



DEVELOPMENT OF A HYBRID METHOD TO DETECT STRUCTURAL DAMAGE

Rafaelle Piazzaroli Finotti

rafaelle.finotti@engenharia.ufjf.br

Flávio de Souza Barbosa

flavio.barbosa@ufjf.edu.ufjf.br

Alexandre Abrahão Cury

alexandre.cury@ufjf.edu.ufjf.br

Programa de Pós-Graduação em Modelagem Computacional

Universidade Federal de Juiz de Fora

Campus Universitário sn 36036-330 Juiz de Fora/MG, Brazil.

Roberto Leal Pimentel

r.pimentel@uol.com.br

Gabriel Soares Ferreira

gabrielsoaresf@gmail.com

Lucas Farias Barbosa Melo

lucasfarias_jp@hotmail.com

Universidade Federal da Paraíba

Campus Universitário sn 50051-900 João Pessoa/PB, Brazil.

Abstract. *Structural damage detection using dynamic measurements has led to the development of several techniques in the last decades. Most of these methods associate modal variations of the structure to damage like methods based on strain energy deviation, methods based on changes in curvature mode shapes, flexibility matrix analysis, etc. Although these techniques aforementioned are mostly efficient to identify structural alterations in numerical models, they have difficulties in practical applications with experimental data. Thus, hybrid methods to detect the presence of damage directly from raw dynamic measurements in*

addition to structural modal characteristics can be a promising field of research, involving strategies based on artificial intelligence and higher-order statistics. This work aims to present the preliminary results of a hybrid method to detect structural damage. Using modal data and also higher-order statistics of structural time histories as inputs of artificial intelligence algorithms, the viability of the proposed methodology is initially evaluated. Two applications are analyzed: a simply supported numerical beam and an experimental tested prototype concrete slab. The good results achieved motivate the continuous development of the proposed hybrid method.

Keywords: *Structural Dynamics, Damage Identification, Computational Intelligence.*

1 INTRODUCTION

Damage in structures can be caused by design flaws, constructive problems, structural overload or natural events. Structural Health Monitoring (SHM) enables damage prevention and structural maintenance to ensure safe conditions for users (Cachot *et al.*, 2015). The interest in structural damage identification is a topic of important engineering researches and has gained increasing attention over the years. In this context, some techniques to detect and evaluate structural changes using vibration data have been discussed, as it can be seen in Alves *et al* (2015) and Alvandi & Cremona (2002).

SHM has as principal aim the development of reliable and robust techniques able to detect, locate and quantify the affected regions of the structure. Due to the fact that structural deterioration process mainly reduces structural stiffness and changes vibrational characteristics, damage identification methods are usually based on modal parameters or on structural dynamic measurements directly. With this approach, several methods were developed: The Modal Assurance Criterion (MAC), employed as a correlation indicator between damaged and undamaged mode shapes; The Strain Energy Method (SEM); Indicator based on mode shapes curvature with and without damage; Analysis of flexibility matrix; Methods based on modal properties changes, etc.

Although these techniques mentioned above are mostly efficient in identifying structural alterations in numerical models, they have difficulties in practical applications with experimental data. For this reason, hybrid methods to detect the presence of damage from experimental data directly using time domain data in addition to structural modal characteristics can be considered a good alternative for this problem. Some new strategies have been proposed using Higher-Order Statistics (HOS) and artificial intelligence, as Artificial Neural Networks (ANN) and Support Vector Machines (SVM). The HOS allows distinguishing apparently similar databases by inferring new statistic properties from higher-order statistic cumulants, whereas the artificial intelligence methods can recognize similar observations in a database and separate them into groups which share the same characteristics.

The focus of this study is to evaluate the viability of a hybrid method to detect structural damage and to present some preliminary results. Thus, modal data and also higher-order statistics of structural time domain series are used as inputs of artificial intelligence algorithms. The results given by ANN and SVM algorithms are analyzed and compared in two applications: a simply supported numerical beam and an experimental tested prototype concrete slab.

2 DAMAGE IDENTIFICATION METHODS BASED ON MODAL CHARACTERISTICS

The fundamental idea of structural damage assessment using modal characteristics is that damage changes the physical properties of the structure, such as mass, stiffness or flexibility, affecting natural frequencies, mode shapes and modal damping. Considering this, methods based on the variation of structural vibration characteristics and on indicators built from these modal parameters have been developed, as shortly described in the following paragraphs.

In general, most studies show that natural frequencies decrease with damage increase. It seems intuitive, since damage reduces the structural stiffness. Cawley & Adams (1979) are one of the pioneers in assessing the integrity of structures using the variation of natural frequencies as a damage indicator. Since then, many techniques and improvements have been proposed over the years. Messina *et al.* (1998) developed a correlation coefficient to detect and quantify damage called Multiple Damage Location Assurance Criterion (MDLAC). This coefficient is based on the sensitivity of the frequency to damage and on a statistical correlation between the predictions of the frequency changes and the measured frequency. In the work of Fox (1992), experimental and numerical results for a beam with a crack-like defect showed that natural frequencies are sensitive indicators of damage for that case. Pimentel *et al.* (2015) evaluated concrete precast slabs and observed a distinct pattern variation of the natural frequencies for cracked and uncracked slabs.

Other works are concerned with the development of techniques based on mode shapes changes. An example is the Modal Assurance Criterion (MAC), proposed by Allemang & Brown (1982), a correlation indicator that can be employed between damaged and undamaged mode shapes. The MAC coefficient varies between 0 and 1, where 0 means no correlation and 1 represents a perfect correlation. A variation of MAC index was presented by Lieven & Ewins (1988), the Coordinate Modal Assurance Criterion (COMAC), that measures the correlation of several mode shapes for each degree-of-freedom. A large deviation from 1 in the COMAC index indicates the structural damage presence. If COMAC is equal 1, it has a perfect correlation for the coordinate displacement. Regarding the damping ratio to identify damage, there are not many studies being done in this subject, but the results of the works done by Ndambi *et al.* (2000) and Kawiecki (2001) suggested the modal damping measurement as a useful parameter.

Another researches have been focused on detecting structural flaws using characteristics that come from modal parameters. Pandey *et al.* (1991) introduced an indicator based on damaged and undamaged mode shape curvatures, which associates the change in flexural stiffness to the change in curvature. A few years later, Kim & Stubbs (1993) proposed the Strain Energy Method (SEM), which detects and locates the structural affected region based on the strain energy deviation before and after the damage occurrence. Most recently, Cury *et al.* (2011) presented a hybrid approach using the SEM method and natural frequencies to locate and quantify damage. The analysis of flexibility is another method concerned with identifying and locating the affected area of the structure, where the presence of damage is associated to the structural stiffness reduction and, therefore, to the increase in flexibility (Pandey & Biswas, 1994).

3 DAMAGE IDENTIFICATION METHODS BASED ON TIME DOMAIN SERIES

Most damage identification techniques mentioned in the previous section have shown to be efficient in numerical models. However, they present some difficulties in practical application with experimental data. Furthermore, the method used to extract the modal parameters from dynamic signatures can considerably affect the results of the damage detection techniques, introducing additional uncertainties (Alvandi & Cremona, 2006). In an effort to give alternatives to these issues, new techniques interpreting the time domain signal directly using statistical analysis and artificial intelligence for pattern recognition have been suggested, such as in Iwasaki *et al.* (2004), Haritos & Owen (2004) and Wen *et al.* (2007).

Optimization algorithms such as Artificial Neural Networks (ANN), Support Vector Machines (SVM) and other artificial intelligence technologies are also considered useful tools for solving structural damage assessment problems. These algorithms work as a classifier, which try to identify damage levels using feature input data extracted from dynamic response measurements. Some concepts of Neural Network and Support Vector Machine are presented in this section. At last, the Higher-Order Statistic to characterize the structural vibration data is approached.

Despite the good results in several cases, time domain measurements are not widely used due to the difficulties in managing a large amount of raw data and the lack of tools to deal with them.

3.1 Artificial Neural Network

Neural networks are adaptive learning machines built from many different processing elements (PE), called neurons. In pattern classification problem, the decision surface is divided into regions representing the classes. The boundary decisions are estimated in a learning process and constructed through the statistic variability among classes. The most common ANN is the Multilayer Perceptron (MLP), a feedforward network composed of interconnected processing elements trained with nonlinear functions. Each PE connection has two associated adjustable parameters, weight and bias, scaled by backpropagation algorithms (learning rules) in order to minimize the error between the predicted and measured output. As explained in Principe *et al.* (1999), the PEs sum all these contributions and produce an output that is a nonlinear function of the result. The training stage is an iterative process that only finishes when a criterion for the error between ANN and data is satisfied. At the end, the perceptron is able to generalize other inputs that belong to the same class but were not used for training.

In order to ensure the generalization ability of the ANN model, a cross-validation method is used on the training stage. The cross-validation consists in partitioning the training set in two subsets, training and validation, and testing the neural network model performance with the validation subset at each interval of iterations. The training phase is interrupted when the error in subset validation starts to increase, in other words, when the maximum point of generalization is found.

There are several cross-validation methods, however, this article is concerned only in k -fold cross-validation (Kohavi, 1995). In this technique, the original data set is randomly divided in k subsets with approximately the same amount of samples, containing examples of all classes. In each iteration, a distinct subset is used for testing and the other $k-1$ subsets are

used for training. The training and test process are repeated several times. The precision estimative is the number of the correct classifications divided by the sample number of the subset k . At the end, each sample from the original dataset was tested once, so there was no overlapping data. The final performance of the model is calculated through the mean of the correct predicts of all k -fold validations.

3.2 Support Vector Machine

Another popular artificial intelligence technology for pattern recognition problem is the Support Vector Machine (SVM). SVM is a statistical learning algorithm trained to determine the boundary between two classes of data in a space, where an optimal separating hyperplane is constructed in order to maximize the margin and minimize the misclassification (Vapnik, 1995). The maximization of the margin is based on an optimization function to minimize the euclidian norm of the vector that defines the direction of the separating hyperplane. The training data points located at the maxim margins are called support vectors, as illustrated in Fig. 1.

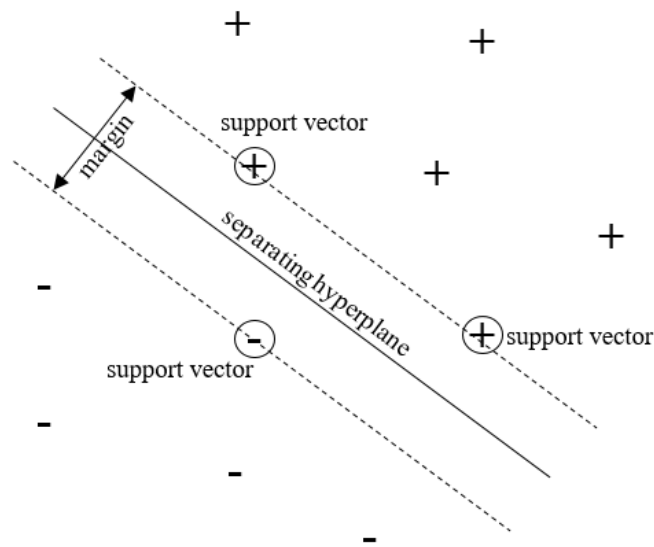


Figure 1. Linear SVM illustration.

For non-linear binary classification, the input is mapped into a high-dimensional feature space through a kernel function. The kernel function used in this article is the Gaussian, also called Radial Basis Function (RBF). In this case, the SVM has two free parameters that need to be specified: σ from the RBF kernel function; and C , a regularization parameter from the formulation of the margin maximizing, used to avoid the data overfitting. These parameters are estimated by training an SVM for multiple values of C and σ . The pair which minimizes the generalization error is chosen.

3.3 Higher-Order Statistics

Most of dataset have Gaussian behavior and are completely characterized by the second-order statistic, which the autocorrelation of two series in the time domain offer a primary characterization of the measured data. However, there are situations in which the product of two sequences of measurements doesn't provide enough information, requiring another technique to distinguish the signal. Higher-Order Statistic (HOS) is a technique that uses

higher-order cumulants to infer new properties about the data, where the time domain estimators have been obtained after multiplied by more than two time-series (De la Rosa *et al.*, 2013). For the case of structural dynamic data, the measurement signals are very similar in the presence or absence of damage. Therefore, the HOS can provide parameters to identify subtle differences among the time domain signals, enabling the detection of structural alterations.

The ten different statistical indicators (first, second, third and fourth order) used to characterize the time domain data in this article are listed below:

Peak:

$$x_{peak} = \max|\mathbf{x}| \quad (1)$$

Mean:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad (2)$$

Mean Square:

$$x_{sq} = \frac{1}{n} \sum_{i=1}^n (x_i)^2 \quad (3)$$

Root Mean Square:

$$rms = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i)^2} \quad (4)$$

Variance:

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \quad (5)$$

Standard Deviation:

$$\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2} \quad (6)$$

Skewness:

$$s = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^3}{\sigma^3} \quad (7)$$

Kurtosis:

$$k = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^4}{\sigma^4} \quad (8)$$

Crest Factor:

$$Cf = \frac{x_{peak}}{rms} \quad (9)$$

K-factor:

$$Kf = x_{peak} \cdot rms \quad (10)$$

4 A HIBRID METHOD TO DETECT STRUCTURAL DAMAGE

In view of the difficulties inherent to the aforementioned methods, the viability of a hybrid method to identify structural damage is evaluated in this work. The idea is to use changes on modal data and higher-order statistic parameters of structural time domain series together, as input of artificial intelligence algorithms. In order to study the viability of the proposed method, an ANN and SVM algorithms were constructed to work as a classifier for damage identification and their results were compared. A simply supported numerical beam and an experimental tested prototype concrete slab were analyzed.

Considering that damage occurrence induces changes on structural stiffness, the modal input for each damage level is characterized by the changes on natural frequencies. Ten statistics parameters are calculated to represent the structural time domain measurements: peak, mean, mean square, root mean square, variance, standard deviation, skewness, kurtosis, crest factor and K-factor, as discussed in section 3.3.

The neural network implemented is a MLP with one hidden layer. The numbers of processing elements in the output layer correspond to the levels of damage. The MLP was trained performing the 10-fold cross-validation method. The data was partitioned as follows: 1/10 for test, 1/10 for validation and 1-2/10 for training. Levenberg-Marquadt optimization method (Hagan & Menhaj, 1994) was chosen as training function, using the mean square to assess the error and a sigmoid hyperbolic tangent as activation function.

Other artificial intelligence method used in the present work is the SVM. The algorithm was trained using Gaussian Radial Basis Function kernel, where the best parameters sigma and C were selected by training an SVM for different values of these parameters in a 10-fold cross-validation. The SVM multi-class classification problem was solved by using one-against-all strategy (Bishop, 2006). This method consists on constructing one binary SVM model per class, training each model to distinguish the samples of one class from the remaining samples of the other classes, as illustrated in Fig. 2.

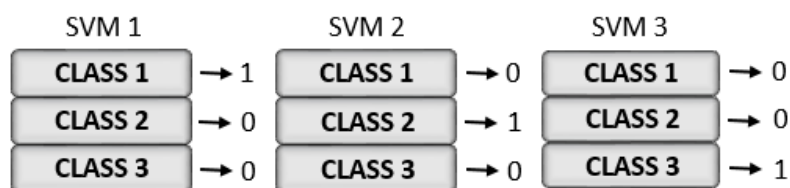


Figure 2. One-against-all strategy.

The ANN and SVM artificial intelligence algorithms, as well as the statistics indicators, were constructed with toolboxes and functions available in Matlab®.

5 SIMPLY SUPPORTED BEAM APPLICATION – NUMERICAL TESTS

5.1 Description of the beam model

The present section analyzes the damage identification algorithms based on a numerical application using a finite element model of a simply supported beam. Structural dynamic responses were obtained by a numerical model of a simply supported beam, made of steel, having I-shaped section and 6 m length (Alves, 2012). The mechanical properties of the beam are:

Young's Modulus (E) = 210 GPa ;

Density = 7850 kg.m⁻³ ;

Cross-Section Area = 2.81x10⁻³m² ;

Moment of Inertia = 2.845x10⁻⁸m⁴.

The finite element model consists of 200 elements of Bernoulli beam formed by two nodes with two degrees-of-freedom each (vertical translation and rotation). This beam was excited by a random force with different frequencies and amplitudes, applied at 0.69 m from the right support, as can be seen in Fig. 3. The dynamic responses were considered as vertical displacements measured in 10 equidistant points (channels) of the beam during 10 seconds, where the sampling rate was 1/100 s.

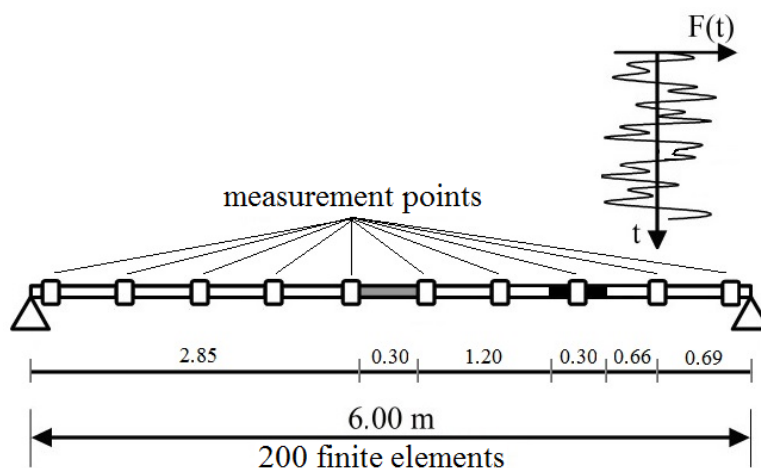


Figure 3. Simply supported beam model.

Three different levels of damage were simulated: Healthy beam (undamaged– Class 1); 20% reduction of young's modulus at the half length of the beam, represented by the gray part in Fig. 3 (damage level 1 – Class 2) and; 10% reduction of young's modulus at the quarter length of the beam, the black part denoted in Fig. 3, in addition to 20% previous young's modulus reduction (damage level 2 – Class 3). Furthermore, noise levels were added to measurements in each structural configuration mentioned above: Noiseless; 5% signal/noise (noise 1) and 10% signal/noise (noise 2). The corresponding noise levels were simulated by Eq. (11):

$$\mathbf{x}_{i,\text{noise}} = \mathbf{x}_i + n_{\text{noise}} \cdot \sigma_{\mathbf{x}_i} \cdot V \sim N(0,1), \tag{11}$$

where $\mathbf{x}_{i,\text{noise}}$ is the vector signal with noise, \mathbf{x}_i is the noiseless vector signal, n_{noise} is the noise level, $\sigma_{\mathbf{x}_i}$ is the standard deviation and $V \sim N(0,1)$ is the gaussian vector with zero mean and unit standard deviation. Ten different dynamic measurements were simulated for each damage and noise level, totalizing 90 signals.

For both ANN algorithms, the output data classes are represented by a target matrix [90x3], where their lines indicate the sample category through the following binary encoding: (1 0 0) – No damage; (0 1 0) – Damage level 1 and, (0 0 1) – Damage level 2. However, for the SVM classifiers, the three damage classes of the input data are represented by a target vector encoded as: 1 – No damage; 2 – Damage 1 and, 3 – Damage 2.

In the present application, the classification is made along the entire length of the beam, so the input dataset was arranged in a matrix [90x100] where the lines are the samples and columns are the statistic indicators (10 indicators x 10 channels = 100). Every ten columns group has an indicator type, in the order described in section 3. The Fig. 4 shows an example of the network architecture for the proposed ANN model.

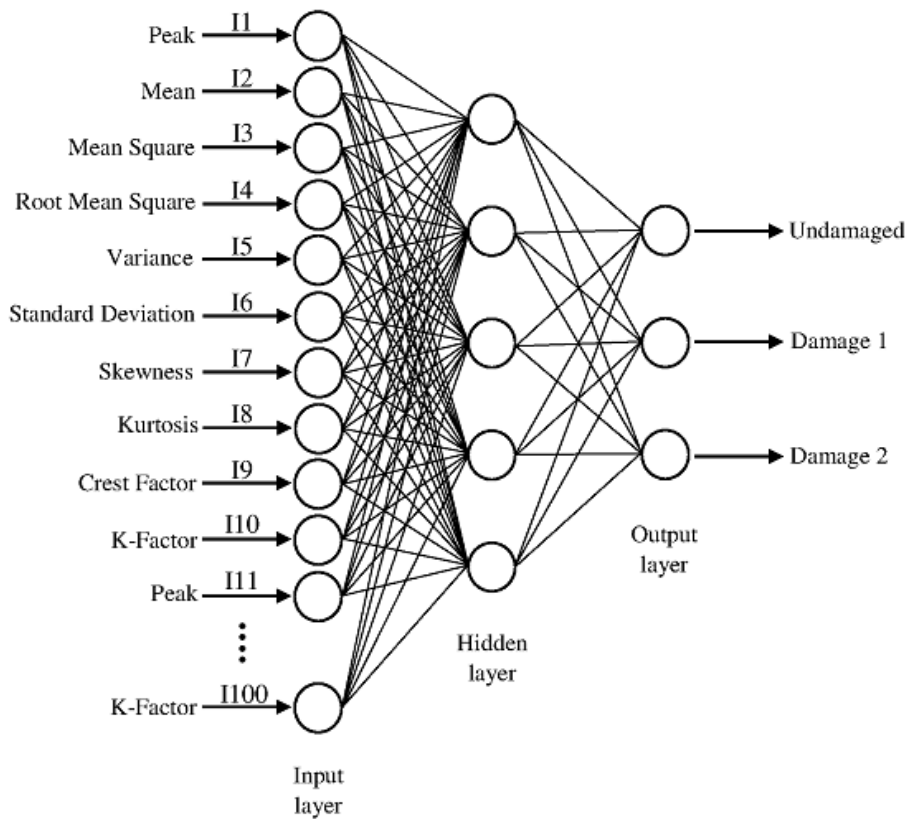


Figure 4. MLP network with 5 neurons in hidden layer.

Only the changes observed on the first three natural frequencies of the dynamic response were considered as ANN and SVM modal input, arranged in a matrix [90x3]. The natural frequencies of the beam were provided by Alves (2012) and were identified through Sys-Ident developed in LCPC (Laboratoire Central des Ponts et Chaussées, Paris, France), which is based on the random decrement technique and Ibrahim method (Barbosa & Cremona, 2001).

6 PROTOTYPE CONCRETE SLAB APPLICATION – EXPERIMENTAL TESTS

6.1 Description of the slab

The dynamic behavior of the concrete slab is analyzed through experimental investigation. For this purpose, a concrete slab prototype of 3.00 m length \times 1.35 m width \times 0.08 thick was built, with steel reinforcement using steel bars with 5.0 mm diameter and yielding stress 60MPa (CA 60 in brazilian codes), equally spaced by 25 cm. The concrete was casted using Portland cement type V-25 equivalent to early age resistant type cements. The slab was designed for a 16kN ultimate load and was simply supported along two steel beams.

Dynamic tests were performed at pre-established points of excitation and measurements, using a 5 kg B&K model 8210 instrumented sledge hammer and an Endevco model 752A13 piezoelectric accelerometer with a sensitivity of 1 V/g. Damages were imposed on the slab by static loads, considering four load stages: 0 kN, 8 kN, 16 kN and 22 kN. The static load was applied by a hydraulic jack and distributed in two lines parallel to the supports, located at one third of the slab span. The dynamic responses were measured after the respective cycle of loading and unloading. The experimental setup is shown in Fig. 5.



(a) Load distribution by a hydraulic jack.

(b) Data acquisition.

Figure 5. Experimental setup of the concrete slab test.

Impulsive excitations (hammer blows) were applied in each point and the responses were measured by an accelerometer, that was fixed at a specific node in the slab. The location of the excitation and measurement points is represented in Fig. 6.

Each excitation and response signals lasted 4.0 secs and had a total of 4,096 data points. The data was acquired and processed by the spectrum analyzer Dataphysics model Quattro, in order to obtain the time domain series and the frequency response functions (FRFs). The frequency resolution was 0.25 Hz (1/ 4.0 secs). To minimize the noise effects, the FRFs were the average of five excitations. With regard to the time domain history, only the last excitation of each point was recorded. The signals obtained from node 5 and 41 were discarded due to data acquisition problems and in order to facilitate the 10-fold cross-validation processing of

computational algorithms, respectively. At the end, 40 samples were obtained for each load stage, totalizing 160 dynamic measurements.

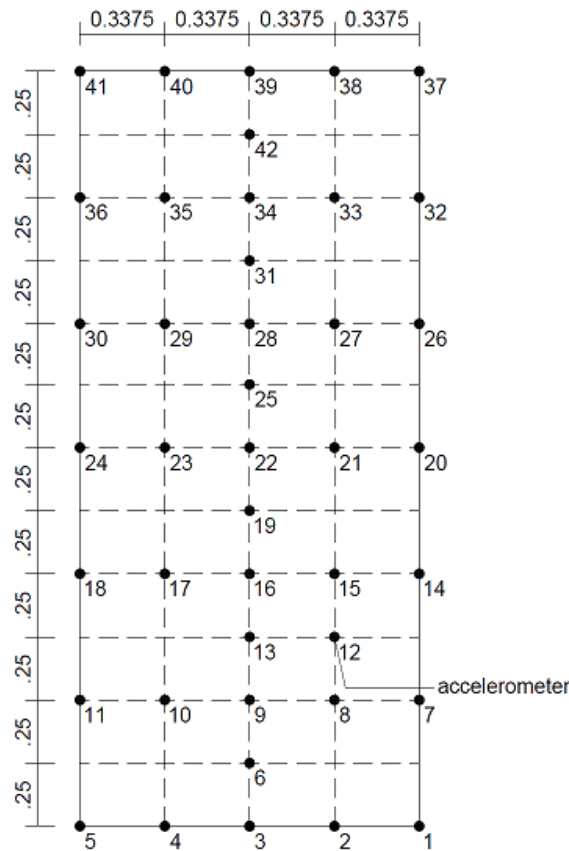


Figure 6. Excitations points of the concrete slab test (dimensions in m).

The targets for the slab were encoded as: No damage – Class 1 (0 kN) – (1 0 0 0) for ANN and 1 for SVM; Damage due the 8 kN static load – Class 2 – (0 1 0 0) for ANN and 2 for SVM; Damage due the 16 kN static load – Class 3 (0 0 1 0) – for ANN and 3 for SVM; Damage due the 22 kN static load – (0 0 0 1) for ANN and 4 for SVM.

As previously mentioned, the 10 statistic indices were calculated for each signal of the simulated vibrational data, giving as input a matrix [160x10]. The MLP network architecture designed for the slab is similar to the beam, except for the number of neurons in the input layer (only 10 statistical characteristics).

The first natural frequency of the slab was identified through the FRFs, as well as damping ratio, arranged in an input matrix [160x2]. The damping ratio for the first mode was obtained from the FRF, employing the half power method.

7 RESULTS

Dynamic behavior analysis was performed for the numerical and experimental applications using artificial intelligence algorithms. The results are the percentages of the correct classifications for the respective damage situations of each case (number of correct classifications divided by the number of the samples). Both ANN and SVM algorithms were

executed 30 times and the classification rate is represented by the mean values of these 30 repetitions.

7.1 ANN algorithms

The results of the ANN models for the two applications (numerical beam and prototype slab) are presented in Tab.1. The neural networks were implemented with 10 neurons in the hidden-layer.

Table 1. Results of correct classifications done by ANN algorithms.

	Simply supported numerical beam		Experimental tested prototype concrete slab	
	HOS Input	Modal Input	HOS Input	Modal Input
Mean	96.70%	98.00%	53.31%	96,97%
Standard Deviation	3.64%	2.31%	4.00%	3.83%

For modal input, the correct classification rates obtained on the ANN are greater than 95%. These good results are achieved because the changes on natural frequencies are well defined for each damage level. However, the ANN failed in identifying damage for the prototype slab using HOS input. This probably happened due the fact that the impulsive load applied by the hammer at the slab produced a short dynamic response, making it hard to characterize the signals among the different levels of damage in the structure. Unlike what happened with the slab, the excitement of the beam is made by a periodic load, where the dynamic signals are longer than the observed for impulsive load.

7.2 SVM algorithms

The results of the SVM models are presented as correct classification rates for each binary classifier of the multi-class model, and are shown in the Tab. 2, Tab. 3 and Tab. 4. The correct classification rates obtained on the SVM for the two applications are higher than 97%, except for the slab application using HOS input, which the percentage is around 77%. Even so, SVM has better performance than ANN in all cases, highlighting the slab application using HOS input where the ANN algorithm didn't achieve good classification rates.

Table 2. Results of correct classifications done by SVM algorithm for the simply supported numerical beam.

	HOS Input			Modal Input		
	Class 1	Class 2	Class 3	Class 1	Class 2	Class 3
Mean	99.11%	99.11%	97.85%	100%	99.93%	99.70%
Standard Deviation	1.44%	1.50%	1.77%	0%	0.28%	0.50%

Table 3. Results of correct classifications done by SVM algorithm for the experimental tested prototype slab using HOS indicators as input.

	Class 1	Class 2	Class 3	Class 4
Mean	79.48%	79.56%	76.21%	75.44%
Standard Deviation	0.90%	1.72%	0.23%	1.71%

Table 4. Results of correct classifications SVM algorithm for the experimental tested prototype slab using the first natural frequency and damping ratio as input.

	Class 1	Class 2	Class 3	Class 4
Mean	99.37%	98.67%	99.25%	99.93%
Standard Deviation	0%	0.32%	0.38%	0.19%

8 DISCUSSIONS AND CONCLUSIONS

In this paper, preliminary studies focusing the development of a hybrid damage detection method were presented. The main idea of this proposed method is to apply HOS and modal data as inputs of artificial intelligence algorithms.

Analyzing the performance of ANN with HOS and Modal Input, it was observed that Modal Input allows better results for short dynamic responses. For a relatively longer time history (simply supported beam), results for HOS or Modal Input have similar performances.

Results for the second analyzed structure (the tested slab) shows that SVM has better performance than ANN. For the same set of HOS input, the correct identification of each class augmented from around 53% (ANN) to around 77% (SVM). Based on these previous observations, one can conclude that, for these preliminary studies, SVM is indicated for the future hybrid damage detection model.

Once the artificial intelligence method is chosen, the definition of the kind of data for the proposed damage detection model demands more studies. For long-term experimental measurements with small differences in terms of modal data, it is expected that the inclusion of HOS data also as input to a SVM may increase the performance of the damage detection model. This hybrid strategy may include weights for each kind of data, focusing the best performance of the damage identification process.

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