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APPLICATION OF ERA'S METHOD FOR THE EXPERIMENTAL MODAL ANALYSIS OF COMPOSITE STRUCTURES

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Abstract. *This work is placed on the scope of characterization and modeling of structures and the central objective is to establish a computer code capable to provide models for structures of interest from practical experimentation, to be used to refine the computer models a posteriori. To realize that project, the experimental modal analysis was applied in the Eigensystem Realization Algorithm – ERA. As well as most of the techniques of identification of dynamic of systems, the ERA also creates the compute modes, being necessary the application of procedures for filtering the real modes before the noises and/or computers modes. In this work, two techniques for the distinction were used: Modal Amplitude Coherence (MAC) and Modal Singular Value (MSV). Beside this, a novel approach to help interpret and filter based in MSV deviation was proposed and tested, producing good results. To achieve this, a computer code that receives experimental data and returns the global properties of the system, i.e., the natural frequencies, the damping factors and the mode shapes was developed. The preliminary results, developed in the know structure, indicate the code has the potential to handle with complex geometries.*

Keywords: *ERA, Composite structures, Modal analysis, Identification of global properties.*

1 INTRODUCTION

In the aeronautical sector, the last decades were marked by the development of new materials and technologies, increasing the technical and economical possibilities for overcoming the new challenges constructive challenges of aircrafts (Lawton, 2015). In this scenario, composite materials have been taking more space, because they have more versatility to match the characteristics necessary for each application. Among the advantages of utilizing this type of material can be cited the increase of resistance of the structure both to corrosion as fatigue, increase of rigidity, weight reduction, enabling greater flexibility of the design. However, one of the great challenges for utilization of composites materials is the difficulty of characterization of their properties.

The recent works involving techniques of experimental characterization of complex structures has received attention in the last years, due to the unsatisfactory results of the finite elements modelling that uses the preliminary knowledge of layers and orientations of the internal fibers to represents composite structures (Chiang, 2010). The composite materials are subject to countless imperfections, as in the fabrication process, that many times it is hand-made, where can show up the defects as bubbles or dross that have big influence in the final properties of composites (Mouritz et. al., 1999).

Besides difficulties of to find the real properties of the material, it is also one the difficulty in make a trustworth representation of the structure, since a great part of the simulations in finite elements are made through simplified elements. From that point of view, Alvin (2003) has described that, although of the unfamiliarity of the mass and of the influence of the intrinsic static coefficients to the experimental test, the modal analysis allows to characterize mass behavior, damping and rigidity, simultaneously. Thus, the abovementioned work concludes that experimental modal analysis is a good tool for characteristics extraction, allowing achieving, simultaneously, mass behavior and parameters of damping and rigidity.

In the literature, several methods are based on calculation of the frequency response function (FRFs) or yet impulse response function (FRIs) for the extraction of the properties of the structure, being that many of these uses the realization algorithms based in the Markov parameters. Ho and Kalman (1965) have introduced the important idea about minimum realization theory, however this possesses noise problems. In the works of Kung (1978) e Zeiger and McEwan (1973) are proposed the adaptations based on the technique of singular value decomposition and, starting from the existing theory, Juang and Pappa (1985) developed the Eigensystem Realization Algorithm (ERA), addressed to be utilized in the aerospace sector, receiving, a posteriori, several contributions in the issue of reduction of noises and distinction of mode shapes.

Of the theoretical point of view, the ERA consists into algorithm of minimum realization, that besides allowing multiple inputs and outputs, it is controllable and observable. Realization is a form of identify a dynamic system preserving basic properties of this system. A dynamic system can have many equivalent realizations, in different forms (Sczibor, 2002). In that context, in this work, it was applied the ERA for to characterize a composite structure and to obtain properties as natural frequencies and damping. For the construction of the computational function, initially, it was realized a comparative analysis of a cantilever beam with his analytical modal parameters, obtaining, after adjustments, a functional computational code, that, soon afterwards, it was adapted to handle two-dimensional structures.

2 THE ERA METHOD

The ERA method was developed by Juang and Pappa (1985) in the context of aerospace engineering, where the structures and systems are more complex when compared to most the applications of engineering. The ERA method is an algorithm of identification, where it is possible determine, from the experimental data the input and the output, the matrix that represents the dynamic behavior the system in state space (Alves, 2005).

2.1 The Markov parameters

The equation of motion of the dynamic system with n degrees of freedom can be written as Eqs. (1) e (2).

$$[M]\{\ddot{w}(t)\} + [C]\{\dot{w}(t)\} + [K]\{w(t)\} = [B_2]\{u(t)\} \quad (1)$$

$$\{y(t)\} = [V_a]\{\ddot{w}(t)\} + [V_v]\{\dot{w}(t)\} + [V_d]\{w(t)\} \quad (2)$$

Where [M], [C] e [K] are the matrix of mass, of damping and of stiffness, respectively, and all with dimension n x n, already the vectors $\{\ddot{w}(t)\}, \{\dot{w}(t)\}$ e $\{w(t)\}$ represent the acceleration, velocity and displacement, respectively, the matrix $[B_2]$ is a matrix of input influence and {u(t)} is the input vector itself. While {y(t)} is the output vector the system and $[V_a]$, $[V_v]$ e $[V_d]$ are the matrix of output influence for acceleration, velocity and displacement, respectively. After obtaining the movement equation, it can be rewritten in the state space, i.e., it is written as a system of equations of the first order. It is important to highlight that starting from the system write in state space it is possible verify, that the states can be controlled and observed.

$$\{\dot{x}(t)\} = [A_c]\{x(t)\} + [B_c]\{u(t)\} \quad (3)$$

Wherein $[A_c]$ is the matrix of state (2n x 2n), $[B_c]$ is the matrix of influence of input (2n x r) and {x(t)} is the matrix of state space. Then, starting from this time-stamp, the equation can be discretized in the time. Give an space of time with points equally spaced of de Δt, t = 0, Δt, $2\Delta t$, $3\Delta t$, …, $k\Delta t$, $(k+1)\Delta t$, the Eq. (3) can rewrite as:

$$\{x((k+1)\Delta t)\} = e^{[A_c]\Delta t}\{x(k\Delta t)\} + \int_{k\Delta t}^{(k+1)\Delta t} e^{[A_c](k+1)(\Delta t-\tau)}[B_c]\{u(\tau)\}d\tau \quad (4)$$

By defining,

$$[A] = e^{[A_c]\Delta t} \qquad [B] = \int_0^{\Delta t} e^{[A_c]\hat{\tau}}[B_c]d\hat{\tau} \qquad \text{e} \qquad \hat{\tau} = (k+1)\Delta t - \tau \quad (5)$$

So, assuming the initial conditions are null, it is can represent the discreet state space as:

$$x(k) = [A]^k\{x(0)\} + \sum_{j=1}^{k}[A]^{j-1}[B]\{u(k-1)\} \quad (6)$$

$$\{y(k)\} = [C][A]^k\{x(0)\} + \sum_{i=0}^{k}[Y_i]\{u(k-i)\} \quad (7)$$

Being,

$$[Y_k] = \begin{cases} [D], & k = 0 \\ [C][A]^{k-1}[B], & k > 0 \end{cases} \quad (8)$$

Where $[Y_k]$ are the matrix of dimension *m x r* and correspond to response of the system in the time instant $k\Delta t$. The column n of $[Y_k]$ is the response of the system to the pulse applied in the input n, staying all the others input same to zero. The matrix $[Y_k]$ are called Markov parameters of the system (Sczibor, 2002).

2.2 The Hankel matrix

The Markov parameters $[Y_k]$ can be used appropriately in the matrix format, as (Sczibor, 2002):

$$[H(k-1)] = \begin{bmatrix} [Y_k] & [Y_{k+1}] & \cdots & [Y_{k+\beta-1}] \\ [Y_{k+1}] & [Y_{k+2}] & & [Y_{k+\beta}] \\ \vdots & & \ddots & \vdots \\ [Y_{k+\alpha-1}] & [Y_{k+\alpha}] & \cdots & [Y_{k+\alpha+\beta-2}] \end{bmatrix} = \begin{bmatrix} CA^{k-1}B & CA^{k}B & \cdots & CA^{k+\beta-2}B \\ CAB & CA^{2}B & & CA^{k+\beta-1}B \\ \vdots & & \ddots & \vdots \\ CA^{k+\alpha-2}B & CA^{k+\alpha-1}B & \cdots & CA^{k+\beta+\alpha-3}B \end{bmatrix} \quad (9)$$

The matrix presenting in Eq. (9) it is called general Hankel matrix of dimension αm x βr, where $\alpha \geq n$ e $\beta \geq n$, being *n* the order of the system and *m* and *r* are the numbers of input and output, respectively. The ERA's formulation begins after the confection of the Hankel matrix.

The first step is the decomposition of Hankel matrix (H_0) in the singular values,

$$H(0) = R\Sigma S^T \quad (10)$$

Where R and S are orthonormal and Σ is a rectangular matrix. Also R_n e S_n are formed by the *n* first columns of the matrix R and S, respectively. From the mathematical manipulations, can be determined the matrix A, B, C and D.

$$\hat{A} = \Sigma_n^{-1/2} R_n^T H(1) S_n \Sigma_n^{-1/2} \quad (11)$$

$$\hat{B} = \Sigma_n^{1/2} S_n^T E_m \quad (12)$$

$$\hat{C} = E_r P_\alpha \quad (13)$$

$$\hat{D} = Y_0 \quad (14)$$

Com,

$$E_m = \begin{bmatrix} I_m \\ O_m \\ \vdots \\ O_m \end{bmatrix}_{m(\beta-1)xm} \quad \text{e} \quad E_r = [I_r \quad O_r \quad \ldots \quad O_r]_{r\,x\,r(\alpha-1)} \quad (15)$$

Where I_m and I_r are the identity matrix and O_m e O_r are the null square matrix. As the matrix A represent the dynamic matrix of the system, from your eigenvalues and eigenvectors, it is possible to extract the properties of the system, i.e., the natural frequencies, the damping factors and the mode shapes.

2.3 Flow diagram of the formulation

Based in the exposed, the method can be represented by the flowchart of Fig. 1.

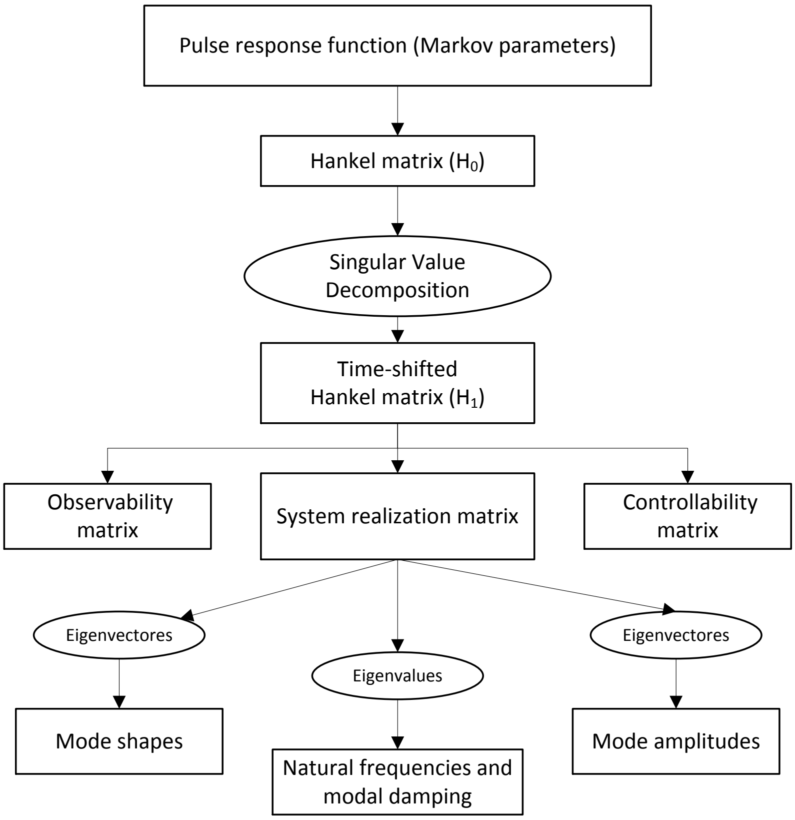


Figure 1 – Flow diagram of ERA's method.

3 EXPERIMENTS

In the experimental point of view, Alves (2005) states that the experiments to being realized must to have good repeatability, once how much larger the amount of data experimental, more realists will be the results. In this way, in the present work was chosen the realization of impact on a clamped-free composite plate with 500 mm of width, 445 mm of length e 3,10 mm of thickness. This plate is composed by 16 layers of carbon fiber, with orientation [0 45 -45 90 0 45 -45 90]s and with a fine layer of Teflon inserted in its centroid.

In order to better organize the locations of impact and acquiring points, the plate was discretized into 90 elements, forming 99 possible points for impact/reading, and the spectral band 0-200Hz of interest, as shown in Fig. 2.

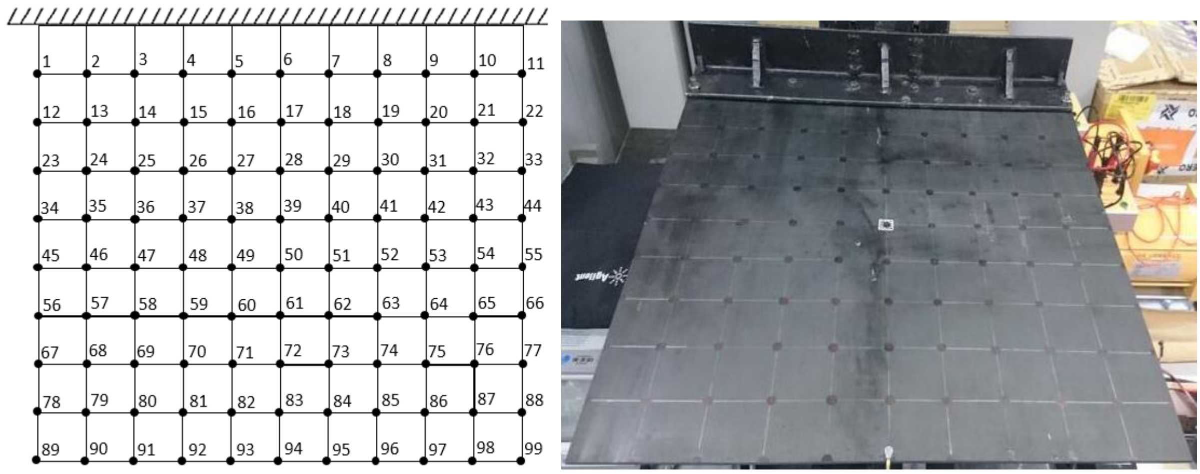


Figure 2 – Representation of the division of nodes in the plate (left) and plate used for experiments (right).

The Fig. 3 shows the outline of the experimental apparatus and his practical assembly. The Tab. 1 relates the instruments used, indicating the sensibility of the transducers.

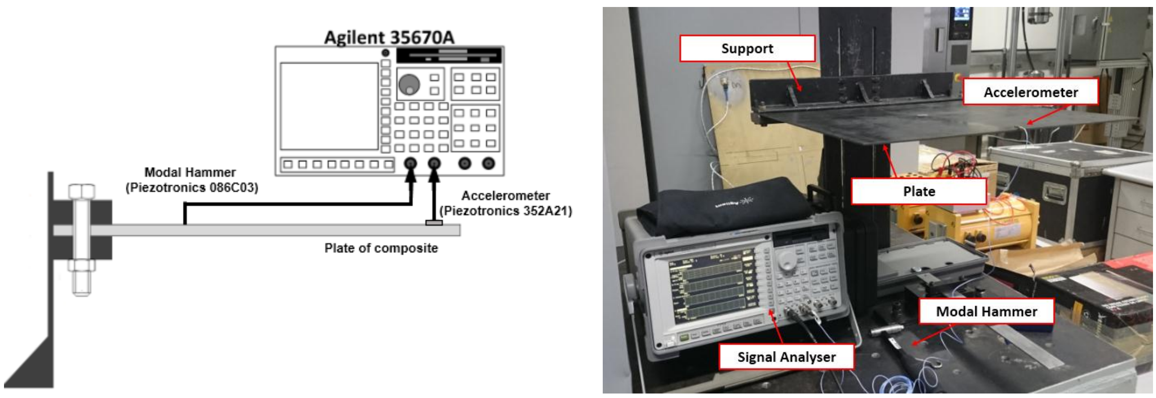


Figure 3 – Scheme of the experiments (left) and the experimental assembly (right).

Table 1 – Instruments used for the experiments.

Instrument	Model	Sensibility
Modal Hammer	Piezotronics 086C03	2.25 mV/N
Accelerometer	PCB Piezotronics 352A21	1.004 mV/(ms²)
Signal Analyser	AGILENT 35670A	-

So, aiming to do a good characterization of the plate, were proceeded various tests to determine the candidates for points to applying impact and acquiring acceleration, always using responses with 30 averages. After thus, it was noticed that how much more close the impact of the clamped edge, more clear is the FRF, i.e., with less noise. Besides, how much more near the acceleration acquisition in relation to the free edge, better are the results for identification, due to the biggest vibrations amplitudes occurs in this region. For the best homogenization, in the Fig4 it was indicated all chosen points and for the frequency responses functions was used the following nomenclature: H + (number of the node where the impact it is applied) + (number of the node where the accelerometer it is placed).

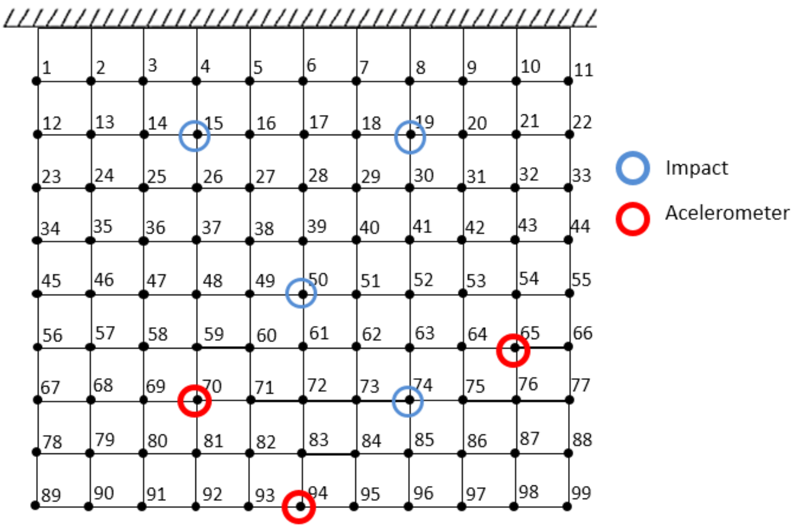
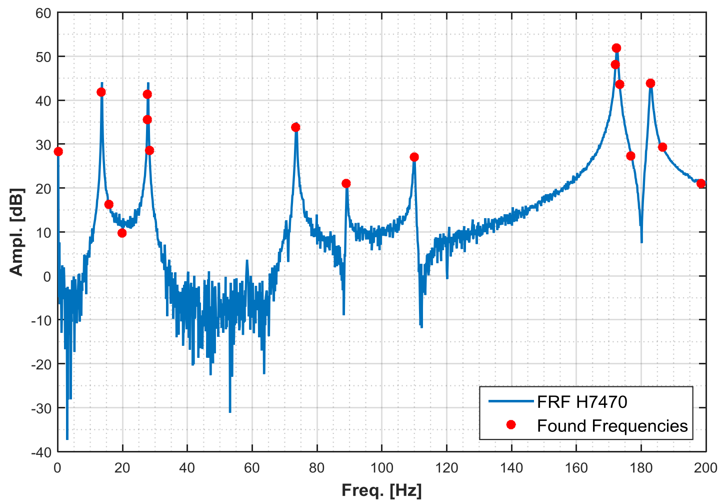


Figure 4 – Representation of the impact points and points of accelerometer.

4 RESULTS

How originally demonstrated by Juang and Pappa (1985), the ERA method is based in the energy method, i.e., the modes with the bigger amplitudes will be more easily identified. However, this caracteristic also causes the emergence of the noise modes, these modes are originated by noise in the experimental response or still the computational modes, that arise due to algebraic operations accomplished by the analytical method (Zeiger & McEwan, 1973). As it can be observed in the Fig. 5, that it presents as example FRF H7470, a great number of frequencies are identified, but the most of these frequencies are not true frequencies of the structure.



Fequencies
0.0324
13.4454
15.8262
19.9915
27.6204
27.7055
28.2396
73.4542
89.0569
109.8609
172.0181
172.4160
172.4160
173.3281
176.9396
182.8040
182.8502
186.6500
198.6228
200.1174

Figure 5 – All frequencies initially calculated by ERA's method.

Thus, to handle the problem outlined, it is needed the application of a distinction procedure (Alves, 2005). For such, three principal methods are used in the literature: the first is called modal singular value (MSV) that quantifies the contribution of the mode for the response of the structure, other method is the modal assurance criterion (MAC), that measure the temporal consistence of the determinate mode and the last distinction method is the selector of order of model (SOM), a combination of MAC with MSV.

The temporal consistence measure by MAC varies between 0 and 1. In general, it is considered a good consistence when the value is above 0.9 and a low consistence when the value is below 0.1 (Sczibor, 2002). This index can be calculated through Eq. (16):

$$MSV_i = \sqrt{|\hat{c}_i|(1 + |\hat{\lambda}_i| + |\hat{\lambda}_i^2| + \cdots + |\hat{\lambda}_i^{l-2}|)|\hat{b}_i|} \approx \sqrt{\frac{|\hat{c}_i|.|\hat{b}_i|}{1-|\hat{\lambda}_i|}} \tag{16}$$

Where $\bar{q}_i$ is the "wait" temporal evolution of the mode i, $\hat{q}_i$ is the identified temporal evolution and the superscript * means transposed.

While the measure value for MSV isn't limited to a defined strip of values, since that technique emphasizes that the frequencies of smaller energy possess a smaller contribution value, therefore it is difficult to obtain a parameter of comparison. Knowing that λ are the identified eigenvalues, $\hat{c}_i$ is the element *i* of matrix of the sensors in modal coordinates and $\hat{b}_i$is the element *I* of matrix of the actuators in modal coordinates.

In this work, with the objective to evaluate the meaning of MSV, it was established as a threshold of minimum value of MSV all results above the subtracted medium value of the standard deviation (σ), meditated by a factor (N), as in Eq. (17):

$$MSV \geq \overline{MSV} - N * \sigma_{MSV} \tag{17}$$

That way, for the MSV, it is becoming possible to compare the modal contribution for every mode shape detected, creating, therefore, a very practical computation rule that facilitates the distinction modal without depending on a specialist's action.

The Tab. 2 shows all the frequencies calculated by ERA on 0 to 200Hz frequency band, with their respective filtering indexes. It is worth to emphasize that the index SOM is the normalization of the combination of MAC with MSV, as in the equation below:

$$SOM = \frac{MSV * MAC}{\max(MSV * MAC)} \tag{18}$$

Table 2 - Frequencies calculated by ERA's method with his respectively filtering indexes values.

Frequencies [Hz]	MSV	MAC	SOM
0.0324	11.7618	0.0021	0.0003
13.4454	16.8284	0.1242	0.0228
15.8262	73.3937	0.0038	0.0030
19.9915	7.1164	0.7773	0.0605
27.6204	3.1121	0.6733	0.0229
27.7055	20.7099	0.0069	0.0016
28.2396	2.5106	0.0593	0.0003
73.4542	22.8701	0.9963	0.2491
89.0569	32.3554	0.9938	0.3514
109.8609	43.3595	0.9966	0.4723
172.0181	1.2911	0.7177	0.0101
172.4160	1.6067	0.9965	0.0175

172.4160	115.2108	0.7941	1.0000
173.3281	66.0391	0.5340	0.3855
176.9396	43.8174	0.6083	0.2913
182.8040	25.7582	0.1720	0.0484
182.8502	42.0510	0.8224	0.3780
186.6500	3.1300	0.0098	0.0003
198.6228	54.3691	0.7391	0.4392
200.1174	7.1658	0.0082	0.0006

From the Tab. 2, it can be observed that the frequency 172.4160Hz was identified twice, however, in one of them MAC is larger than 0.9 and the other has a value of MAC much smaller and, according to Alvin (2003), it is caused by computational mode introduced by the algorithm of the ERA.

4.1 Analysis from MAC index

How it said before, the MAC is a method that measure the temporal consistency of a mode, i.e., how much larger is this value, larger is the possibility of the mode can be a true mode of the structure.

Table 3 - Evaluation of variation of the index MAC for the distinction of modes.

MAC ≥0.6	MAC ≥ 0.7	MAC ≥ 0.8	MAC ≥ 0.9
19.9915	19.9915	73.4542	73.4542
27.6204	73.4542	89.0569	89.0569
73.4542	89.0569	109.8609	109.8609
89.0569	109.8609	172.4160	172.4160
109.8609	172.0181	182.8502	
172.0181	172.4160		
172.4160	172.4160		
172.4160	182.8502		
176.9396	198.6228		
182.8502			
198.6228			

The Tab. 3 presents the calculated frequencies after the application of distinction of modes, it is possible to see that for different criteria, the number of the filtered frequencies changes. In the first case (MAC ≥ 0.6), it has 11 natural frequencies of the structure, however, it is possible to note that an another parameter of modal filtering is required, since exist many identified frequencies in a very close band, as can be observing the Fig. 6 and Fig. 7.

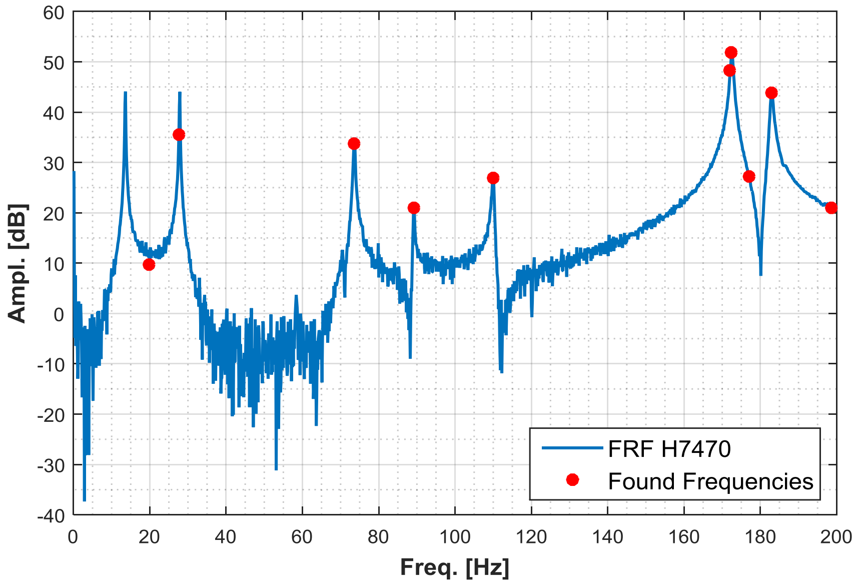


Figure 6 – Response for MAC ≥ 0.6.

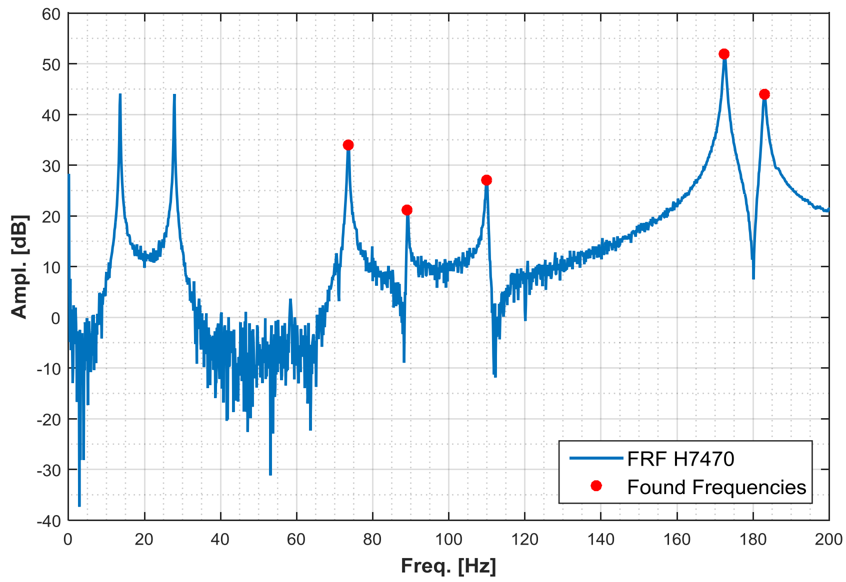


Figure 7 – Response for MAC ≥ 0.8.

The true frequencies of the system have MAC above of 0.8, but it is necessary to note the existence of frequencies below 60Hz which were not identified. This can be explained by the energy theory, in which if the low-frequency modal shapes has less energy, therefore, they are more difficult to be characterized.

4.2 Analysis from MSV index

Another criterion for distinction of vibration modes is the MSV, in which, to have a better evaluation, it is necessary to realize calculations for different values of N, as shown in Tab. 4.

Table 4 - Evaluation of the influence of N factor weighting on variation of the index MSV filtering.

N=0.2	N=0.4	N=0.5	N = 0.65	N=0.8	N = 1
15.8262	15.8262	13.4454	0.0324	0.0324	0.0324
89.0569	27.7055	15.8262	13.4454	13.4454	13.4454
109.8609	73.4542	27.7055	15.8262	15.8262	15.8262
172.4160	89.0569	73.4542	27.7055	19.9915	19.9915
173.3281	109.8609	89.0569	73.4542	27.7055	27.6204
176.9396	172.4160	109.8609	89.0569	73.4542	27.7055
182.8040	173.3281	172.4160	109.8609	89.0569	28.2396
182.8502	176.9396	173.3281	172.4160	109.8609	73.4542
198.6228	182.8040	176.9396	173.3281	172.4160	89.0569
	182.8502	182.8040	176.9396	173.3281	109.8609
	198.6228	182.8502	182.8040	176.9396	172.0181
		198.6228	182.8502	182.8040	172.4160
			198.6228	182.8502	172.4160
				198.6228	173.3281
				200.1174	176.9396
					182.8040
					182.8502
					186.6500
					198.6228
					200.1174

It is notable that with N value larger, more natural frequencies are found. This is because, as shown in Tab. 2, these values are widely dispersed, exerting remarkable influence on average value and, therefore, on the standard deviation. Another important conclusion is that for values of N upper 0.65, the variety of frequencies does not change, only have an augmentation of the computational/noise modes. For a better observation, the graphical representation for two cases (N=0.2 and N=1) it is shown below.

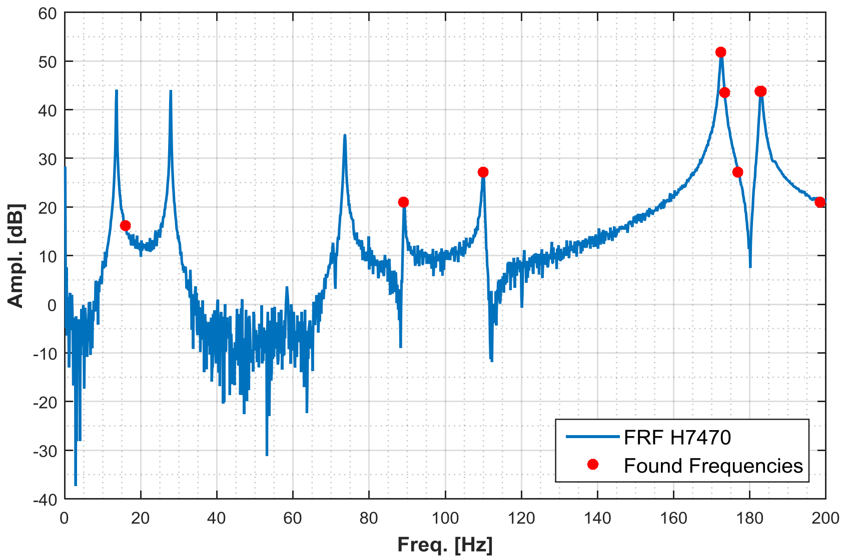


Figure 8 – Response for MSV for N = 0.2.

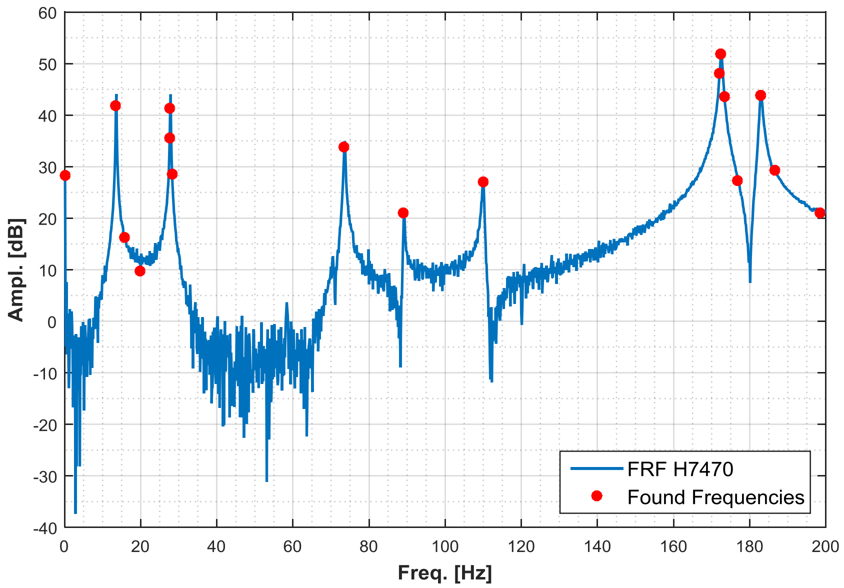


Figure 9 - Response for MSV for N = 1.

Noted above, it can be observed in Fig. 8 the same problem of MAC index, in which the frequencies below 60Hz are not identified. It must be said that the frequency found around 20Hz is due to noises in this band of frequencies and it isn't a true mode.

4.3 Analysis by SOM index

The SOM index is a combination of the MAC and MSV indexes, but how the index acts in distinction of modes is different, a procedure to evaluation of this behavior is necessary.

Table 5 - Evaluation of variation of the index SOM for different criteria.

SOM > mean(SOM)	SOM ≥ 0.7	SOM ≥ 0.8
73.4542	19.9915	73.4542
89.0569	73.4542	89.0569
109.8609	89.0569	109.8609
172.4160	109.8609	172.4160
173.3281	172.0181	182.8502
176.9396	172.4160	
182.8502	172.4160	
198.6228	182.8502	
	198.6228	

Table 6 - Evaluation of the impact of N weight on variation of the index SOM filtering

N=0.2	N=0.4	N=0.5	N = 0.65	N=0.8	N = 1
73.4542	73.4542	19.9915	13.4454	0.0324	0.0324
89.0569	89.0569	73.4542	19.9915	13.4454	13.4454
109.8609	109.8609	89.0569	27.6204	15.8262	15.8262
172.4160	172.4160	109.8609	73.4542	19.9915	19.9915
173.3281	173.3281	172.4160	89.0569	27.6204	27.6204
176.9396	176.9396	173.3281	109.8609	27.7055	27.7055
182.8502	182.8502	176.9396	172.4160	28.2396	28.2396
198.6228	198.6228	182.8502	173.3281	73.4542	73.4542
		198.6228	176.9396	89.0569	89.0569
			182.8040	109.8609	109.8609
			182.8502	172.0181	172.0181
			198.6228	172.4160	172.4160
				172.4160	172.4160
				173.3281	173.3281
				176.9396	176.9396
				182.8040	182.8040
				182.8502	182.8502
				186.6500	186.6500
				198.6228	198.6228
				200.1174	200.1174

As SOM has the values between 0 and 1, it is possible use the same criteria used in the MAC (Tab. 5) and the true frequencies are present for values above 0.8 and for MAC this value is 0.9. Also, the same strategy applied for MSV can be applied for SOM, as presented on Tab. 6 and the conclusions are similar of the MSV, i.e., for N up to 0.65 (Fig. 10) only computational/noise modes are added. In the Fig. 11, it can be observing that even with N = 0.2 some frequencies are not true modes.

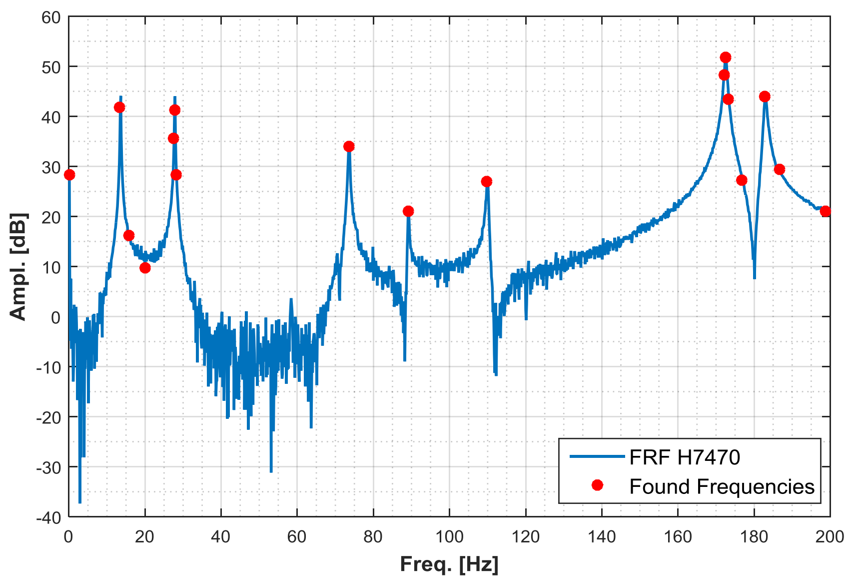


Figure 10 – Responde for SOM for N=1.

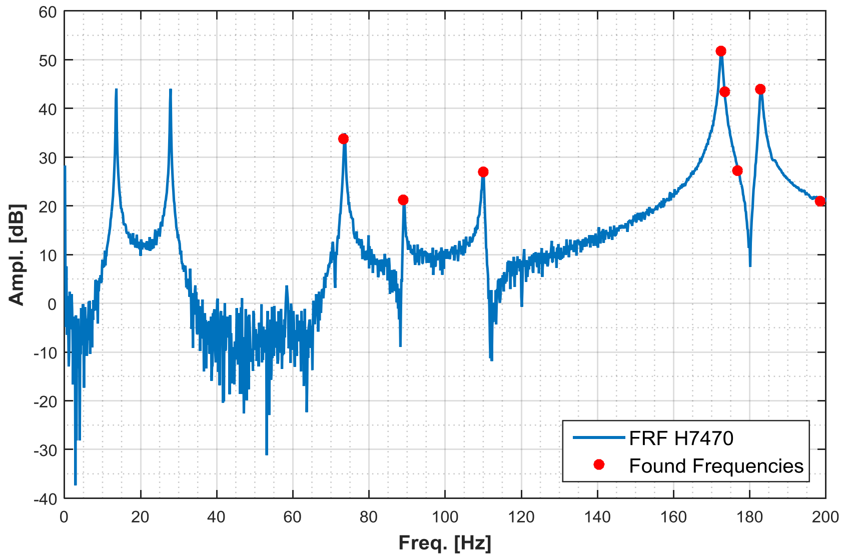


Figure 11 - Responde for SOM for N=0.2.

5 CONCLUSIONS

Among the distinguishing methods studied, each has a characteristic to distinguish the true modes of computer and/or noise modes, however the most suitable for use is the SOM as it aggregates the other two values in order to quantify the contribution of each mode for the final response of the structure.

The study of the influence of the N factor value considering the standard deviation proved to be a good alternative to the distinction of the modes, especially for MSV, where there is a large standard deviation of the values caused by the distribution of excitation energy over spectrum of vibrate modes. This strategy reduces the dependence of interference and appears as a possibility to automate the filtering modes.

At this point, it should be noted that the SOM is a normalized value, which makes it easier filtering. As can be seen above even for small values of N, as frequencies was identified. Therefore, to obtain a better distinction should be used similar method with the MAC, i.e. how much closer to the value to 1, greater the contribution and thereby greater the probability of being a true vibrate mode.

As seen in all methods, all of them have difficulty in characterizing modes below 60 Hz, which can be a symptom of direct interference of the electrical installation. Furthermore, in order to improve the performance in this frequency range, can perform tests on more narrowband range, also modifying the tip of impact hammer, in order to deliver more energy in the spectrum start.

In all cases studied, it was clear that the ERA is an important tool for the characterization of structures, but is necessary, however, the development and application of filtering tools for noise and/or computational vibrate modes, that arise from the mathematical calculations procedures.

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