AN IMPROVED BEM-LSM COUPLING-BASED TOPOLOGY OPTIMIZATION

Hugo Luiz Oliveira

Edson Denner Leonel

hugoitaime@usp.br

edleonel@sc.usp.br

University of São Paulo, School of Engineering of São Carlos, Department of Structural Engineering. http://www.set.eesc.usp.br

Av. Trabalhador São Carlense, 400. São Carlos-SP, Brazil. 13.566-590.

Abstract. *Topology Optimization (TO) is recognized as an important approach during early stages of structural concept. It allows the designer for finding higher performance solutions taking into account the limitation of natural resources. Most computational TO procedures are based on domain methods, in which feasible solutions are searched in relaxed design space. In such a case, jagged faces and grey-scale interpolations often lead to artificial stresses along the optimization. This study presents an algorithm for two-dimensional structural analysis, which overcomes such a difficulty. In addition it allows addressing both shape and topology changes. The coupling between Level Set Method and Boundary Element Method provides precise topologies along the whole optimization process. A benchmark example is used to illustrate its accuracy. The advantages of the proposed procedures are summarized as follows. Firstly, it is a gradient-based approach requiring information only at the boundary. Secondly, it leads to lower computational effort if compared to other available methodologies. The presented formulation shows efficiency and brings out new research perspectives.*

Keywords: *BEM-LSM coupling, Topology Optimization, Boundary Element Method.*

1 INTRODUCTION

Engineers are often asked for finding the best compromise between structural performance and resources application. Trial and error based on experience has often been adopted as solution method. In early stages of design, Topology Optimization (TO) has proven to be a more productive approach. TO is efficient for finding the structural material distribution that minimizes some appropriate cost function. In this study, TO is addressed on its classical version, in which compliance is adopted as performance function.

The seminal work by Bendsoe and Kikuchi (1988) opened the way for numerical applications in TO. Their formulation transforms TO into a material distribution problem of an anisotropic material, in which the effective properties calculated by the Homogenization method. Since then, new approaches have been proposed including SIMP (Bendsoe, 1989) and Level Set Method (Sethian and Wiegmann, 2000). These efforts have led TO for reaching the necessary maturity to be utilized for industrial purposes. The majority of the algorithms are based on domain methods as Finite Element Method (FEM). The interested reader is referred to (Rozvany, 2009; Sigmund and Maute, 2013) for comprehensive details. In the present study, focus is dedicated to Boundary Element Method (BEM) formulations.

Level Set Method (LSM) was introduced by Osher and Sethian (1988). It is an efficient formulation for simulating movements of curves. It allows complex topology changes, as merging and breaking, to occur naturally. Among many applications achieved in sciences, it is also possible to use it for TO (Sethian and Wiegmann, 2000).

The first coupling between BEM and LSM was presented by Abe et al (2007). The authors utilize design sensitivity analysis to find the velocity field along the boundary. Its elegant formulation is based on derivatives of matrixes H and G previously introduced by Tai and Fenner (1996). The results are dependent on the ground structure, since it cannot nucleate holes.

Marczak (2007) presented a new numerical approach capable to nucleate holes. The author utilises topology derivatives for finding the best place for removing a fixed amount of material. No domain mesh is required. Through an iterative material elimination (hard-kill) process, optimal shapes are achieved. Topology derivative is a consistent way to alter topology class and has been found frequently in FEM framework.

Yamasaki et al (2013) introduced the immersion BEM concept based on the reaction-diffusion equation (Yamada, 2010). This formulation replaces BEM nodes by nodal level-set values. It permits directly linking sensitivities of mechanical cost functionals to level-set function. As in Abe et al (2007), the final topology may be affected by the initial structure.

A BESO-BEM approach was presented by Ullah et al (2014). The authors' formulation is based on heuristic relation between von Mises stresses and boundary velocities. This idea was first introduced by Sethian and Wiegmann (2000) under the Finite Differences framework. A stress threshold is assumed as rejection criterion, whereas Level-set equation handles topology changes. The idea can also be applied to three dimensional structures, using marching cubes to track the zero-level-curve (Ullah et at, 2015).

The present study addresses the coupling between BEM and LSM. The key difference from available procedures is the way the level set velocities are calculated. Here, shape derivative introduced by Allaire et al (2004) are used to guide the algorithm. Some advantages are highlighted. Firstly, clear boundary definition is provided during the whole optimization process. It is important for future extensions, especially for design dependent problems. In addition, since BEM provides information directly on the boundary, it is possible to define boundary velocities without extra cost.

The classical two-bars structural problem is used as testing case. The results show that the proposed algorithm can handle properly complex topology changes, even material detachment without extra-effort. It is also observed shape smoothness along the whole process avoiding numerical issues related to normal derivatives.

2 BOUNDARY ELEMENT METHOD

Problems formulated in terms of boundary integrals are solved by BEM as long as the fundamental solution is known. BEM is applied into several fields of science, including Elasticity problems (Brebbia, Dominguez, 1989; Aliabadi, 2002). Assuming linear constitutive law and small displacements and rotations, Betti's theorem can be used to derive the following expression:

$$c_{lk}(s,f)u_k(s)+\int_{\Gamma} p^*_{lk}(s,f)u_k(f)d\Gamma=\int_{\Gamma} u^*_{lk}(s,f)p_k(f)d\Gamma \tag{1}$$

It is a common practice naming *s* and *f* as *source* and *field* points respectively. Eq. 1 assumes that no body forces are present, and it is valid for points along the boundary. If Γ is differentiable at *s* then $c_{lk}=\delta_{lk}/2$. Let μ and ν be material parameters. u^*_{lk} and p^*_{lk} are Kelvin fundamental solutions given by:

$$u^*_{ij}=\frac{1}{8\pi\mu(1-\nu)}\left[(3-4\nu)ln\left(\frac{1}{r}\right)\delta_{ij}+r,_i r,_j\right] \tag{2}$$

$$p^*_{ij}=-\frac{1}{4\pi(1-\nu)r}\left\{\frac{\partial r}{\partial n}\left[(1-2\nu)\delta_{ij}+2r,_i r,_j\right]+(1-2\nu)\left(n_i r,_j-n_j r,_i\right)\right\} \tag{3}$$

Frequently it is not possible to find boundary fields u_k and p_k that represents all possible combinations of fixing and load conditions. Nevertheless Eq. 1 is used for generating a computational procedure for finding boundary unknowns. This is the essence of BEM. In order to calculate the integrals along the boundary, it is convenient to divide the entire boundary into N_e finite sized elements. Using lagrangean polynomials of order o follows: $u_k=\phi_m(\xi)u^m_k$, $p_k=\phi_m(\xi)p^m_k$. Making substitution in Eq.1 and rearranging terms follows:

$$\begin{aligned}&c_{ij}(s,f)u_j(s)+\sum_{e=1}^{N_e}\sum_{l=1}^{m}\left[\int_{-1}^{1}p^*_{ij}(s,f(\xi^e))\phi(\xi^e)J(\xi^e)d(\xi^e)\right]u^{el}_j=\\&\sum_{e=1}^{N_e}\sum_{l=1}^{m}\left[\int_{-1}^{1}u^*_{ij}(s,f(\xi^e))\phi(\xi^e)J(\xi^e)d(\xi^e)\right]p^{el}_j\end{aligned} \tag{4}$$

where *l* is the number of nodes varying from $m=1,\dots,o+1$. u^{el}_j and p^{el}_j are displacements and tractions, respectively, in the local node *l* on the element *e*. ξ is a dimensionless coordinate J stands for Jacobian transformation. In the isoparametric approach, boundary fields are approximated by the same set of basis functions used for approximate geometry. One can apply Eq. 4 to all nodes that define the geometry and obtain the system of equations which can be expressed in a matrix form as follows:

$$[H]\{U\}=[G]\{P\} \tag{5}$$

Note that Eq. 2 and Eq. 3 are the kernels of Eq. 4, therefore singular. It requires careful treatment before calculation. Subtraction Singularity Method (Aliabadi, 2002) can be used for this end. Boundary conditions are imposed in Eq. 5 by appropriate columns interchange. It permits Eq. 5 to be rewritten as:

$$[A]\{X\}=\{b\} \tag{6}$$

The vector b is related to the prescribed values for tractions and displacements. Vector X contains the unknowns of the problem. Matrix A is fully populated and non-symmetric. When further information about the interior domain is needed (stresses, strains, displacements), a post-processing step is required.

3 LEVEL SET METHOD

The reason for using LSM is twofold. Firstly it permits handle complex topology changes in a natural way. In addition, it is a convenient way to parameterize the design domain using only boundary information. This flexibility is accomplished by setting a family of functions defined along a fictitious time axis. Structural domain is represented implicitly by the Level Set function $\Psi : \mathbb{R}^2 \times \left[0,\infty\right) \rightarrow \mathbb{R}$ defined such that $\Psi < 0$ denotes structural domain, $\Psi > 0$ indicates empty part of the design domain. Instead of following the boundary movement as in lagrangian formulations, the evolution is governed by a function Ψ whose zero-level-set represents the structural boundary. This eulerian description of motion is governed by the following partial differential equation (Osher and Sethian, 1988):

$$\frac{\partial \Psi}{\partial \tau} + F\left|\nabla \Psi\right| = 0 \tag{7}$$

In Eq. 7, *F* represents the velocity field normal to the level set function at the referred point of the design domain. Equation 7 is a Hamilton-Jacobi type equation which was investigated along the twentieth century. As so, several numerical schemes are known for solving them. Classical upwind solution is adopted using Finite Differences under strict Courant-Friedrichs-Lewy conditions (Sethian, 1999).

4 COUPLING BEM AND LSM

Considering $\overline{V}$ as required volume, the minimum compliance problem is formulated as follows:

$$\min_{\Gamma=\partial\Omega} C = \int_{\Gamma} u_i t_i \, d\Gamma \tag{8}$$

$$\text{s.t. } \int_{\Omega} d\Omega - \overline{V} = 0 \tag{9}$$

Instead of analysing this problem through the classical KKT condition, Allaire et al (2004) suggested replacing it by a series of convergent topologies. In this study, a computational procedure is proposed for generating each topology of the feasible space. BEM and LSM are coupled through the following tasks:

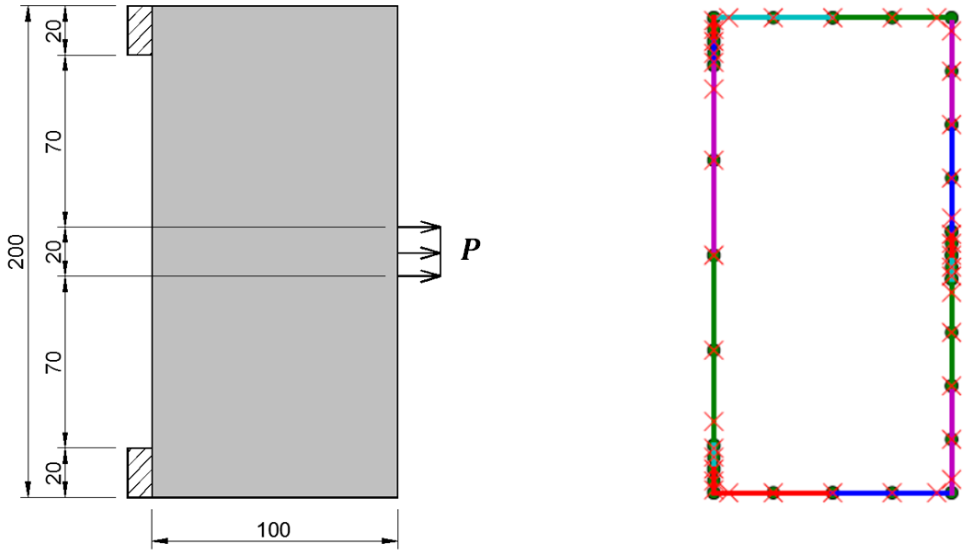
1. Level Set Initiation. Signed distance function is used to calculate the initial value of level set function (Sethian, 1999);
2. BEM mesh definition. This step is responsible for generation of classical polynomial elements, along the boundary. Higher order elements may also be utilized. BEM convention needs to be respected (clockwise for material and counter clockwise for void);
3. BEM analysis. The direct BEM analysis is performed, as presented in section 2;
4. Compute objective function. Cost function is admitted to be the external energy (Eq. 8);
5. Check convergence. If convergence for compliance is observed, then finish;
6. Check for holes. Calculate von Mises stress field among all internal points. If the neighbourhood of some internal point present stresses below a threshold then a circular hole is created in this area;
7. Calculate velocities. The velocities needed in Eq. 7 is obtained from the shape sensitivity analysis for the augmented lagrangian form (Allaire et al, 2004);
8. Solve Level Set equation. Following explicit time algorithm given by Osher and Sethian (1988), it is possible to determine the next position of the zero-level curve;
9. Remeshing. Determine the zero-level points using linear interpolation. Rebuild geometry by using NURBS fitted to this set of points. In this way it is faster to recreate boundary elements avoiding mesh instabilities. Return to step 2.

The proposed algorithm has some interesting features that will be explored in the next item. They can be summarized as follows:

- ✓ Since the boundary is implicitly represented by the level set function, it is known at each fictitious time step. It can be advantageous, especially for engineering minimization problems where load actions depend on the actual boundary.
- ✓ The algorithm is gradient based. The velocities are dependent on stresses along the boundary, therefore can be calculated without extra computational cost (step 7).
- ✓ The algorithm can be used either for shape or topology optimization.
- ✓ It is independent from the initial structure because holes may automatically be nucleated based on stresses, as suggested by Ullah et al (2014).
- ✓ The use of NURBS facilitates remeshing task. NURBS are used only for geometry representation. It means that isogeometric analysis is not carried out, although it represents a good opportunity for improvements.

5 APPLICATION

In this example a benchmark well documented in the literature is presented. Consider a plane structure fixed on one edge and loaded on the opposite one. Material is assumed as linear elastic with Young modulus $E=210$ GPa and Poisson ratio $\upsilon=0.3$. Let the load $P=100$ kPa. Thickness is considered 1mm under plane stress assumption. Fixing and load conditions, as well as structural geometry are illustrated in Fig. 1.



(a) (b)

Figure 1 Initial structure (a) Load, Fixing and Geometry conditions [unities - mm] (b) Initial BEM mesh (quadratic elements)

One searches for the stiffest structure that can be conceived using only 30% of the initial material. There are two ways to obtain the same result for this example. One of them is via Shape Optimization. The second one is via TO. The difference between them concerns hole nucleation into TO. Fig. 2 shows the convergence history for shape optimization.

In Fig. 3, the set of feasible shapes is illustrated. The boundary is clearly represented in each time step. The evolution is symmetric since the problem has symmetry fixing and load conditions. The optimizable boundary pieces are smooth. The final result resembles a two bars structure (truss) and is well agreed to literature. On the other hand, topology feasible set is showed in Fig. 4. Note that holes appear in early stages of the process. It represents regions where material has low stresses, and can arbitrary be removed. In such a case, the use of LSM reveals its advantage, since merging and breaking of curves can occur naturally. The algorithm stability is shown by the final result. Although different in essence, both optimization procedures arrive at the same optimal structure.

When the topology optimization is carried out by LSM, some part of the structural material can detach from the main structure. Abe et al (2007) alerts for *material islands*, and suggest that it should be eliminated before BEM analysis for sake of stability. In the present algorithm, *islands* can also appear (note Fig. 3 – Iteration 12) but no numerical instabilities were observed. BEM/LSM handles it properly.

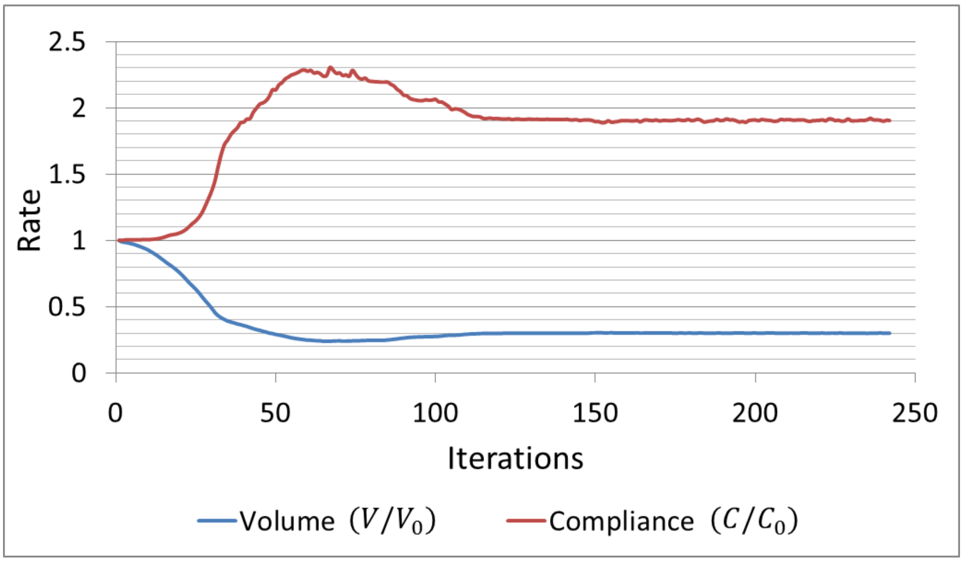


Figure 2 Convergence history

6 CONCLUSION

In this study, the coupling between BEM and LSM was addressed. This coupling originates a numerical procedure able for dealing with Topology Optimization. The key point is the use of shape derivative to guide boundary evolution in each time step. The procedure does not represent extra computational effort, since all the necessary information is provided by BEM analysis. The chosen upwind Level Set discretization is a fast way to solve evolution partial differential equation. For practical purposes, it represents computational efficiency. Topology class changes are made possible due to local material removing procedure inspired by the Bubble Method. It is remarked the clear definition of the boundary along all optimization process. It opens a new branch of investigations, especially in cases where one is interested in design dependent variables, and boundary uncertainties. The present work enlarges and opens new perspectives for BEM applications.

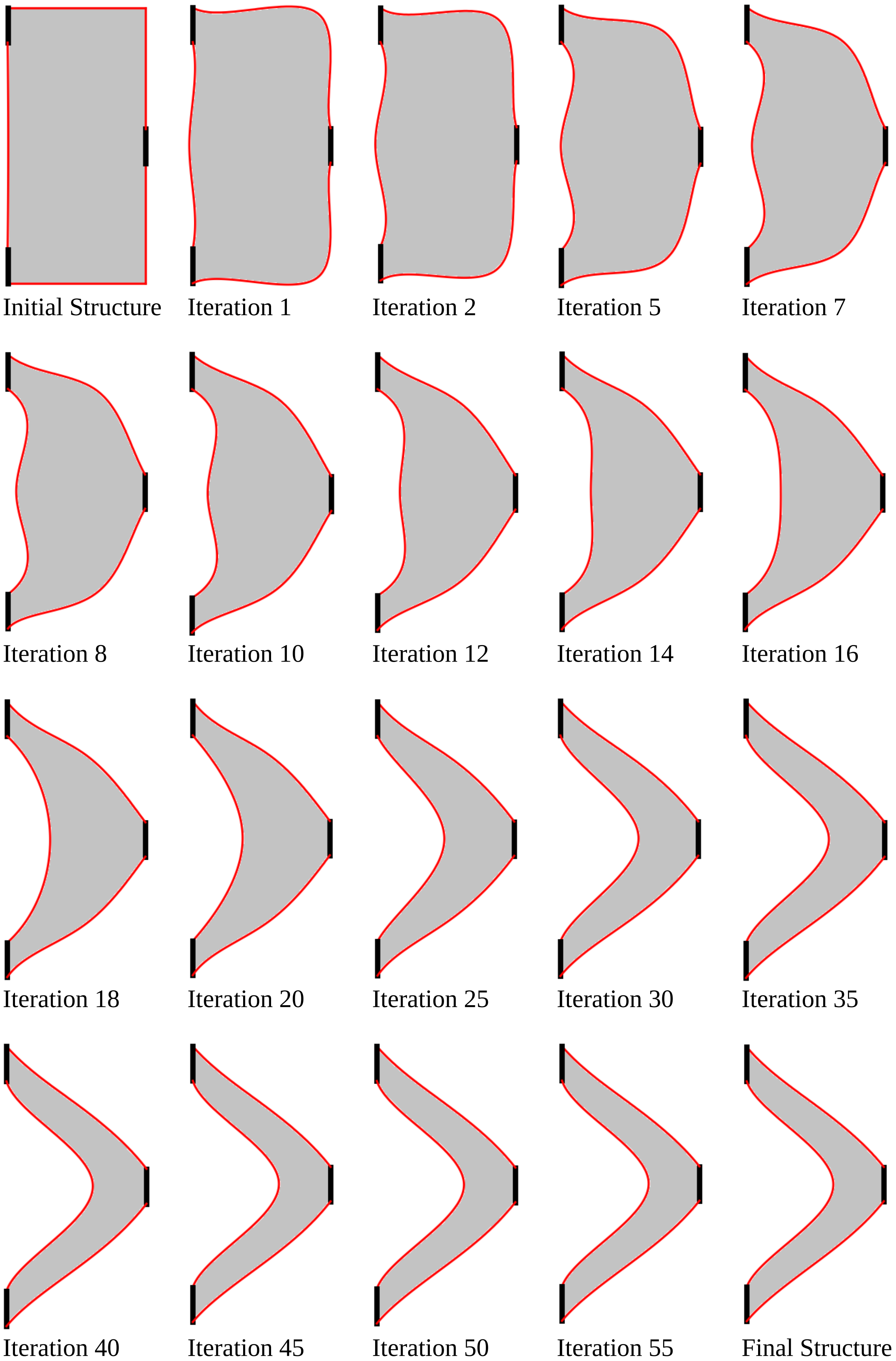


Figure 3 Shape History

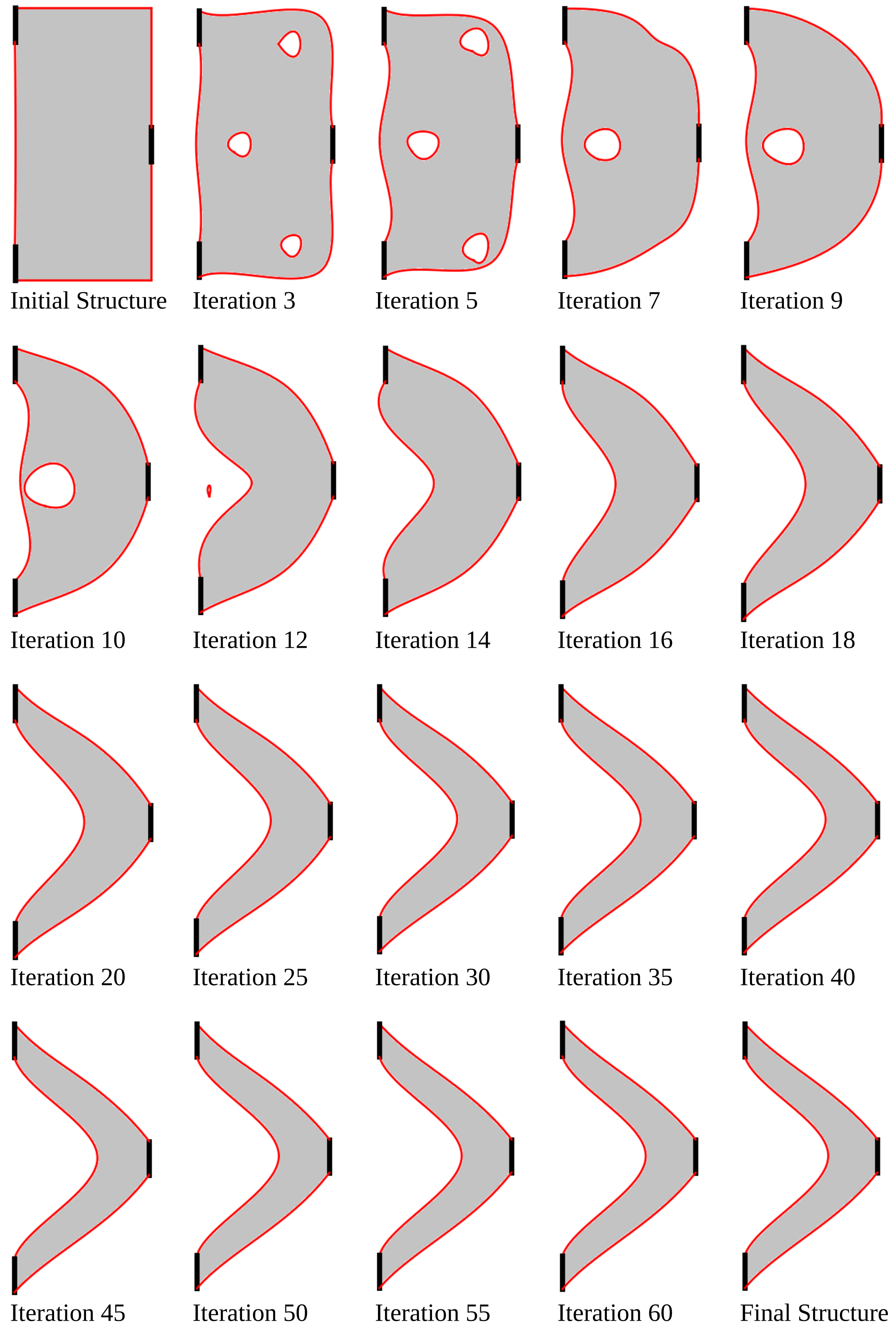


Figure 4 Topology History

ACKNOWLEDGEMENTS

Funding provided by São Paulo Research Foundation (FAPESP), grant number 2012/24944-5 and 2015/07931-5, is greatly appreciated.

REFERENCES

Abe K., Kazama S., Koro K., 2007. A boundary element approach for topology optimization problem using the level set method. *Communications in numerical methods in engineering*, v. 23, p. 405-416.

Allaire G., Jouve F., Toader A-M., 2004. Structural optimization using sensitivity analysis and a level-set method. *Journal of Computational Physics*, v.194, p.363-393.

Alliabadi M.H. *The boundary element method. Applications in solids and structures.* v. 2, New York: J. Wiley, 2002.

Bendsoe M.P., 1989. Optimal shape design as a material distribution problem. *Structural Optimization*, v.1, p.193-202.

Bendsoe M.P., Kikuchi N., 1988. Generating optimal topologies in structural design using a homogenization method. *Computer Methods in Applied Mechanics and Engineering*, v. 71, p.197-224.

Brebbia C.A., Dominguez J. *Boundary Elements: An Introductory Course*, Southampton: McGraw Hill, 1989.

Marczak R.J., 2007. Topology optimization and boundary elements – a preliminary implementation for linear heat transfer. *Engineering Analysis with Boundary Elements*, v.31, p.793-802.

Osher S., Sethian J.A. Fronts Propagating with Curvature-Dependent Speed: Algorithms Based on Hamilton-Jacobi Formulations. *Journal of Computational Physics*, v.79, p. 12-49, 1988.

Rozvany G.I.N., 2009. A critical review of established methods of structural topology optimization. *Struct Multidisc Optim*, v. 37, p.217-237.

Sethian J.A., 1999. *Level Set Methods and Fast Marching Methods: Evolving interfaces in computational geometry, fluid mechanics, computer visions, and material science*, ed.2, Cambridge Universty Press, New York.

Sethian J.A., Wiegmann A., 2000. Structural Boundary Design via Level Set and Immersed Interface Methods. *Journal of Computational Physics*, v.163, p.489-528.

Sigmund O., Maute K., 2013. Topology optimization approaches - A comparative review. *Struct Multidisc Optim*, v. 48, p.1031-1055.

Tai K., Fenner R.T., 1996. Optimum shape design and positioning of features using the boundary integral equation method. *International Journal for Numerical Methods in Engineering*, v.39, p.1985-2003.

Ullah B., Trevelyan J., Ivrissimtzis I., 2015. A three-dimensional implementation of the boundary element and level set based strucutral optimization. *Engineering Analysis with Boundary Elements*, v. 58, p.176-194.

Ullah B., Trevelyan J., Matthews P.C., 2014. Structural optimization based on the boundary element and level set methods. *Computer and Structures*, v.137, p.14-30.

Yamada T. et al., 2010. A topology optimization method based on the level set method incorporating a fictitious interface energy. *Comput. Methods Appl. Mech. Engrg*. v.199, p.2876-2891.

Yamasaki S., Yamada T., Matsumoto T., 2013. An immersed boundary element method for level-set based topology optimization. *International Journal for Numerical Methods in Engineering*, v. 93, p.960-988.