h_p/a_p=5. The pile is under a vertical load of unit intensity.

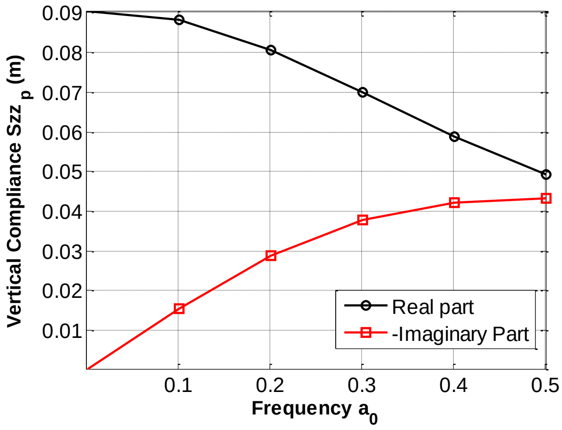


Figure 5. Response of the embedded pile

Figure 4b shows the forces acting on the structure. The equilibrium equation of the structure, as shown in Eq. (9), is:

$$\vec{F}_{ext} - \vec{F}_p = -\omega^2 M_f U_f \tag{9}$$

The pile response to external excitation is determined through a flexibility matrix, S_{zzp}, as shown in Eq. (10)

$$U_p = S_{zzp}\vec{F}_p \tag{10}$$

In this case, F_{rea} is equal to F_p.

The only force that acts on the interface is F_p, which is responsible for pile displacement U_p.

By replacing Eq. (9) into equation Eq. (10), it is possible finds an equation that relates the displacement of pile and foundation, as shown in Eq. (11):

$$U_p = \omega^2 M_f U_f S_{zzp} + S_{zzp}\vec{F}_{ext} \tag{11}$$

In order to satisfy the criteria of kinematic compatibility, the displacement of the soil must be equal to the displacement of the foundation, as shown in Eq (12):

$$U_p = U_f \tag{12}$$

Applying the Eq. (12) into Eq. (11) and isolating U_f yields:

$$U_f = \left(\frac{S_{zzp}\vec{F}_{ext}}{1 - S_{zzp}\omega^2 M_f} \right) \tag{13}$$

Equation (13) gives the displacement of the foundation that is supported entirely by a pile embedded in the soil.

Notice that Eq. (13) is similar to Eq. (8), in one equation flexibility matrix corresponds to the foundation and another corresponds to the pile.

3.3 Soil-pile-foundation interaction

In this section, the soil-pile-foundation system is investigated. In this case, the foundation interacts partially with the soil and partially with the pile, as shown in Fig. 1a.

Figure 1b shows the forces acting on the structure. The equilibrium equation of the structure, as shown in Eq. (14), is:

$$\vec{F}_{ext} - \vec{F}_{rea} = -\omega^2 M_f U_f \tag{14}$$

The reaction force F_{rea} is the sum of the soil force F_{hs} and the pile force F_p, as shown in Eq. (2).

By replacing Eq. (2) into equation Eq. (14), one finds an equation that relates the displacement of the foundation with the forces acting on the pile and the soil, as shown in Eq. (15):

$$\vec{F}_{ext} + \omega^2 M_f U_f = \alpha \vec{F}_p + (1-\alpha)\vec{F}_{hs} \tag{15}$$

Substituting Eqs. (5) and (10) into Eq. (15), it is possible finds an equation that relates the displacement of the foundation, the pile and the soil, as shown in Eq. (16):

$$\vec{F}_{ext} + \omega^2 M_f U_f = \alpha\left(U_p S_{zzp}^{-1}\right) + (1-\alpha)\left(U_{hs} S_{zzhs}^{-1}\right) \tag{16}$$

This system must satisfy the criteria of kinematic compatibility, which states that the displacement of the foundation must be equal to the displacement of the pile and to the displacement to the soil, as shown in Eq. (17):

$$U_f = U_p = U_{hs} \tag{17}$$

Applying Eq. (17) into Eq. (16) and isolating U_f yields

$$U_f = \frac{\vec{F}_{ext}}{\left[\alpha S_{zzp}^{-1} + (1-\alpha) S_{zzhs}^{-1} - \omega^2 M_f\right]} \tag{18}$$

Equation (18) gives the foundation response in which the plate is partially supported by the pile and the soil.

4 NUMERICAL RESULTS AND DISCUSSION

This section presents numerical results to analyze the displacement of the foundation shown in Fig. 1a and study the influence of α (Eq. 18). For the purpose of presentation of numerical results, the following normalizations are defined: ratio of modulus of elasticity $E' = E_p/E_{hs}$ and mass ratio $B = m_f/m_{hs}$, in which m_f is the mass of the foundation and m_{hs} is the mass of the soil. The soil mass, m_{hs}, is defined as the mass comprised by a volume formed by the area of the soil-foundation interface possessing a unit depth.

The soil-pile-foundation system is subjected to an axial load F_{ext}=1N. In this section, the following parameters are considered: for the half-space, E_{hs}=2.5Pa, η_{hs}=0.01, υ_{hs}=0.25, ρ_{hs}=1kg/m³; for the foundation, a_f=1m, and for the pile: h_p/a_p=35, ρ_p=1kg/m³. The proposed formulation in which the foundation interacts only with the soil (Fig. 2a), and the case in which the foundation interacts only with the pile (Fig. 3a), have been compared with a solution by Labaki, Mesquita and Rajapakse (2014) and Labaki, Mesquita and Rajapakse (2015), respectively.

4.1 Soil-foundation interaction (α=0): validation

In Fig. 6, the real and imaginary parts of the foundation displacement are shown for B=0 (massless foundation). Figure 7 shows the corresponding results for the case in which B=50.

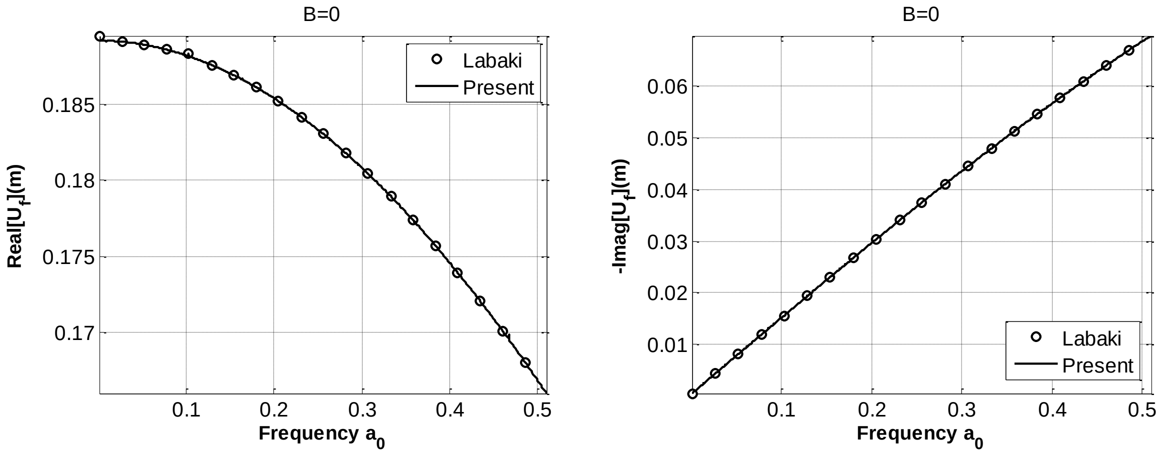


Figure 6. Foundation displacement for B=0, Soil-only support.

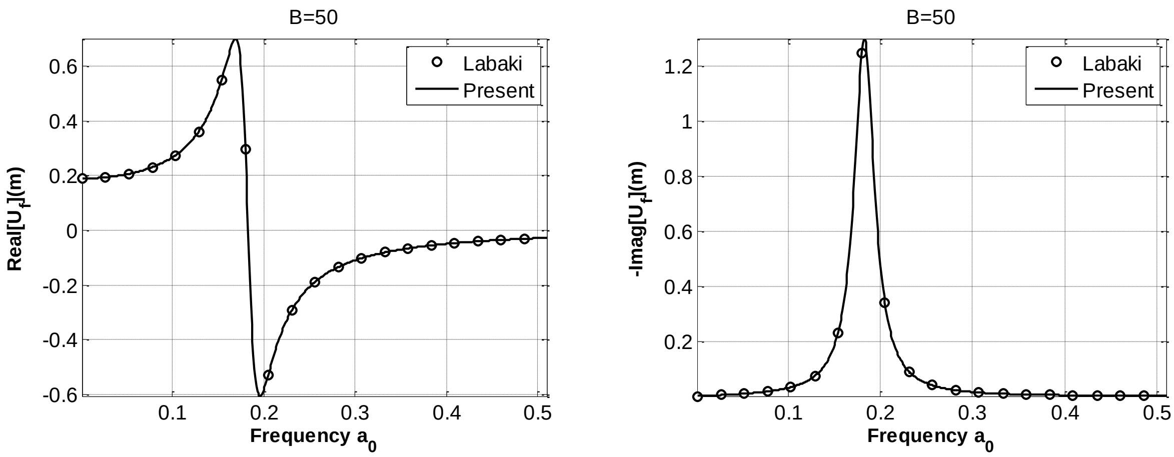


Figure 7. Foundation displacement for B=50, Soil-only support.

4.2 Pile-foundation interaction (α=1): validation

Figures 8 and 9 show the real and imaginary part of the foundation displacement for B=0 and varying E'.

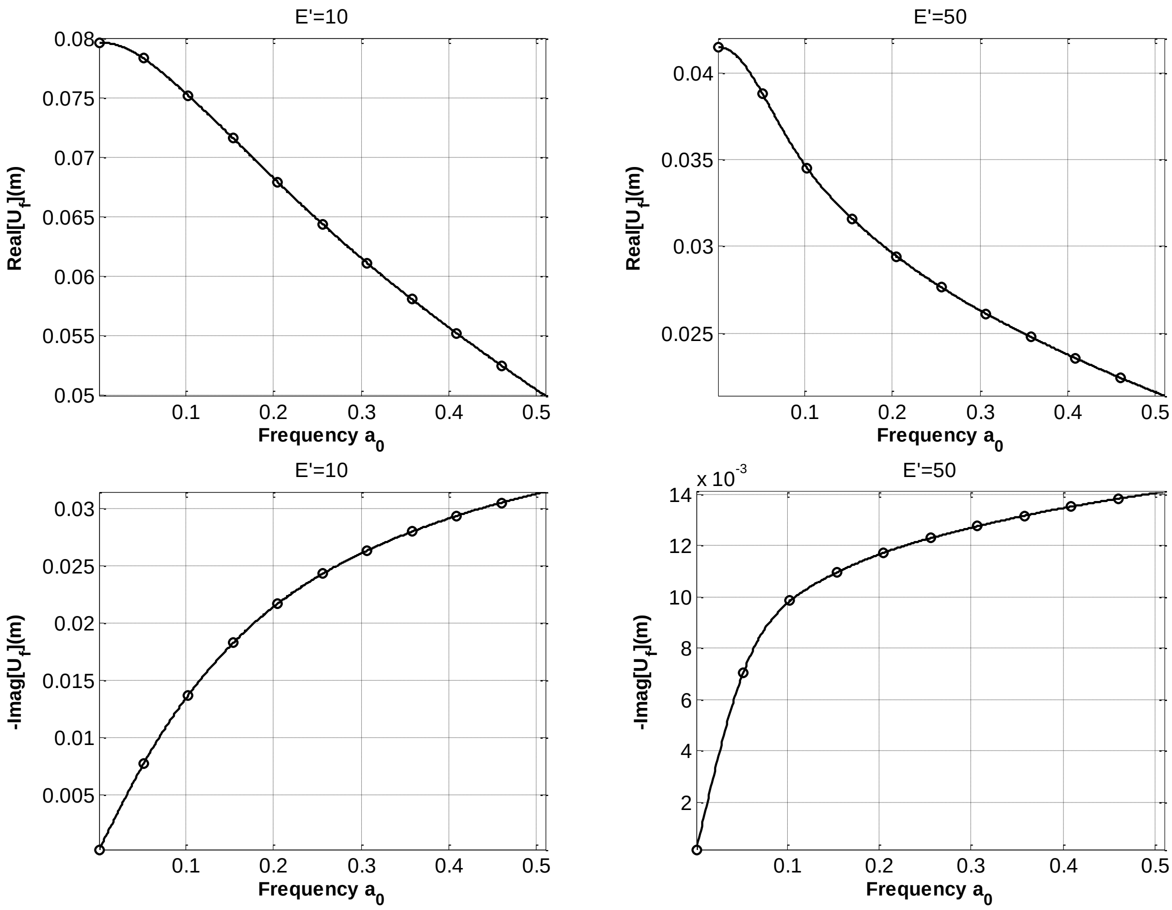


Figure 8. Real and Imaginary value of the foundation displacement (E´=10 and 50) for B=0

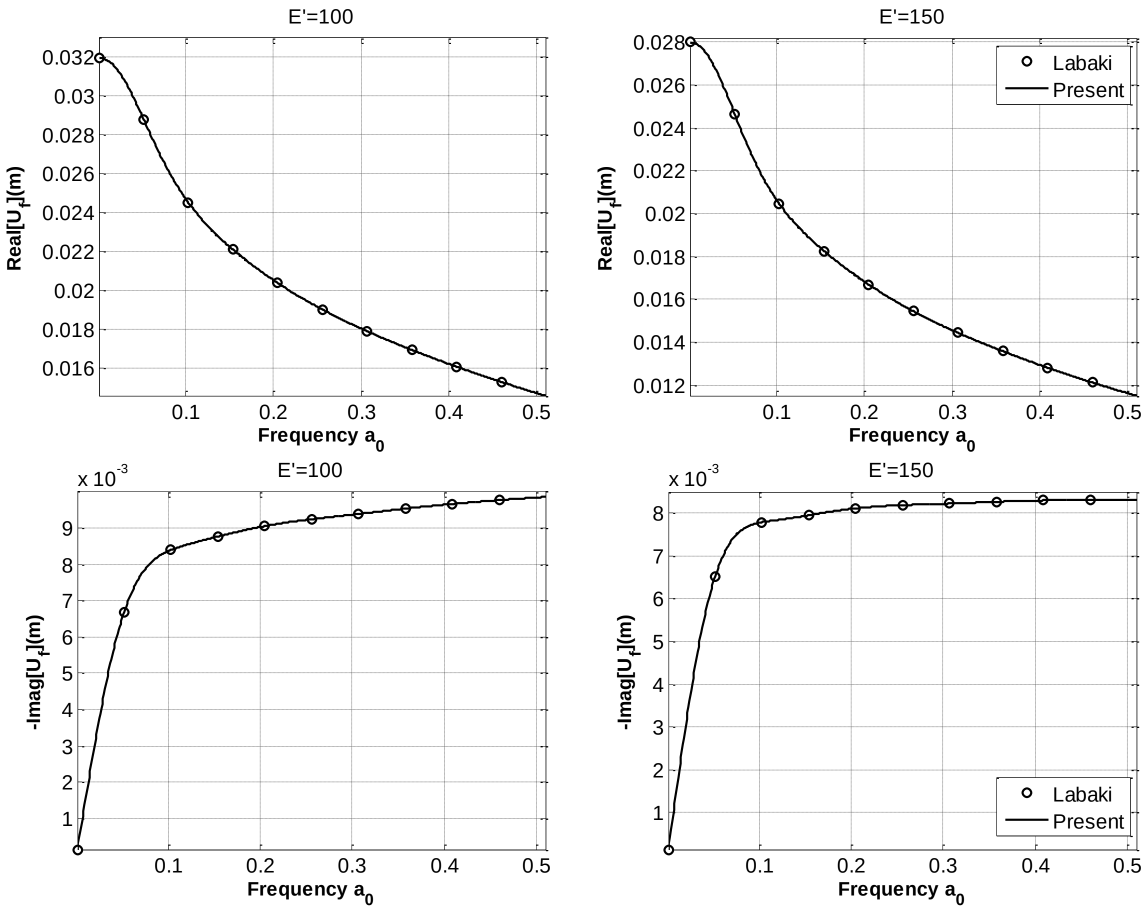


Figure 9. Real and Imaginary value of the foundation displacement (E´=100 and 150) for B=0

4.3 Pile-foundation interaction (α=1): results

Figures 10 and 11 show the real and imaginary part of the foundation displacement for B=0 and varying E'.

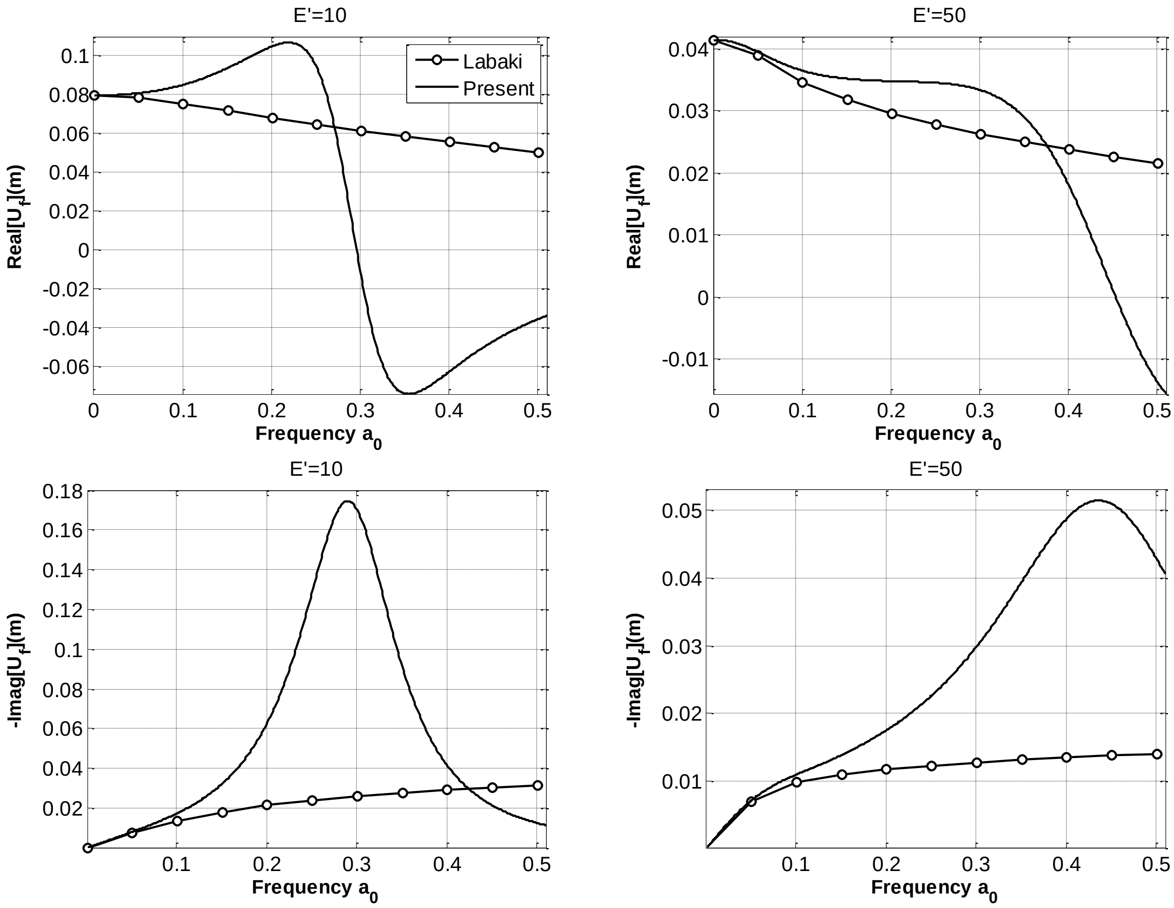


Figure 10. Real and Imaginary value of the foundation displacement (E´=10 and 50) for B=50

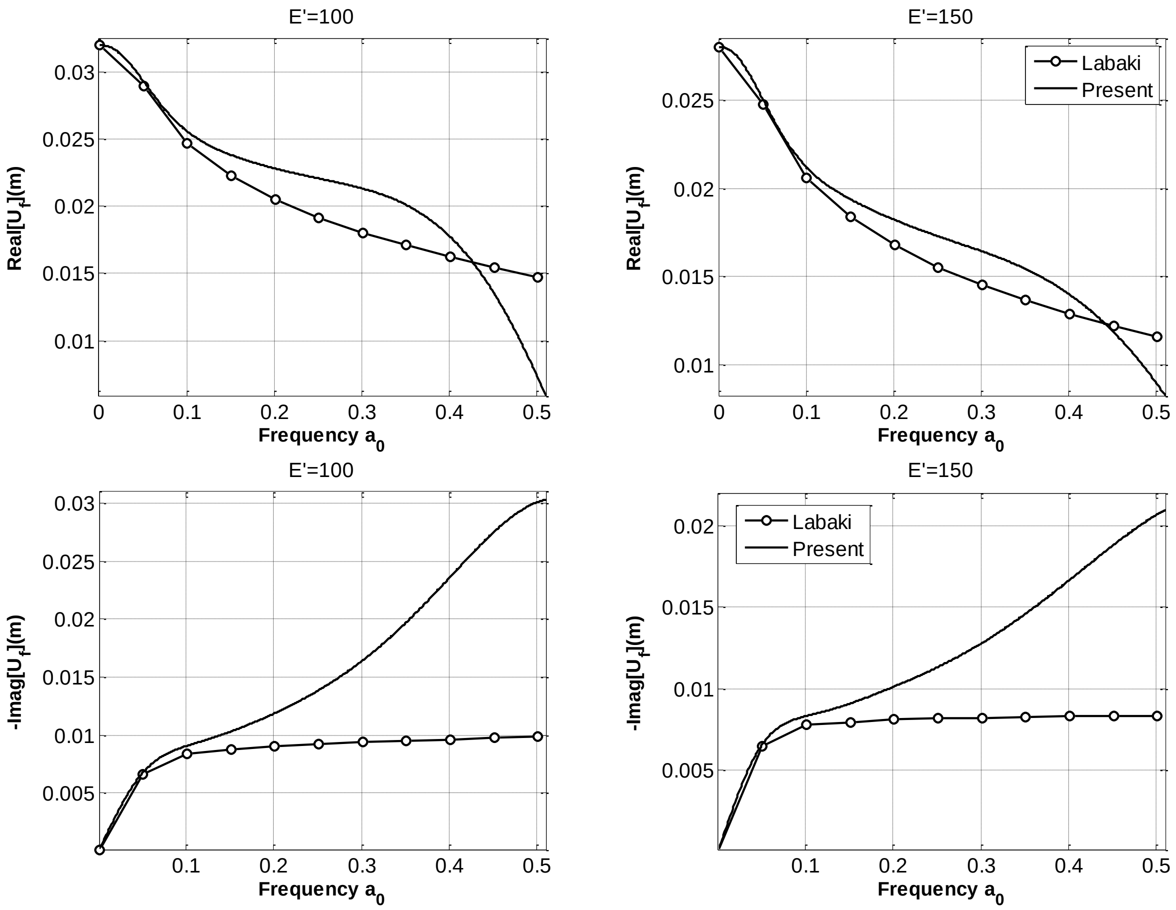


Figure 11. Real and Imaginary value of the foundation displacement (E´=100 and 150) for B=50

Figures 10 and 11 show that the presence of a foundation with mass (B>0) on top of the pile affects the vibration of the pile significantly. This is an important result of coupled foundations presented in this paper.

4.4 Soil-pile-foundation interaction: results

Analyzing Figs. 6 to 11, the static value of displacement of the foundation interacting only with the soil is different from the static value of displacement of the foundation interacting only with the pile.

In order to model the coupling of the foundation with the underlying pile and soil, the value of the stationary soil displacement should be close to the value of the stationary pile displacement. A correction factor c_f, as shown in Eq. (19), is introduced to adjust the compliance of the two support mechanisms, so that their static values coincide:

$$c_f = S_{zzhs}(\omega = 0) / S_{zzp}(\omega = 0) \tag{19}$$

Thus, as shown in Eq. (20), S_{zzp} becomes:

$$S^{*}_{zzp} = S_{zzp} c_f \tag{20}$$

in which ‘*’ indicates a corrected static pile displacement. This corrected factor is used in Eqs. (18) instead of S_{zzp}.

Figures 12 and 13 show the displacement of the system for the case of a massless foundation (B=0). Different values of E' are considered.

When α=0, it means that the external force is transferred from the foundation directly into the soil (Fig. 2a). Conversely, when α=1 the external force is transferred from the foundation to the soil entirely through the pile. There is no direct load transfer from the foundation to the soil (Fig. 4a).

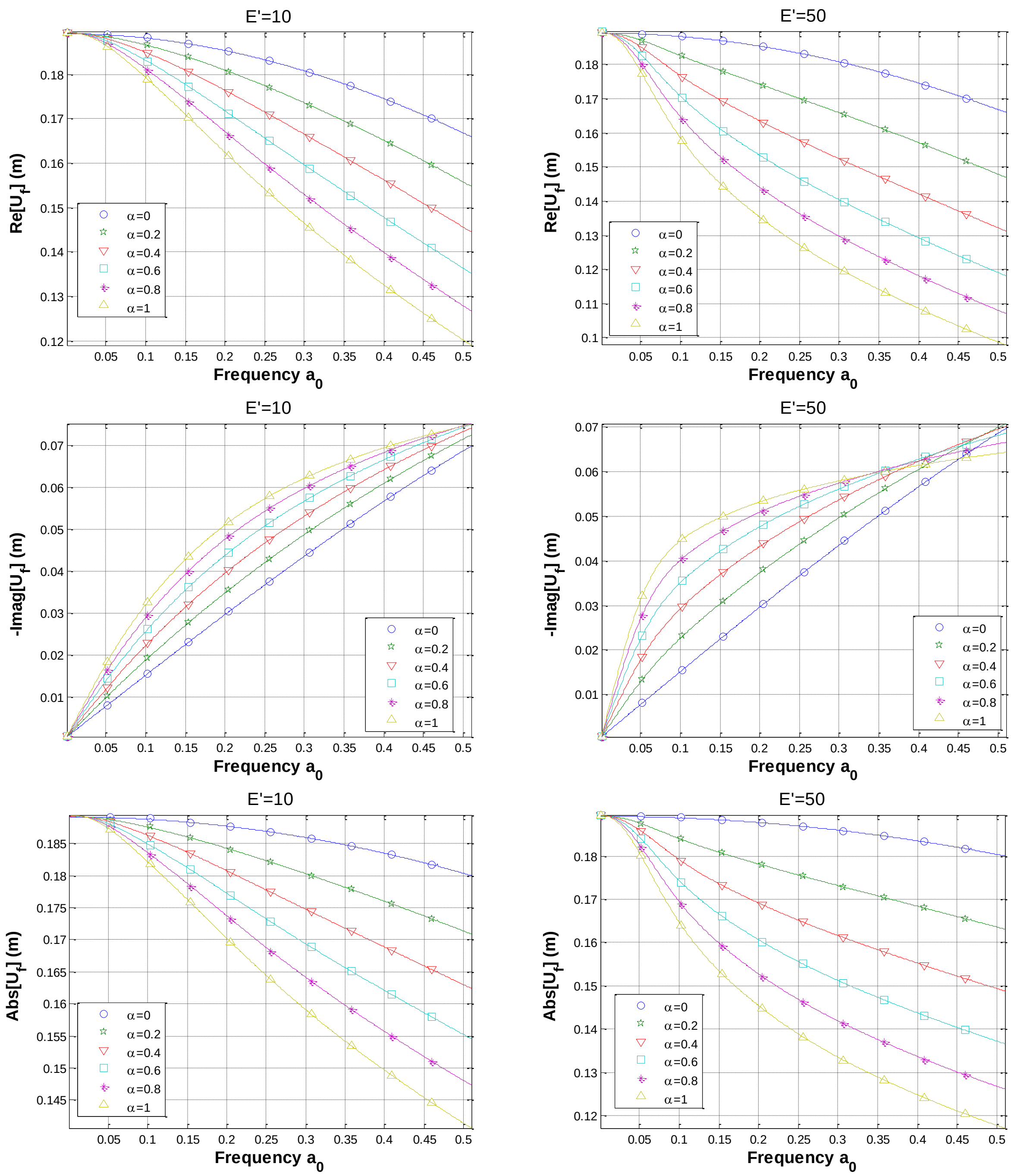


Figure 12. Real, Imaginary and Absolute value of the foundation displacement (E´=10 and 50)

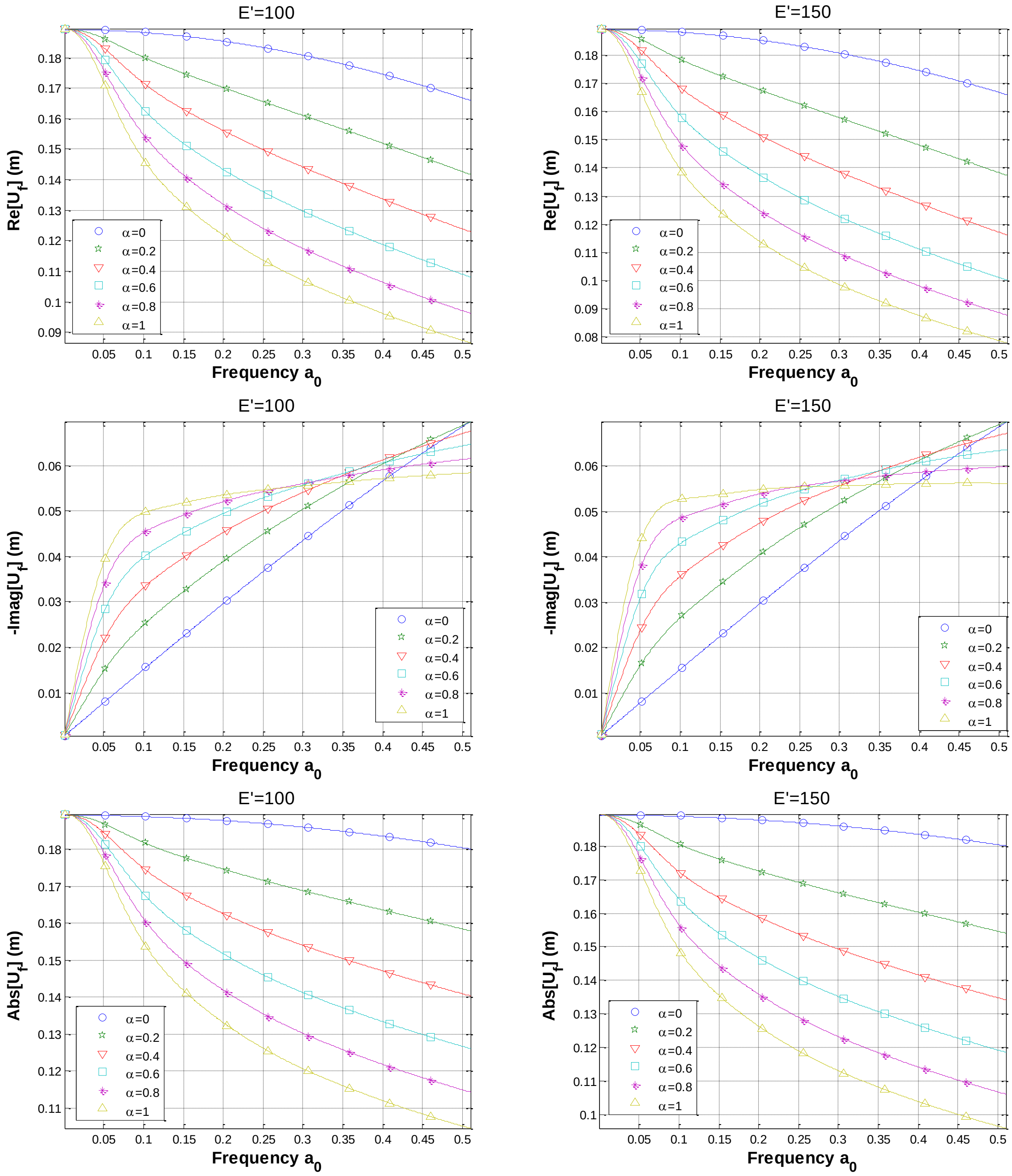


Figure 13. Real, Imaginary and Absolute value of the foundation displacement (E´=100 and 150)

An analysis of Figs. 12 and 13 show that when α=0, the displacements of the foundation do not depend on the properties of the pile. That is physically consistent because when α=0, the pile it not present in the coupled system. On the other hand, it is observed that an increase in α corresponds to a decrease in the displacement of the foundation. That is also physically consistent, since the presence of the pile in the system makes the system stiffer (Rangwala et al., 2012).

Figures 14 and 15 show the corresponding results for the case in which B=50.

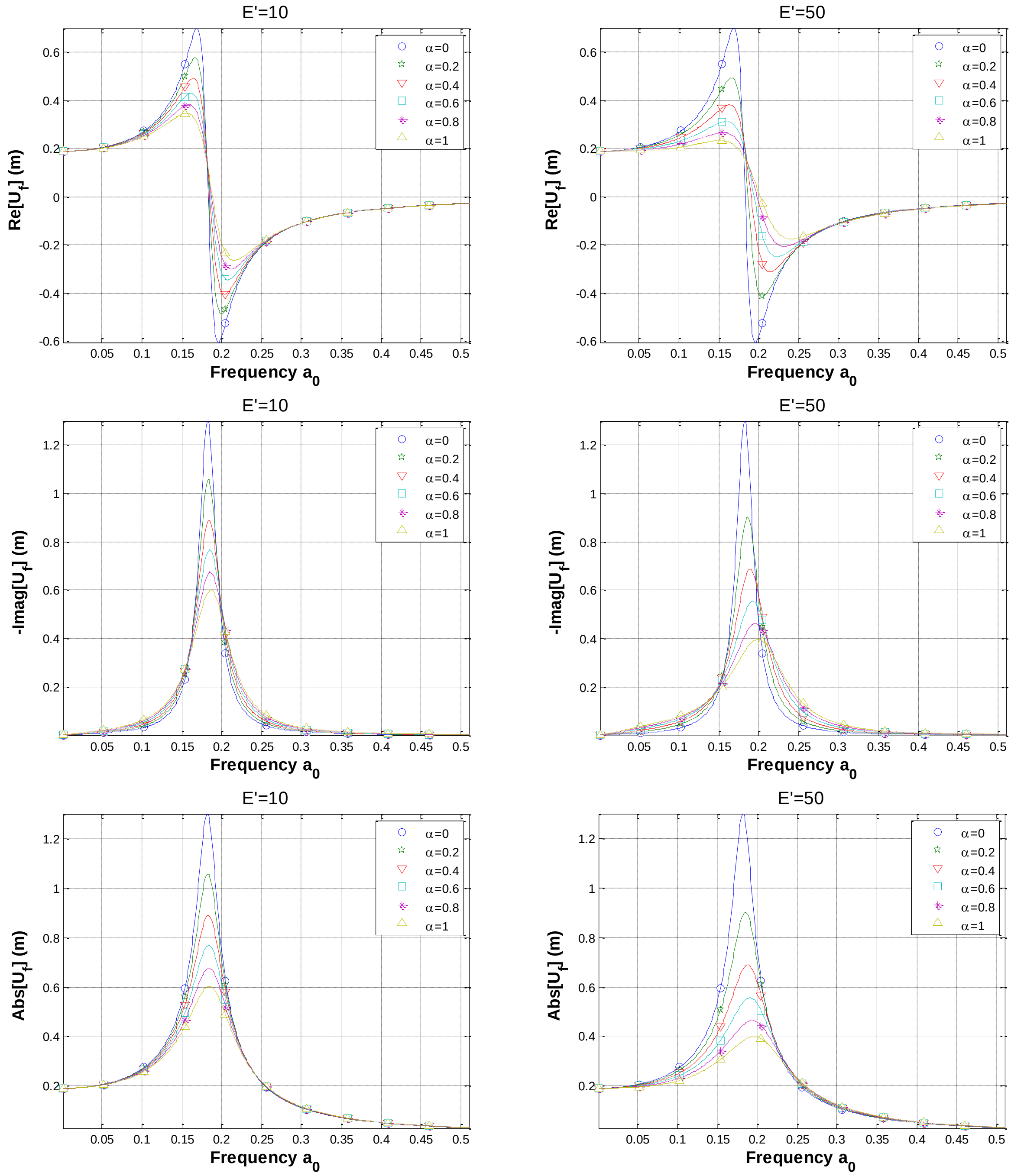


Figure 14. Real, Imaginary and Absolute value of the foundation displacement (E´=10 and 50)

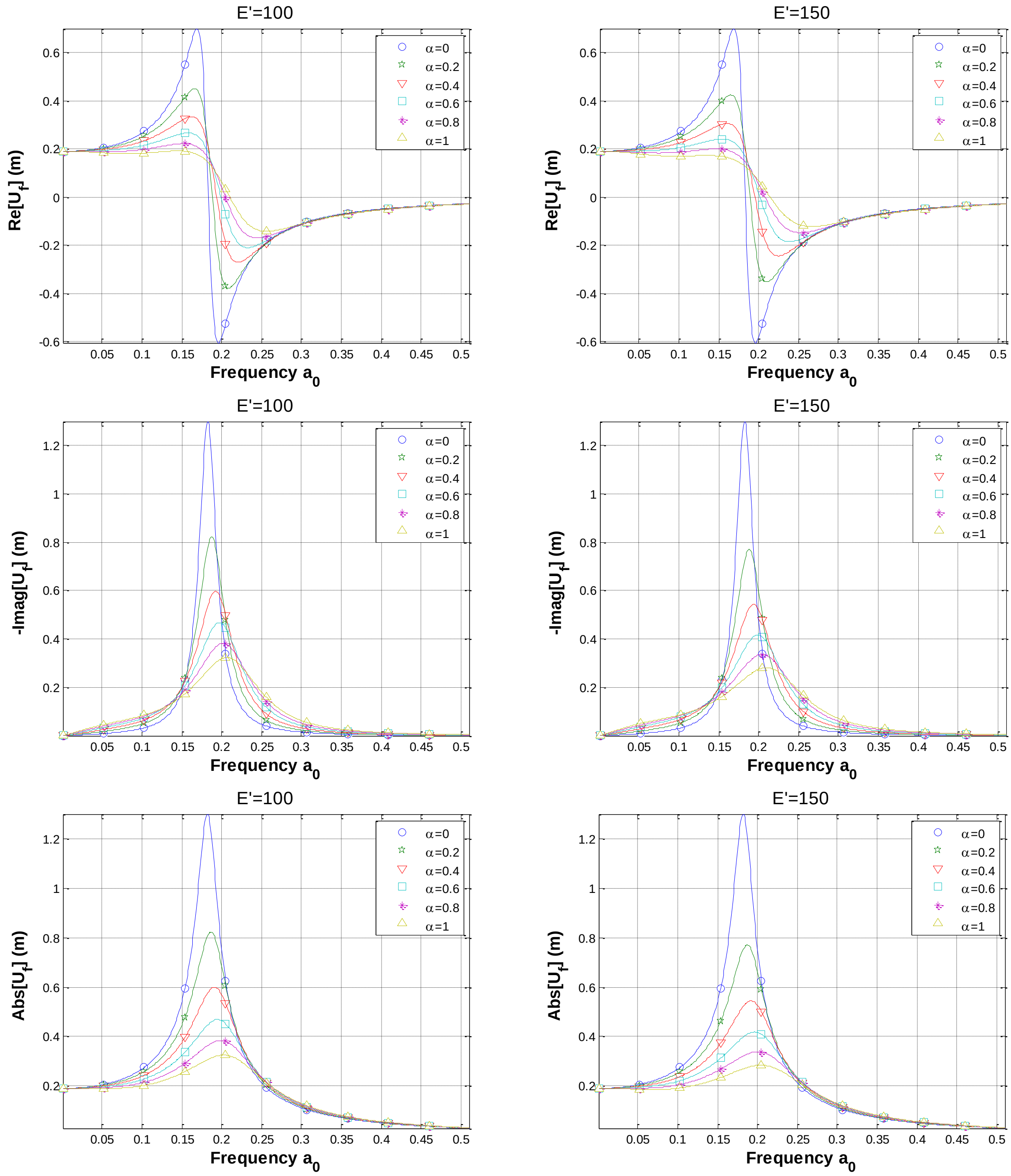


Figure 15. Real, Imaginary and Absolute value of the foundation displacement (E´=100 and 150)

A comparison of these sets of results (B=50) with the previous ones (B=0) in Figs. 12 and 13 shows that B=50 shows a well-defined resonance region which did not occur when B=0. Figures 14 and 15 show that α and the modulus of elasticity of the pile both have a marked influence on the displacement of the coupled system. An increase in the modulus of elasticity of the pile is observed to correlate with a descrease in the displacement of the foundation-pile-soil system.

5 CONCLUSIONS

This paper presented a formulation for analyzing the dynamic response of rigid foundations interacting with an underlying pile and its surrounding soil. A parameter was introduced, which allows an investigation of coupling configuration between two extremes: the case in which a rigid foundation is fully supported by its underlying soil, and the case in which the foundation is fully supported by a pile embedded within the soil. The study has shown that the response of the coupled system depends significantly on the amount of load that is transferred from the foundation to the pile and the soil. The introduction of pile results in an overall decrease in the vibration amplitude of the system. The extreme in which that amplitude is smallest corresponds to the case in which the system is only a pile interacting with the soil. Regardless of the composition of the system, an increase in the stiffness of the pile causes a slight shift forward in the resonance frequency of the system. It can, potentially, also be used in an inverse study to determine with which degree each supporting mechanisms is acting on the foundation.

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