REVISTA INTERDISCIPLINAR DE PESQUISA EM ENGENHARIA

CILAMCE 2016

XXXVII IBERIAN LATIN AMERICAN CONGRESS ON COMPUTATIONAL METHODS IN ENGINEERING
BRASÍLIA - DF - BRAZIL

APPROXIMATED EVALUATION OF A RIGID ACOUSTIC CAVITY COUPLED ON A FLEXIBLE BOARD IN A FREE VIBRATION

Agnaldo Antonio Moreira Teodoro da Silva

Marcus Vinícius Girão de Morais

agnaldoantoniomtsilva@gmail.com

mvmorias@unb.br

Universidade de Brasília, Integridade de Materiais da Engenharia

Campus UnB Gama, Área Espacial Industria de Projeção A – 72.444-240, Brasília DF Brasil

Abstract. *The study of the acoustic behavior of a cavity depends on factors such as acoustic wavelength, cavity dimensions, the effect of flexible interface structures, making it relatively complex problem. Due to the complexity of the problem it's sought simplified methods for the purpose techniques analysis applied with greater ease. Pretlove and Craggs (1969) presented a two-dimensional analysis of a cavity coupled to a plate using the normal mode as an approximation for explaining experimental results and then compared with results obtained by other authors. Recently Rojas (2015) presented an analytical solution to a rigid acoustic cavity coupled to a flexible plate in free vibration using an approximation by weighted residuals (Galerkin Ritz). The present study performed an analysis adding another modal forms to the Pretlove and Craggs' works (1969) in order to evaluate the method's efficiency. The comparisons with the Pretlove's (1965) semi-analytical results and with the Rojas' (2015) analytical results are the validation of this present study, checking a good approximation with the results.*

Keywords: *acoustic cavity rigid, flexible plate, approximation normal mode, free vibrations.*

1 INTRODUCTION

The development of analytical methods aiming the acoustic behavior analysis of cavities was the goal of a several studies. Reference texts can be founded in the works from the authors Rayleigh (1945), Dowell and Voss (1963), Lyon (1963), Pretlove (1965), Gerges e Fahy (1976), Morse and Ingard (1986), Junger and Feit (1993), Howe (1998), Fahy (2007) among others. Between them it can be mentioned some authors who motivated the present study.

Pretlove and Craggs (1969) present a simple approach using just two in vacuo vibration modes in order to define the natural frequencies and the modal forms in an acoustic cavity coupled on a flexible plate.

Ribeiro (2010) proposed two alternatives to the solution of frequencies and coupled system modes. The first one is a methodology named Pseudo-Coupled method, which depends on an imposition of certain deformed modal for the construction of the frequency equation in an associated way. The second one is an exact approach, with the solution of the differential involved equation (beam equation), resulting in frequencies and coupled modes.

Ferreira (2012) developed a methodology for the comparison between analytical and numerical solutions for acoustic and vibro-acoustic cavities using the pseudo-coupled technique for both the development of approximated analytical solutions and to compare the numerical model. Then, it was applied some techniques for the fluid-structure coupling, modal analysis, harmonic and in response to the frequency analyzing and comparing the result of these techniques through charts and modal forms and in frequency answer.

Rojas (2015) presented an analytical study of the vibro-acoustic problem of an rigid acoustic cavity coupled on a flexible plate, which the goal was to understand the behavior of an coupled system that permits experimental numerical comparisons, and to study more complex problems. The present study adopted the method used by Pretlove and Crags (1969) adding modal forms in order to evaluate the convergence of the method in comparison to the results achieved from other authors.

2 APPROACH TO COUPLED PANEL+CAVITY

The picture 1 represents the vibroacoustic system, flexible plate coupled on an acoustic cavity. It will be considered a cavity of dimensions $L_x \times L_y \times L_z$. The flexible plate is coupled to the system on the cavity's superior parto on the XY plan and the walls are taken as rigids. The differential equation governing the transverse plate subject to distributed loads is described in Eq. (1).The interior fluid of the cavity is the air.

$$D\left(\frac{\partial^4 w(x,y)}{\partial x^4} + 2 \cdot \frac{\partial^4 w(x,y)}{\partial x^2\, \partial y^2} + \frac{\partial^4 w(x,y)}{\partial y^4}\right) = p_z(x, y) \tag{1}$$

Or using biharmonic, it has been:

$$D \cdot \nabla^4 w(x, y) = p_z(x, y) \tag{2}$$

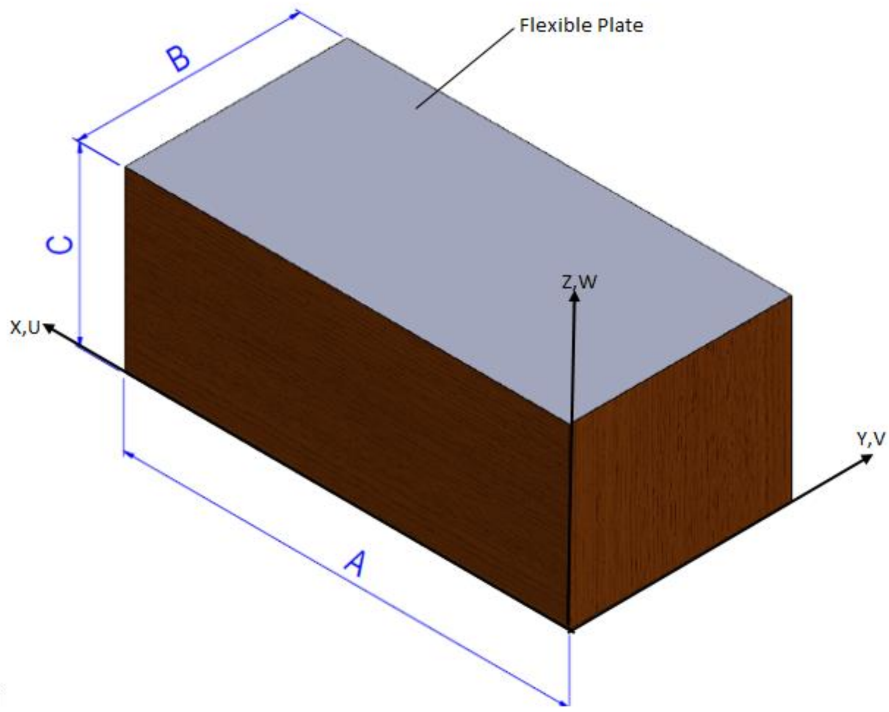

Figure 1. Rectangular box with one flexible plate

To the analysis of the coupled system it was performed a study about the free or natural vibration of the system in order to determinate a relation between the coupling among the structural and acoustic models. The equation of the acoustic wave used to describe the pressure p on the inside of the system is described on Eq. (3).

$$\nabla^2 p(x,y,z,t) = \frac{1}{c^2}\cdot\frac{\delta^2 p(x,y,z,t)}{\delta t^2} \qquad (3)$$

Where:

p is the pressure on the coupled system

c is the speed of the sound

The equation must satisfies the following contour conditions:

$$x = 0, L_x: \frac{\delta p(x,y,z,t)}{\delta x} = 0 \qquad (4a)$$

$$y = 0, L_y: \frac{\delta p(x,y,z,t)}{\delta y} = 0 \qquad (4b)$$

$$z = 0, L_z: \frac{\delta p(x,y,z,t)}{\delta z} = 0 \qquad (4c)$$

$$z = L_z, L_z: \frac{\delta p(x,y,z,t)}{\delta x} = -\rho_{ar}\cdot\frac{\delta^2 w(x,y,t)}{\delta t^2} \qquad (4a)$$

Where:

$w(x,y,t)$ is the transversal vibration of the flexible plate.

ρ_{ar} is the environment density of fluid (air).

To the solution of the coupled system, Pretlove and Craggs (1969) assumed the deformed coupled as equal as the vacuum coupled in order to simplify the solution, resulting in practical analytical solutions to the coupled frequencies for a certain way of vibration.

The solution presented by Pretlove and Craggs (1969) and by the present study are showed in the following sections.

2.1 Approximation toward two modes (PRETLOVE and CRAGS, 1969)

The aim of this section is to present a theorical analysis developed by Pretlove and Crags (1969). It was performed a simple approach to analyzing the vibrations of a system coupled on a plate-cavity. The presented theory predicts a few spots for maximum tensions and it was performed an tridimensional analysis of two vacuum's vibration modes only. The first one is the fundamental mode and the second one has three half waves in the longest direction of the board.

The assumed notation to the study is presented in Figure 1, wherein the depth of the board is represented in the z direction. Thus it is described the deformation of the board by the Eq. (5) using the Galerkin's method.

$$W = \sum_{i=1}^{2} C_i \cdot \varphi_i = C_1 \cdot \varphi_1(x, y) + C_2 \cdot \varphi_2(x, y) \tag{5}$$

Where C_i is the plate mode coordinates as a function of time defined by:

$$C_1 = q_{11} \; e \; C_2 = q_{31} \tag{6}$$

And the displacement functions for the two methods adopted for a simply supported plate are:

$$\varphi_1(x, y) = \mathrm{sen}\frac{1 \cdot \pi \cdot x}{L_x} \cdot \mathrm{sen}\frac{1 \cdot \pi \cdot y}{L_y} \tag{7}$$

$$\varphi_2(x, y) = \mathrm{sen}\frac{3 \cdot \pi \cdot x}{L_x} \cdot \mathrm{sen}\frac{1 \cdot \pi \cdot y}{L_y} \tag{8}$$

The displacement volume due to deflection is given by:

$$\delta V = \int_0^a \int_0^b \mathrm{sen}\frac{1 \cdot \pi \cdot x}{a} \cdot \mathrm{sen}\frac{1 \cdot \pi \cdot y}{b} dxdy \cdot q_{11} + \int_0^a \int_0^b \mathrm{sen}\frac{3 \cdot \pi \cdot x}{a} \cdot \mathrm{sen}\frac{1 \cdot \pi \cdot y}{b} dxdy \cdot q_{31} \tag{9}$$

Or after the resolution:

$$\delta V = \frac{4 \cdot a \cdot b}{\pi^2} \cdot q_{11} + \frac{4 \cdot a \cdot b}{3 \cdot \pi^2} \cdot q_{31} = \frac{4 \cdot a \cdot b}{\pi^2} \cdot \left(q_{11} + \frac{q_{31}}{3}\right) \tag{10}$$

Disregarding the dynamic pressures and assuming the compressible adiabatic state has to equation represented by:

$$\delta p = p_0 \cdot \gamma \cdot \frac{\delta V}{V} \quad \mathrm{com}\; \rho \cdot c^2 = p_0 \cdot V \tag{11}$$

Substituting (11) into (12) gives:

$$\delta p = \rho \cdot c^2 \cdot \frac{4}{\pi^2 \cdot h} \cdot \left(q_{11} + \frac{q_{31}}{3}\right) \tag{12}$$

For each mode assuming the acoustic generalized forces are given by:

$$F_{11} = -\int_0^a \int_0^b \delta p \cdot \mathrm{sen}\left(\frac{\pi \cdot x}{a}\right) \cdot \mathrm{sen}\left(\frac{\pi \cdot y}{b}\right) dxdy = -\delta p \cdot \frac{4 \cdot a \cdot b}{\pi^2} \tag{13}$$

$$F_{31} = -\int_0^a \int_0^b \delta p \cdot \mathrm{sen}\left(\frac{3 \cdot \pi \cdot x}{a}\right) \cdot \mathrm{sen}\left(\frac{\pi \cdot y}{b}\right) dxdy = -\delta p \cdot \frac{4 \cdot a \cdot b}{3 \cdot \pi^2} \tag{14}$$

Adopting:

$$K_A = \frac{16\cdot\rho\cdot a\cdot b\cdot c^2}{\pi^4\cdot h} \tag{15}$$

And substituting (15) into the equations (13) and (14):

$$F_{11} = -\left(K_A \cdot q_{11} + \frac{K_A}{3} \cdot q_{31}\right) \tag{16}$$

$$F_{31} = -\left(K_A \cdot \frac{q_{11}}{3} + \frac{K_A}{9} \cdot q_{31}\right) \tag{17}$$

The equations of motion of the two modes are:

$$\begin{cases} M_{11} \cdot \ddot{q_{11}} + K_{11} \cdot q_{11} = F_{11} \\ M_{31} \cdot \ddot{q_{31}} + K_{31} \cdot q_{31} = F_{31} \end{cases} \tag{18}$$

The dynamic equation of the plate is given by:

$$D \cdot \nabla^4 w(x,y) - m \cdot \ddot{w}(x,y) = 0 \tag{19}$$

Where M and K are the mechanical values of generalized mass and generalized stiffness.

Applying weighted residuals in Equation (19):

$$\int_\Omega \left(D \cdot \nabla^4 w(x,y) - m \cdot \ddot{w}(x,y)\right) \cdot \varphi d\Omega = 0 \tag{20}$$

Solving Equation (20) obtain:

$$M_{11} = \rho_{m\cdot} t \cdot \frac{a\cdot b}{4} \tag{21}$$

$$M_{31} = \rho_{m\cdot} t \cdot \frac{a\cdot b}{4} \tag{22}$$

$$K_{11} = D \cdot \frac{a\cdot b}{4} \cdot \left[\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2\right] \tag{23}$$

$$K_{31} = D \cdot \frac{a\cdot b}{4} \cdot \left[\left(\frac{3\cdot\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2\right] \tag{24}$$

Then the coupled system is given by:

$$\begin{cases} -w^2 \cdot M_{11} \cdot q_{11} + K_{11} \cdot q_{11} + K_A \cdot q_{11} + \frac{K_A}{3} \cdot q_{31} = 0 \\ -w^2 \cdot M_{31} \cdot q_{31} + K_{31} \cdot q_{31} + \frac{K_A}{3} \cdot q_{11} + \frac{K_A}{9} \cdot q_{31} = 0 \end{cases} \tag{25}$$

Dividing the two equations by K_{11}:

$$\begin{cases} -w^2 \cdot \frac{M_{11}}{K_{11}} \cdot q_{11} + \frac{K_{11}}{K_{11}} \cdot q_{11} + \frac{K_A}{K_{11}} \cdot q_{11} + \frac{K_A}{3\cdot K_{11}} \cdot q_{31} = 0 \\ -w^2 \cdot \frac{M_{31}}{K_{11}} \cdot q_{31} + \frac{K_{31}}{K_{11}} \cdot q_{31} + \frac{K_A}{3\cdot K_{11}} \cdot q_{11} + \frac{K_A}{9\cdot K_{11}} \cdot q_{31} = 0 \end{cases} \tag{26}$$

$$\alpha = \frac{a}{b} \tag{27}$$

$$\Omega = \frac{w}{w_0} \tag{27}$$

$$w_0 = \sqrt{\frac{K_{11}}{M_{11}}} \tag{28}$$

$$\eta = \frac{K_A}{K_{11}} \tag{29}$$

$$\beta = \frac{K_{31}}{K_{11}} = \frac{D \cdot \frac{a \cdot b}{4} \cdot \left[\left(\frac{3 \cdot \pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2\right]}{D \cdot \frac{a \cdot b}{4} \cdot \left[\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2\right]} = \left(\frac{9+\alpha^2}{1+\alpha^2}\right)^2 \tag{30}$$

Where:

α the aspect ratio of the panel

Ω it is the ratio between the natural frequency and the natural frequency in the fundamental mode

η it is the ratio between acoustic generalized stiffness in the first in vacuo mode and mechanical generalized stiffness in the first in vacuo mode

β it is the ratio between mechanical generalized stiffness in third mode and the first mode.

Writing in matrix form we have:

$$\begin{bmatrix} 1 + \eta - \Omega^2 & \frac{\eta}{3} \\ \frac{\eta}{3} & \beta + \frac{\eta}{9} - \Omega^2 \end{bmatrix} \cdot \begin{bmatrix} q_{11} \\ q_{31} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{31}$$

To satisfy the equation of motion is necessary that:

$$\begin{vmatrix} 1 + \eta - \Omega^2 & \frac{\eta}{3} \\ \frac{\eta}{3} & \beta + \frac{\eta}{9} - \Omega^2 \end{vmatrix} = 0 \tag{32}$$

Solving equation (32) is the characteristic polynomial given by:

$$\Omega^4 - \left(1 + \beta + \frac{10 \cdot \eta}{9}\right) \cdot \Omega^2 + \left(\beta + \eta \cdot \beta + \frac{\eta}{9}\right) = 0 \tag{33}$$

Whose following solution presented are the eigenvalues of the equation used to determine the natural frequency of the modes adopted.

$${\Omega_1}^2 = \frac{\left(1+\beta+\frac{10 \cdot \eta}{9}\right) + \sqrt{\left(1+\beta+\frac{10 \cdot \eta}{9}\right)^2 - 4 \cdot 1 \cdot \left(\beta+\eta \cdot \beta+\frac{\eta}{9}\right)}}{2} \tag{34}$$

$${\Omega_2}^2 = \frac{\left(1+\beta+\frac{10 \cdot \eta}{9}\right) - \sqrt{\left(1+\beta+\frac{10 \cdot \eta}{9}\right)^2 - 4 \cdot 1 \cdot \left(\beta+\eta \cdot \beta+\frac{\eta}{9}\right)}}{2} \tag{35}$$

2.2 Approximation toward “n” modes

In order to evaluate the efficiency of the proposed method by Pretlove and Craggs (1969) and also in a seeking for a method’s convergence, will be presented a method’s generalization to several vibration modes. The assumed notation to the study is represented in Figure 1 and the board deformation, for which is using the Garlekin’s method, can be described as in the form of a finite series given by:

$$W = W_{\mathbf{rs}} = \sum_{r=1}^{r=\bar{r}} \sum_{s=1}^{s=\bar{s}} q_{rs} \cdot \varphi_{rs} \tag{36}$$

Or expandable form:

$$W = q_{11} \cdot \varphi_{11}(x, y) + C_{12} \cdot \varphi_{12}(x, y) + C_{13} \cdot \varphi_3(x, y) + \cdots + C_{\bar{r}\bar{s}} \cdot \varphi_{\bar{r}\bar{s}}(x, y) \tag{37}$$

Where in the symbol q_{ij} represents the movements' coordinates generalized for each mode due the time. The values of φ_{ij} represent the displacement functions to the "n" modes assumed for a board wich is just leaning, as described next.

$$\varphi_{11}(x,y) = \text{sen}\,\frac{1\cdot\pi\cdot x}{L_x}\cdot\text{sen}\,\frac{1\cdot\pi\cdot y}{L_y} \tag{38a}$$

$$\varphi_{12}(x,y) = \text{sen}\,\frac{1\cdot\pi\cdot x}{L_x}\cdot\text{sen}\,\frac{2\cdot\pi\cdot y}{L_y} \tag{38b}$$

$$\varphi_{13}(x,y) = \text{sen}\,\frac{1\cdot\pi\cdot x}{L_x}\cdot\text{sen}\,\frac{3\cdot\pi\cdot y}{L_y} \tag{38c}$$

$$\vdots$$

$$\varphi_{\bar{r}\bar{s}}(x,y) = \text{sen}\,\frac{\bar{r}\cdot\pi\cdot x}{L_x}\cdot\text{sen}\,\frac{\bar{s}\cdot\pi\cdot y}{L_y} \tag{38d}$$

The displacement volume due to deflection is given by:

$$\delta V = \int_A \; W dA = \int_0^{L_x}\int_0^{L_y}\sum_{r=1}^{r=\bar{r}}\sum_{s=1}^{s=\bar{s}} W_{rs}\, dxdy \tag{39}$$

After the resolution of the equation we have:

$$\delta V = \frac{4\cdot a\cdot b}{\pi^2}\cdot\sum_{r=1}^{r=\bar{r}}\sum_{s=1}^{s=\bar{s}}\frac{q_{rs}}{r\cdot s} \qquad (34)$$

Disregarding the dynamic pressures and assuming the compressible adiabatic state has to equation represented by:

$$\delta p = p_0\cdot\gamma\cdot\frac{\delta V}{V} \quad \text{com } \rho\cdot c^2 = p_0\cdot V \tag{40}$$

Substituting (40) and (41) we have:

$$\delta p = \rho\cdot c^2\cdot\frac{4}{\pi^2\cdot h}\cdot\sum_{r=1}^{r=\bar{r}}\sum_{s=1}^{s=\bar{s}}\frac{q_{rs}}{r\cdot s} \tag{41}$$

For each mode assuming the acoustic generalized forces are given by:

$$F_{\bar{m}\bar{n}} = -\int_0^{L_x}\int_0^{L_y}\delta p\cdot\text{sen}\left(\frac{\bar{m}\cdot\pi\cdot x}{a}\right)\cdot\text{sen}\left(\frac{\bar{n}\cdot\pi\cdot y}{b}\right) dxdy = -\delta p\cdot\frac{4\cdot L_x\cdot L_y}{\pi^2\cdot m\cdot n} \tag{42}$$

With:

$$m = 1,2,\cdots,r \qquad e \qquad n = 1,2\cdots,s.$$

Solving equation (42):

$$F_{\bar{m}\bar{n}} = -\frac{16\cdot\rho\cdot L_x\cdot L_y\cdot c^2}{\pi^4\cdot h\cdot m\cdot n}\cdot\sum_{r=1}^{r=\bar{r}}\sum_{s=1}^{s=\bar{s}}\frac{q_{rs}}{r\cdot s} \tag{43}$$

Adopting K_A as acoustic generalized stiffness in the first in vacuo mode given by:

$$K_A = \frac{16\cdot\rho\cdot a\cdot b\cdot c^2}{\pi^4\cdot h} \tag{44}$$

Substituting (44) in (43) we have:

$$F_{\bar{m}\bar{n}} = -\frac{K_A}{m\cdot n}\cdot\sum_{r=1}^{r=\bar{r}}\sum_{s=1}^{s=\bar{s}}\frac{q_{rs}}{r\cdot s} \tag{45}$$

The dynamic equilibrium equations for the assumed modes are represented by:

$$\begin{cases} M_{11} \cdot \ddot{q_{11}} + K_{11} \cdot q_{11} = F_{11} \\ M_{31} \cdot \ddot{q_{31}} + K_{31} \cdot q_{31} = F_{31} \\ \vdots \\ M_{\overline{r}\overline{s}} \cdot \ddot{q_{\overline{r}\overline{s}}} + K_{\overline{r}\overline{s}} \cdot q_{\overline{r}\overline{s}} = F_{\overline{r}\overline{s}} \end{cases} \quad (46)$$

The dynamic equation of the plate is given by:

$$D \cdot \nabla^4 w(x, y) - m \cdot \ddot{w}(x, y) = 0 \quad (47)$$

Where D and M are respectively the flexural stiffness and mass of the plate.

$$D = \frac{E \cdot t^3}{12 \cdot (1 - \nu^2)} \quad (48)$$

Applying weighted residuals in Equation (47):

$$\int_\Omega \left(D \cdot \nabla^4 w(x, y) - m \cdot \ddot{w}(x, y) \right) \cdot \varphi d\Omega = 0 \quad (49)$$

Integrating by parts we have:

$$\int_\Omega D \cdot \nabla w^2 \cdot \nabla^2 \varphi d\Omega - \int_\Omega m \cdot \ddot{w} \cdot \varphi d\Omega = 0 \quad (50)$$

First analyze and calculate the mass of the array elements, by:

$$M_{ij}^{kl} = \int_\Omega m_{ij} \cdot \ddot{w}_{ij} \cdot \varphi_{ij} d\Omega = \int_0^{L_x} \int_0^{L_y} \rho_{m\cdot} \cdot t \cdot \ddot{q_{ij}} \cdot \varphi_{ij} \cdot \varphi_{kl}\, dxdy \quad (51)$$

Replacing the scroll function in Equation (51) we have:

$$M_{ij}^{kl} = \rho_{m\cdot} \cdot t \cdot q_{ij} \cdot \int_0^{L_x} \int_0^{L_y} \left(sen\left(\frac{i \cdot \pi \cdot x}{L_x}\right) \cdot sen\left(\frac{j \cdot \pi \cdot y}{L_y}\right) \right) \cdot \left(sen\left(\frac{k \cdot \pi \cdot x}{L_x}\right) \cdot sen\left(\frac{l \cdot \pi \cdot y}{L_y}\right) \right) dxdy \quad (52)$$

Or

$$M_{ij}^{kl} = \rho_{m\cdot} \cdot t \cdot q_{ij} \cdot \int_0^{L_x} \left(sen\left(\frac{i \cdot \pi \cdot x}{L_x}\right) \cdot sen\left(\frac{k \cdot \pi \cdot x}{L_x}\right) \right) dx \cdot \int_0^{L_y} \left(sen\left(\frac{j \cdot \pi \cdot y}{L_y}\right) \cdot sen\left(\frac{l \cdot \pi \cdot y}{L_y}\right) \right) dy \quad (53)$$

Solving the integral present in Equation (53):

$$\int_0^{L_x} \left(sen\left(\frac{i \cdot \pi \cdot x}{L_x}\right) \cdot sen\left(\frac{k \cdot \pi \cdot x}{L_x}\right) \right) dx = \begin{cases} \frac{L_x}{2} & se\ i = k \\ 0 & se\ i \neq k \end{cases} \quad (54)$$

$$\int_0^{L_x} \left(sen\left(\frac{j \cdot \pi \cdot y}{L_y}\right) \cdot sen\left(\frac{l \cdot \pi \cdot x}{L_y}\right) \right) dy = \begin{cases} \frac{L_y}{2} & se\ j = l \\ 0 & se\ j \neq l \end{cases} \quad (55)$$

Substituting in Equation (53):

$$M_{ij} = \begin{cases} \frac{\rho_{m\cdot} \cdot t \cdot q_{ij} \cdot L_x \cdot L_y}{4} & se\ i = j \\ 0 & se\ i \neq j \end{cases} \quad (56)$$

With the analysis and calculation of elements of stiffness matrix we have:

$$K_{rs}^{\overline{r} \cdot \overline{s}} = \int_\Omega D \cdot \nabla^2 w_{rs} \cdot \nabla^2 \varphi_{\overline{r}\overline{s}} d\Omega \quad (57)$$

Replacing the scroll function in Equation (57) we have:

$$K_{rs}^{\bar{r}\cdot\bar{s}} = D \cdot \int_0^{L_x}\int_0^{L_y}\left[\left(\frac{\partial}{\partial x^2}+\frac{\partial}{\partial y^2}\right)\times\left(\text{sen}\left(\frac{\bar{r}\cdot\pi\cdot x}{L_x}\right)\cdot\text{sen}\left(\frac{\bar{s}\cdot\pi\cdot y}{L_y}\right)\cdot q_{rs}\right)\right]\cdot\left[\left(\frac{\partial}{\partial x^2}+\frac{\partial}{\partial y^2}\right)\times\left(\text{sen}\left(\frac{r\cdot\pi\cdot x}{L_x}\right)\cdot\text{sen}\left(\frac{s\cdot\pi\cdot y}{L_y}\right)\right)\right]dxdy \tag{58}$$

Or

$$K_{rs}^{\bar{r}\cdot\bar{s}} = D\cdot q_{rs}\cdot\left[\left(\frac{\pi}{L_x}\right)^2+\left(\frac{\pi}{L_y}\right)^2\right]^2\cdot\int_0^{Lx}\left(\text{sen}\left(\frac{\bar{r}\cdot\pi\cdot x}{L_x}\right)\cdot\text{sen}\left(\frac{r\cdot\pi\cdot y}{L_y}\right)\right)dx\cdot\int_0^{Lx}\left(\text{sen}\left(\frac{\bar{s}\cdot\pi\cdot x}{L_x}\right)\cdot\text{sen}\left(\frac{s\cdot\pi\cdot y}{L_y}\right)\right)dy \tag{59}$$

Solving Equation (59) we have:

$$K_{rs}^{\bar{r}\cdot\bar{s}} = D\cdot q_{rs}\cdot\left[\left(\frac{\pi}{L_x}\right)^2+\left(\frac{\pi}{L_y}\right)^2\right]^2\cdot\begin{Bmatrix}\frac{L_x}{2}\text{ se } r=\bar{r}\\ 0\text{ se } r\neq\bar{r}\end{Bmatrix}\cdot\begin{Bmatrix}\frac{L_y}{2}\text{ se } s=\bar{s}\\ 0\text{ se } s\neq\bar{s}\end{Bmatrix} \tag{60}$$

Or

$$K_{rs}^{\bar{r}\cdot\bar{s}} = \begin{cases}\frac{D\cdot q_{rs}\cdot L_x\cdot L_y}{4}\cdot\left[\left(\frac{\pi}{L_x}\right)^2+\left(\frac{\pi}{L_y}\right)^2\right]^2 & \text{ser } r=\bar{r} \text{ e } s=\bar{s}\\ 0\end{cases} \tag{61}$$

So, in the matrix form we have:

$$\left[-w^2\cdot M+K-A\right]\cdot\{q\}=\{0\} \tag{62}$$

Where M is the modal mass matrix, K the modal stiffness matrix plate, A is the acoustic stiffness matrix and q is the column vector of the generalized coordinates. The matrix forms are as follows:

$$M=\begin{bmatrix}M_{11} & 0 & \cdots & 0\\ 0 & M_{22} & \cdots & 0\\ \vdots & \vdots & \ddots & 0\\ 0 & 0 & \cdots & M_{nn}\end{bmatrix} \tag{63}$$

$$K=\begin{bmatrix}K_{11} & 0 & \cdots & 0\\ 0 & K_{22} & \cdots & 0\\ \vdots & \vdots & \ddots & 0\\ 0 & 0 & \cdots & K_{nn}\end{bmatrix} \tag{64}$$

$$A=\begin{bmatrix}K_{A11} & K_{A12} & \cdots & K_{A1n}\\ K_{A21} & K_{A22} & \cdots & K_{A2n}\\ \vdots & \vdots & \ddots & \vdots\\ K_{An1} & K_{An2} & \cdots & KA_{nn}\end{bmatrix} \tag{65}$$

Replacing the matrices in Equation (62):

$$\begin{bmatrix}-w^2M_{11}+K_{11}+K_{A11} & K_{A12} & \cdots & K_{A1n}\\ K_{A21} & -w^2M_{22}+K_{22}+K_{A22} & \cdots & K_{A2n}\\ \vdots & \vdots & \ddots & \vdots\\ K_{An1} & K_{An2} & \cdots & -w^2M_{nn}+K_{nn}+K_{Ann}\end{bmatrix}\cdot\begin{Bmatrix}q_{11}\\ q_{12}\\ \vdots\\ q_{rs}\end{Bmatrix}=\begin{Bmatrix}0\\ 0\\ \vdots\\ 0\end{Bmatrix} \tag{66}$$

Dividing the equation by modal stiffness K_{11} in the first mode we have:

$$\begin{bmatrix} \phi_1 + \eta - \Omega^2 & \frac{\eta}{2} & \cdots & \frac{\eta}{n} \\ \frac{\eta}{2} & \phi_2 + \frac{\eta}{4} - \Omega^2 & \cdots & \frac{\eta}{2 \cdot n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\eta}{n} & \frac{\eta}{2 \cdot n} & \cdots & \phi_n + \frac{\eta}{n \cdot n} - \Omega^2 \end{bmatrix} \cdot \begin{Bmatrix} q_{11} \\ q_{12} \\ \vdots \\ q_{rs} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{Bmatrix} \quad (67)$$

Where:

$$\alpha = \frac{a}{b} \quad (68)$$

$$\Omega = \frac{w}{w_0} \quad (69)$$

$$w_0 = \sqrt{\frac{K_{11}}{M_{11}}} \quad (70)$$

$$\eta = \frac{K_A}{K_{11}} \quad (71)$$

$$\phi_i = \frac{K_{rs}}{K_{11}} = \left(\frac{r^2 + s^2 \cdot \alpha^2}{1 + \alpha^2}\right) \quad (72)$$

To satisfy the equation of motion is necessary that:

$$\begin{vmatrix} \phi_1 + \eta - \Omega^2 & \frac{\eta}{2} & \cdots & \frac{\eta}{n} \\ \frac{\eta}{2} & \phi_2 + \frac{\eta}{4} - \Omega^2 & \cdots & \frac{\eta}{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\eta}{n} & \frac{\eta}{2n} & \cdots & \phi_n + \frac{\eta}{n \cdot n} - \Omega^2 \end{vmatrix} = 0 \quad (73)$$

The solution of Equation (67) determines the, values representing the eigenvalues of matrix used to determine the natural frequencies of each mode by the equation:

$$f = \frac{\Omega \cdot w_0}{2 \cdot \pi} \quad (74)$$

By replacing the values of Ω represented in the matrix in Equation (73) determines that the eigenvectors are the generalized coordinates used to define the mode shapes.

3 COMPUTATIONAL IMPLEMENTATION

For the development of the equations presented in this work it was used the mathematical software MAPLE 2016. Initially, it was defined the entrance values described by the acoustic cavity's dimensions in study and by the characteristics of the cavity and the board such as: Young module, flexible rigidity, specific mass. Then, it was accomplished the mass calculation and the acoustic rigidity. After the determination of the characteristics and of the constants presented in the section 2.2, it was created the matrix presented in the Equation (67) which satisfies the movement equation. Finally, it was determined the eigenvalues and the eigenvectors for each modal form trough the commands "EigenVectors" and "EigenValues" of the MAPLE 2016. With the determination of the eingenvalues and eigenvectors, the next step was to implement on the MATLAB 2015 a function in order to create charts which represent the modal forms. It is important to mention that the implemention was not accomplished only through the MATLAB 2015 due the packaging problems of the matrix founded by the author.

4 RESULTS

The cavity studied in this work was proposed by Pretlove (1965) shown in Figure (1). Data from the cavity proposed by Pretlove (1965) are presented in the following Tables 1-3:

Table 1. Data fluid

Sound's speed	337.33 m/s
Specific mass' air	1.2466 kg/m³
Temperature	10ºC

Table 2. Characteristics of Cavity Acustic

Dimensões (metros)	
L_x	0.3048
L_y	0.1524
h	0.1524

Table 3. Characteristics of Plate

Analyzed plate	
Material	Aluminum
L_x	0.3048 m
L_y	0.1524 m
Thickness	0.0016256 m
Specific Mass	2700 kg/m³
Elasticity Modulus	67 GPa
Poisson's Coefficient	0.33

In this section it is presented the achieved results to the natural frequencies and the vibration modes, using the method presented by Pretlove and Craggs (1969) with a implementation of six modal forms

4.1 Natural Frequency

Aiming a evaluation of the convergence of the results, calculations were done using implementation from each modal form. During the calculations it was noticed that as from seven modes can occur a problem involving a bad conditioning in the matrix, disturbing the achievements of the eigenvalues and eigenvectors. Therefore, for getting results it was used six modal forms, as it is demonstrated in Table 4.

Tabela 4. Modal forms adopted by the study

Vibration modes			
X	**Y**	**X**	**Y**
1	1	1	3
3	1	3	3
5	1	5	3

To evaluate the efficiency of the method results obtained were compared with the results obtained by (PRETLOVE, 1965) and (ROJAS, 2015) described below. Table 6 only shows results of four modal forms in accordance with the results reported by these authors.

Table 5. Natural frequencies of the plate in a vacuum and coupled (plate + cavity).

Modes	1º mode	2º mode	3º mode	4º mode	5º mode	6º mode
(1,1)	217,5459	217,5331	217,5323	217,5309	217,5308	217,5308
(3,1)	-	544,4775	544,4775	544,4774	544,4774	544,4774
(5,1)	-	-	1.213,8468	1.213,8467	1.213,8467	1.211,8467
(1,3)	-	-	-	1.548,7535	1.548,7535	1.578,4090
(3,3)	-	-	-	-	1.883,4764	1.819,3769
(5,3)	-	-	-	-	-	2.613,0268

The tables 6 and 7 present respectively the natural frequencies obtained for the plate-cavity system coupled to six modes and the comparison of the achieved results by Pretlove and Craggs (1969) and Rojas (2015). It was choosed for presentation only the four first ones modal forms due the comparison among other authors.

Table 6. Natural Frequencies - comparison of results

Modes	Analytical 6 modes Uncoupled	Analítico 6 modes Coupled	Semi-Analytical Pretlove(1965)	Analytical Rojas(2015)
(1,1)	209,27	217,53	216,39	216,19
(3,1)	544,11	544,48	541,60	544,22
(5,1)	1213,79	1.211,85	1.212,40	1.214,17
(1,3)	1548,63	1.578,41	1.543,82	1.549,23

Table 7. Natural Frequencies - comparison of results

Modes	Comparison (%)		
	Coupled/Uncoupled	Coupled/Pretlove	Coupled/Rojas
(1,1)	3,800	0,524	0,616
(3,1)	0,068	0,528	0,047
(5,1)	-0,160	-0,043	-0,189
(1,3)	1,887	2,191	1,849

4.2 Mode Shapes

The pictures presented in this section represent the determined modal forms with implementation of six modal forms.

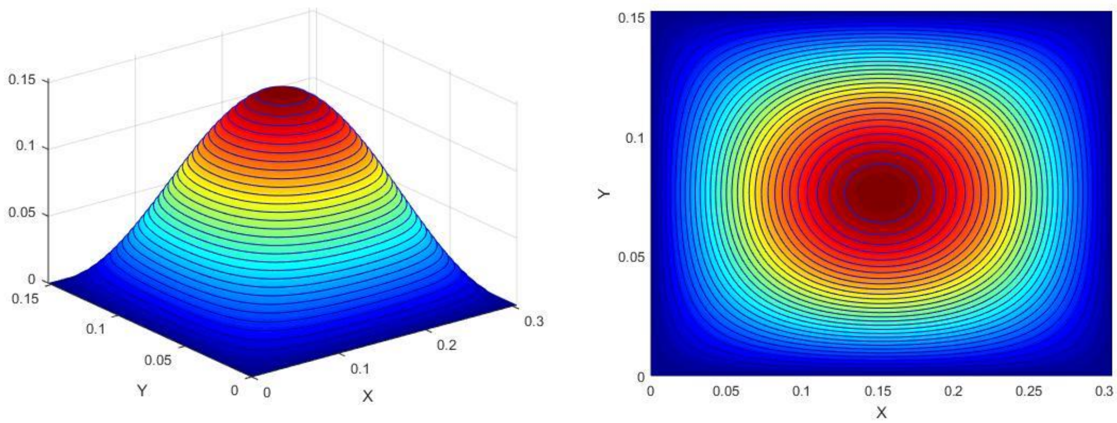

Figure 3. Modal form (1.1) with natural frequency of 217.53 Hz

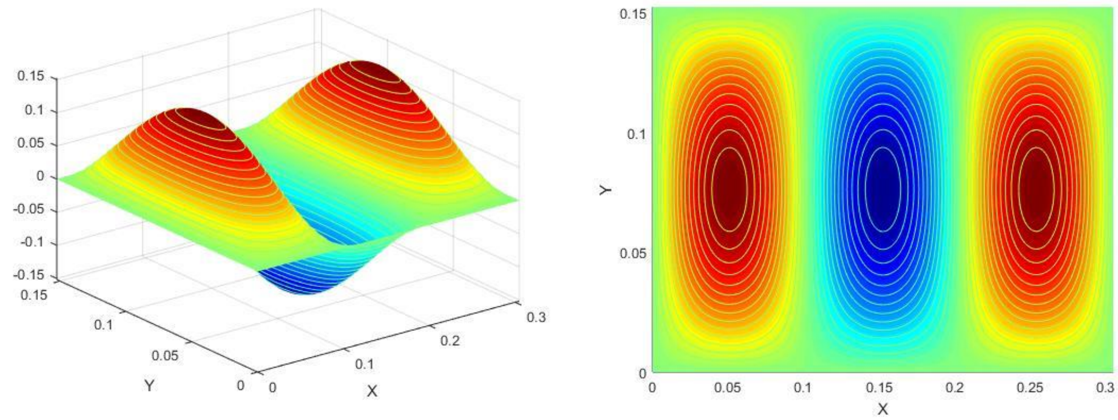

Figure 4. Modal form (3.1) with natural frequency of 544.48 Hz

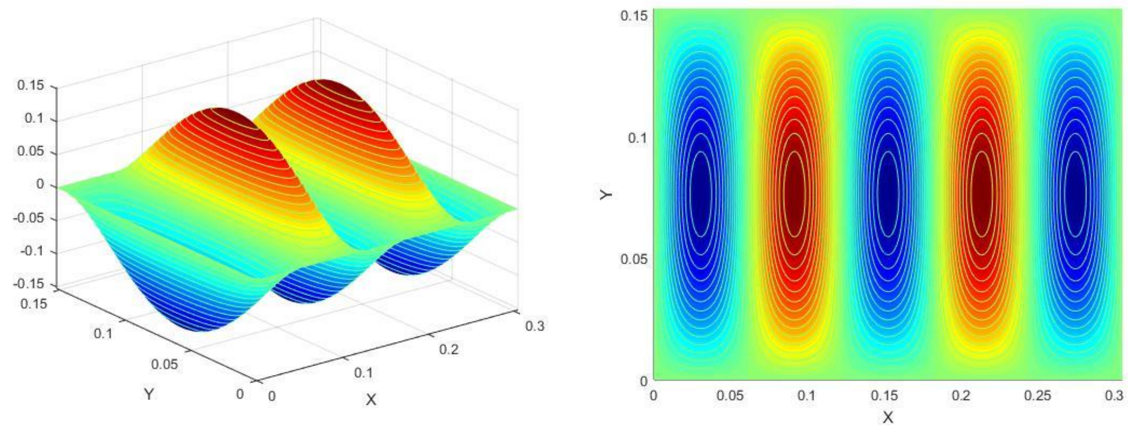

Figure 5. Modal form (5.1) with natural frequency 1211.85 Hz

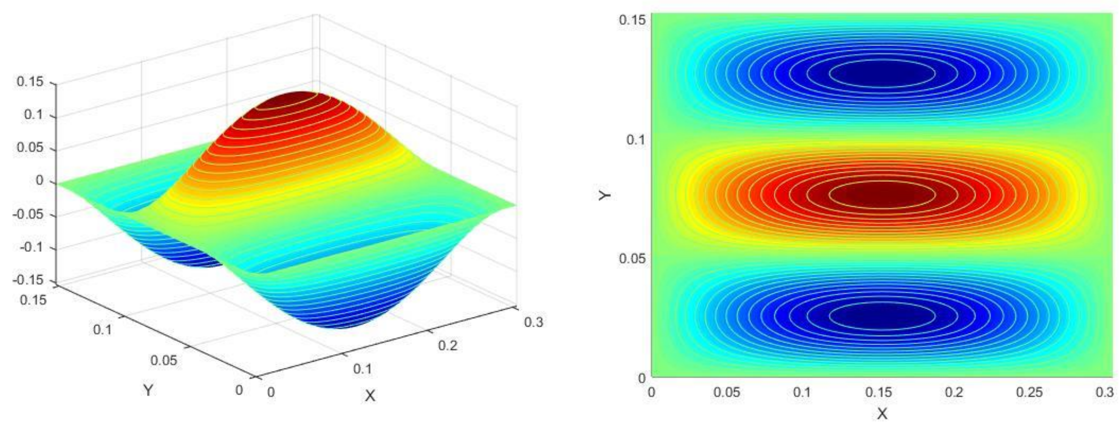

Figure 6. Modal Form 61.3) with natural frequency 1578.41 Hz

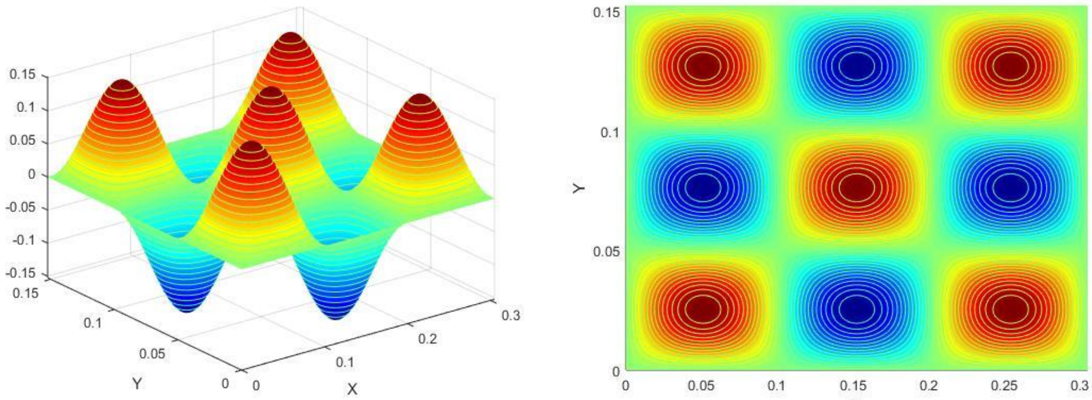

Figure 7. Modal form (3.3) with natural frequency 1819.78 Hz

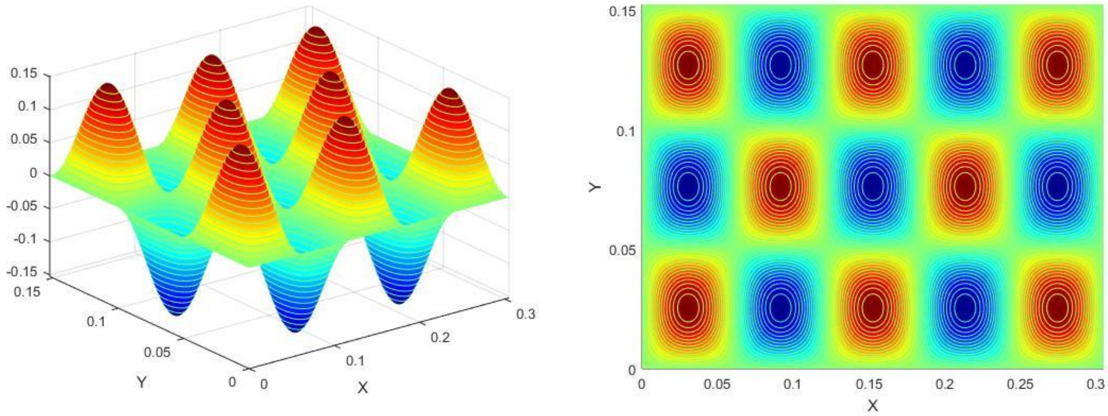

Figure 8. Modal form (5.3) with natural frequency 2613.03 Hz

As figuras 9 a 11 apresentam as deformadas acopladas e desacopladas da estrutura para $x = \frac{L_x}{4}$

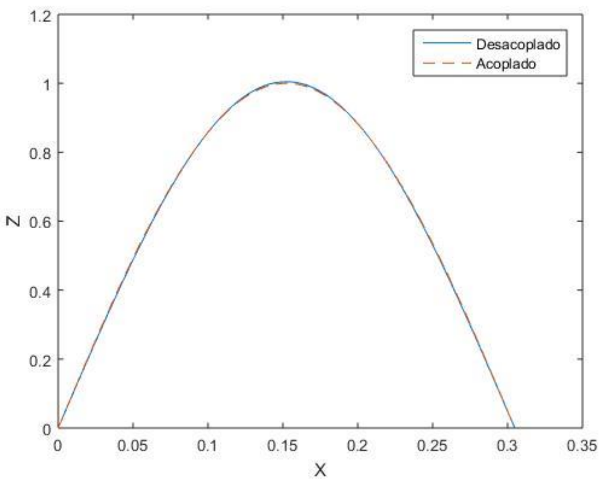

Figure 9. Deformed coupled and uncoupled (1,1)

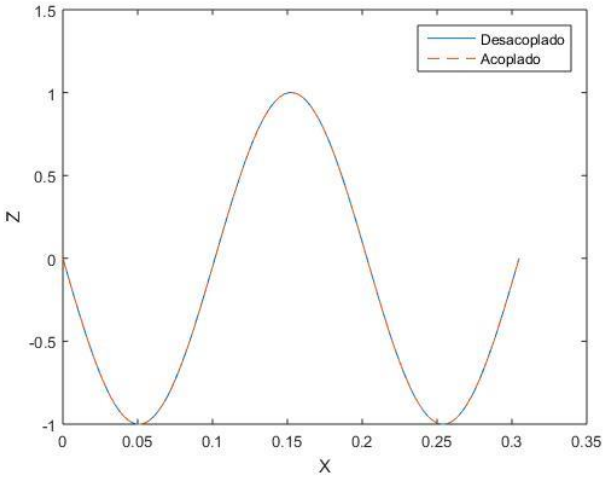

Figure 10. Deformed coupled and uncoupled of mode structure (3,3)

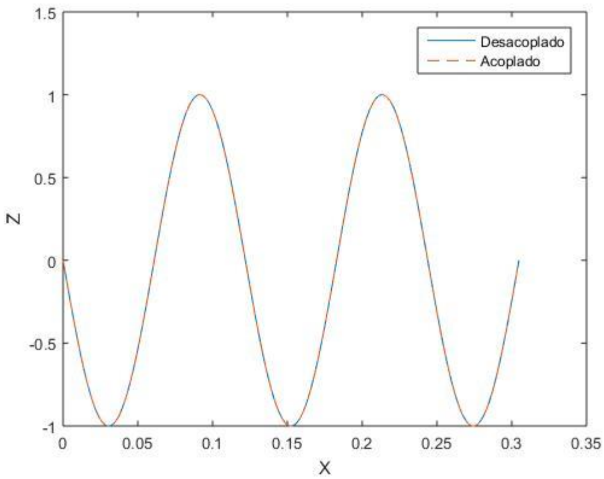

Figure 11. Deformed coupled and uncoupled of mode structure (5,3)

5 CONCLUSION

In this work it was studied the analytical solution for the problem of the acoustic vibro of the system, rigid acoustic cavity coupled on a flexible board presented by Pretlove (1969). The presented method is an simple approach which allowed a prediction of the system's natural frequency and determination of the vibration modes. For the calculations it was done an implementation of several modal forms aiming an evaluation of the method's convergence. It was noticed that the presented method is efficient for the implementation of six vibration modes. As from the implementation of seven vibration modes an error of bad conditioning occurred, this was justified by the fast growing of the elements from the main diagonal inregard to the other elements. Comparing the achieved results using the coupled and the uncoupled mode it was noticed that the approximated results present a maximum difference of 3,8%. The achieved results were put in comparison to the results achieved by Pretlove (1965) and Rojas (2015), aiming an evaluation of the method's efficiency. It was noticed that the achieved results demonstrated, for four modal forms, a maximum difference of 2.19%

when in comparison to Pretlove (1965) and of 1.849% when in comparison to Rojas (2015), such difference can be reduced with a implementation of more vibration modes. Comparing only the first three modal forms it is perceptible that the maximum error can reduce about 0.6% in both cases, proving that this method is a simple and efficient method for the validation of the studied cavity. The pictures 9 and 11 indicate that the coupled and uncoupled settings are practically identical, which is a contribution to the adoption of a simplified analytical procedure, considering that in this cases it is adopted deformed coupled equal as the respective deformed in vacuum. It is important to emphasize that the assumption of a deformed coupled equal as a vacuum coupled it is not totally valid, although in the proposed problem it has demonstrated satisfied results. It is suggested the search for a method in order to solve the problem of the bad conditioning of the matrix aiming a implementation of more modal forms and also aiming a precise study for the method's convergence

ACKNOWLEDGEMENTS

The second author thank to the FAP-DF for the financial supporting (CarAcous2 Project, process number 2010/00351-0).

REFERENCES

Ferreira, A. **Comparação analítica numérica de cavidades acústicas e vibroacústicas.** Dissertação de Mestrado, Universidade de Brasilia, Brasilia DF FT Departamento de Engenharia Mecânica, 2012.

Pretlove, A. J. **Free vibrations of a rectangular panel backed by a closed rectangular.** Journal of Sound Vibration, England, v. 3(2), p. 197 - 209, 1965.

Pretlove, A. J.; Craggs, a. **A Simple Aproach To Coupled Panel-Cavity Vibrations**. Journal of Sound Vibration, England, v. 11 (2), 207 - 215 1969.

Ribeiro, P. **Soluções Analíticas para Cavidades Acústicas Bidimensionais com aplicação ao Estudo de Problemas de Interação Dinâmica Barragem-Reservatório.**Tese de Doutorado, Universidade de Brasilia, Brasilia DF, FT Departamento de Engenharia Civil e Ambiental, 2010.

Rojas Huacanca, J. R. **Solução Analítica de uma Cavidade Acústica Rígida Acoplada a uma Placa**. Universidade de Brasília. Faculdade UnB Gama. Distrito Federal, p. xvi, 118 p. 2015.