REVISTA INTERDISCIPLINAR DE PESQUISA EM ENGENHARIA

CILAMCE 2016

XXXVII IBERIAN LATIN AMERICAN CONGRESS ON COMPUTATIONAL METHODS IN ENGINEERING
BRASÍLIA - DF - BRAZIL

A $FOSM$-BASED RELIABILITY ASSESSMENT OF STEAM GENERATOR TUBES UNDER CRACKING DEGRADATION

Alexandre Santos Francisco

Ronan Berthou

afrancisco@metal.eeimvr.uff.br

berthouronan.br@gmail.com

Universidade Federal Fluminense

Avenida dos Trabalhadores 420, 27255-125, Volta Redonda, Rio de Janeiro, Brazil

Abstract. *Steam generator tubes are critical parts in the safety and operability of nuclear power plants. Thus, the degradation in steam generator tubes must be prevented. These tubes are subjected to the degradation mechanism of stress corrosion cracking, which is characterized by several uncertainties. The assessment of structural reliability of the steam generator tubes is essential for preventing the occurrence of failure events. In the present work, a study is carried out for assessing the structural reliability of the steam generator tubes containing cracks, by using first order second-moment ($FOSM$) concepts. The limit state function is based on a limit load model provided by Eletric Power Research Institute (2001), which describes the failure criterion. The applicability of the $FOSM$ method as a methodology for predicting the reliability of steam generator tubes is checked by comparing results of calculated failure probability with ones obtained from Monte Carlo simulation. The $FOSM$ method has provided a non-conservative reliability assessment for the steam generator tubes containing cracks, and are eligible for estimating the failure probability of this safety-critical structure.*

Keywords: *Fracture mechanics, Structural reliability, $FOSM$ method*

1 INTRODUCTION

The steam generator (SG) tubes are critical parts in nuclear power plants because they are a barrier against radioactive releases from the primary circuit. The SG tubes are subjected to degradation mechanisms that give rise to crack-like defects in the material. The most frequently degradation mechanism is the stress corrosion cracking occuring outside the tubes. This outside diameter stress corrosion cracking ($ODSCC$) consists of the combined action of tensile stress, material susceptibility and aggressive environment. The observed damage is the presence of deep, through-wall axial cracks, which have high rate of propagation.

In order to retain the structural integrity of the SG tubes, repair and plugging of excessively degraded tubes are carried out. This maintainance procedure allows the SG to be operated in a failure-tolerance condition. The structural reliability is the most adequate theory applied for establishing an acceptable safety condition. In this theory, failure events are achieved by the violation of limit state functions, which consider applied loading and structural resistences. The failure probability of the SG tubes is evaluated as the probability of violation of a limit state function.

The failure criterion for SG tube is traditionally based on the minimum wall thickness requirement. However, this criterion presents limitations related to the assumptions made for its formulation. It work well for degradation mechanisms as such fretting wear or fatigue, but can be overly conservative in case of crack-like defects due to $ODSCC$.

Cizelj et al. (1994) have applied the First-Order Reliability Method ($FORM$) and Second-Order Reliability Method ($SORM$) in reliability assessment of SG tubes under stress corrosion cracking. They demonstrated the applicability in safety purposes of the $FORM$ and $SORM$ in comparison with the Monte Carlo simulation. They considered that the failure of the tube containing through-wall axial cracks is defined by plastic limit load model (Erdogan, 1976).

Recently, new failure models have been proposed for the reliability assessment. Some models presented in the literature consider the linear elastic fracture mechanics (Wang and Reinhardt, 2003) and the elastic-plastic fracture mechanics (Tonkovic et al., 2006). Bergant et al. (2006) has evaluated the structural reliability based on the Failure Assessment Diagram (FAD), incorporating both plastic limit load and fracture mechanics. They could indicate the potentiality of the FAD as a comprehensive methodology for predicting the failure loads and failure modes of flawed SG tubes. Ghoshua and Shuho (1996) have already calculated the failure probability based on the FAD for pressure vessels containing cracks. They used the $FOSM$ and the Monte Carlo in the calculation.

According to Maneschy and Miranda (2014), the assessment of structural reliability of the SG tubes of Inconel alloy 600 is based on the following guides: Nuclear Energy Institute (NEI) and Eletric Power Research Institute ($EPRI$). For through-wall axial cracks that initiate and propagate under the effect of the $ODSCC$, the failure criterion is established by a correlation between crack sizes, material strength, and other relevant properties. This failure criterion considers that the plastic collapse is the unique responsable for the structural failure, since the fracture thoughness value for the tube material is very high.

Our main purpose is to perform a reliability assessment based on the failure criterion established by Nuclear Energy Institute (2005) and Eletric Power Research Institute (2006). In the present work, we obtain the failure probability of the SG tubes by considering the $ODSCC$ as the prevalent degradation mechanism. In the reliability assessment, we use the $FOSM$ method to calculate numerically the failure probability. In this $FOSM$-based assessment, we employ a first order iterative method for finding the so-called design point. The results calculated by the $FOSM$ method are compared with ones obtained by Monte Carlo simulaton. The $FOSM$ results demonstrate the applicability of the method in the reliability assessment of the SG tubes degraded by $ODSCC$.

In the Section 2, the structural integrity of the SG tubes is devised according to the failure criterion established by Eletric Power Research Institute (2001). The uncertainties of variables that are involved in the failure criterion are presented in the Section 3. Next, the reliablitlity assessment methodology is described in the Section 4. Results obtained from examples are presented and discussed in the Section 5. Finally, conclusions are drawn from the results, in the Section 6.

2 STRUCTURAL INTEGRITY OF SG TUBES

The Nuclear Energy Institute (2005) and Eletric Power Research Institute (2006) are documents that guide the assessment of the structural integrity of the SG tubes, taking into account inspection informations. This assessment aims to show that the structural resistence has been assured before the last inspection, and will continue to be until the next inspection. The structural assessment based on the last inspection is performed in condition monitoring (CM), as found by the inspection.

The structural resistence of the SG tubes is assured when the structural integrity attends to some safety requirements. According to Nuclear Energy Institute (2005), the tubes should support to three times the pressure difference across the tube wall, in normal conditions. And according to Eletric Power Research Institute (2006), the tube should fail at pressures less than three times the pressure difference across the tube wall, with probability of 5%.

The main loading that leads the SG tube to the failure is the pressure difference across the tube wall. As the fracture thoughness is very high, the failure occurs by plastic collapse. Several variables contribute to the failure: pressure difference, material strength, crack morphology, tube geometry, and crack location. A testing program conducted by Eletric Power Research Institute (2001) has proposed a correlation of these variables for predicting the structural integrity of the SG tubes.

Considering a through-wall axial crack, located outside an Inconel alloy 600 tube of the steam generator, between two support plates or between the first support plate and the tube sheet, Eletric Power Research Institute (2001) establishes a failure criterion that is given by the following correlation:

$$p_b = 0.58(S_y + S_u)\frac{t}{r_i}(\phi - \frac{l}{l+2t}h), \tag{1}$$

where p_b is the burst pressure, S_y is the yield stress, S_u is the ultimate stress, t is the tube wall thickness, r_i is the inlet tube radius, ϕ is a correlation coefficient, l is the crack lengh, and h is the ratio between crack depth and tube wall (TW).

The assessment of the structural integrity is performed by attending the so-called CM structural limit, which is expressed by the curve of h as a function of l, as found by the inspection. The CM structural limit is attended when points (h, l) are plotted bellow the curve. The curve is obtained by using the Eq. (1) rewritten to explicit h in terms of the l as follows:

$$h = (\frac{l + 2t}{l})[\phi - \frac{p_b}{0.58(S_y + S_u)}\frac{r_i}{t}]. \tag{2}$$

3 UNCERTAINTIES OF THE VARIABLES

In order to attend also the safety requirement posed by Eletric Power Research Institute (2006), the failure criterion established by Eletric Power Research Institute (2001) should take into accout the uncertainties of the correlative variables. In the correlation, the main variables that represent the relevant uncertainties influencing the failure of the SG tubes are: material strength, correlation coefficient, crack length and relative crack depth.

3.1 The material strength

The material of the SG tube is not uniform for a lot of reasons, such as metallurgical processing and forming. Even for a tube made of Inconel alloy 600, there is a larger scatter among yield stress and ultimate stress values obtained in tests. Therefore, the material strength should be treated as a random variable. Its distribution can be obtained by analyzing the test data, which can show a good fit for truncated normal distribution.

3.2 The correlation coefficient

The testing program conducted by Eletric Power Research Institute (2001) has considered several cracking conditions and tube geometries. This dispersion on test data reflects in errors in the correlation. Thus, the coefficient in the correlation is to be treated as a random variable. Its distribution may be well adjusted by truncated normal distribution.

Uncertainties of tube geometries are not considered explicitly since they are incorporated by the uncertainty of the correlation coefficient.

3.3 The crack sizes

The inspections in the SG tubes can be carried out by the eddy current test (ECT). The ECT is an efficient technique for detect and caracterizing cracks, however uncertainties of crack size are associated with the inspection. Thus, it is reasonable to assume that actual mean crack sizes are related to their measured crack sizes by the following straight-line relationship:

$$x = ax_{ECT} + b, \tag{3}$$

where x is an actual mean crack size, x_{ECT} is an measured crack size in the ECT, and a and b are the slope and intercept of the line, respectively.

For a single crack, an actual size can be treated as a random variable adjusted by truncated normal distribution, regarding all the uncertainties in the ECT. The regression coefficients for the crack length and relative crack depth are presented in the Table 1.

Table 1: Uncertainties of crack size in the ECT

x	a	b
l	1.0	0.0
h	1.01	0.0

4 $FOSM$-BASED RELIABILITY ASSESSMENT

The assessment of structural reliability of the SG tubes is essential for preventing the occurrence of failure events. Failure events are described in terms of functional relations regarding the failure criteria, the so-called limit state functions. The probability of failure is a measure of the chance of violation of the limit state function.

Considering a through-wall axial crack, located outside an Inconel alloy 600 tube of the steam generator, between two support plates or between the first support plate and the tube sheet, the limit state function is given by

$$g(\mathbf{x}) = (\frac{l+2t}{l})[\phi - \frac{p_b}{0.58(S_y + S_u)}\frac{r_i}{t}] - h, \tag{4}$$

in which the components of the vector $\mathbf{x} = (S_y + S_u, \phi, l, h)$ are realizations of the so-called basic random variables, which represent all the relevant uncertainties influencing the failure of the SG tubes. Thus, the failure probability is determined by the following integral (Melchers, 1987):

$$P_f = Pr\{g(\mathbf{x}) \leq 0\} = \int_{g(\mathbf{x})\leq 0} f_{\mathbf{X}}(\mathbf{x})d\mathbf{x}, \tag{5}$$

where $f_{\mathbf{X}}(\mathbf{x})$ is the joint probability density function of the basic random variables.

We have proposed to use the $FOSM$ for calculating the above failure probability, since the basic random variables are uncorrelated normally distributed variables. As the limit state function $g(\mathbf{x})$ is a non-linear function of the basic random variables, a linearization of the limit state function in the design point is performed. The design point is the point on the failure surface $g(\mathbf{x}) = 0$ with the smallest distance to the origin of space of the standardized normal variables. The design point is found by using a first order iterative method. The smallest distance determined by the scheme is defined as reliability index. Finally, the failure probability is calculated by

$$P_f = \Phi(-\beta), \tag{6}$$

where $\Phi(.)$ is a standardized normal cumulative distribution function, and β is the reliability index.

The $FOSM$ methodology for computing the failure probability is implemented according to an iterative scheme, listed in the algorithm as follows.

- Given $g(\mathbf{x})$ and $\mathbf{u}^0$:
- compute $G(\mathbf{u}^0)$ and $\nabla G(\mathbf{u}^0)$;
- for $k = 1$ until convergence do

– compute $\mathbf{u}^k = \frac{\nabla G(\mathbf{u}^{k-1})}{\nabla G(\mathbf{u}^{k-1}).\nabla G(\mathbf{u}^{k-1})}[\nabla G(\mathbf{u}^{k-1}).\mathbf{u}^{k-1} - G(\mathbf{u}^{k-1})]$;

– compute $G(\mathbf{u}^k)$ and $\nabla G(\mathbf{u}^k)$;

– update $\mathbf{u}^{k-1}$;

- end for
- compute $\beta = (\mathbf{u}^k.\mathbf{u}^k)^{1/2}$;
- compute $P_f = \Phi(-\beta)$.

One of the most accurate method for calculating the failure probability is the Monte Carlo simulation. In this case, the Eq. (2) should be rewritten to incorporate the uncertainties of the basic variables. In the normalized coordinate system, the Eq. (2) is given by (Hasofer and Lind, 1974)

$$h = (\frac{l + z_3\sigma_L + 2t}{l + z_3\sigma_L})[\phi - z_2\sigma_\Phi - \frac{p_b}{0.58(S_y + S_u - z_1\sigma_S)}\frac{r_i}{t}] - z_4\sigma_H, \tag{7}$$

where z_1, z_2, z_3 and z_4 are the standardized normal variables for S_y+S_u, ϕ, l and h, respectively, and σ_S, σ_Φ, σ_L and σ_H are the corresponding standard deviations.

In the Monte Carlo method, the four values of z are randomically generated in each one of thousand trials. Once the four values are substituted into Eq. (7), thousand values of h are obtained as a function of l. The CM structural limit is defined by the value of h that corresponds to the cumulative frequency equal to 5%.

5 RESULTS AND DISCUSSIONS

In order to check the applicability of the $FOSM$-based reliability assessment, first we solve a simple structural integrity problem for validating the proposed methodology. Next we perform the reliability assessment of the SG tubes, in which comparisons between results of the $FOSM$ method and the Monte Carlo simulation are done.

5.1 The steel rod problem

Consider a steel rod under pure tension loading. The rod will fail if the applied stress on the rod cross-sectional area exceeds the steel yield stress. The yield stress R of the rod and the loading stress on the rod S are assumed to be uncertain modeled by uncorrelated normally distributed variables. The mean values and the standard deviations of the yield and the loading stresses are given as $\mu_R = 350$, $\sigma_R = 35$ MPa and $\mu_S = 200$, $\sigma_S = 40$ MPa, respectively.

The limit state function describing the failurre event may be written as

$$g(r, s) = r - s, \tag{8}$$

Considering the random variable $M = R - S$, often referred to as the safety margin, the reliability index is calculated directly by

$$\beta = \frac{\mu_M}{\sigma_M} = 2.84. \tag{9}$$

Performing the $FOSM$-based reliability assessment of the steel rod, the reliability index is defined as the smallest distance between the origin of space of the standardized normal variables and the desing point. Using this proposed methodology, we also find the reliability index equal to 2.84. This problem does not offer major dificulties, since the limit state function is linear and the basic random variables are normally distributed. This comparison is useful for providing a preliminary validation to the $FOSM$ method.

5.2 The SG tube problem

Consider the SG tubes made of Inconel alloy 600. The data on cracking conditions and tube geometries are found in the Table 2. The SG tubes are subjected to stress corrosion cracking, occuring outside. Through-wall axial cracks are found between two support plates for the tubes. The data on uncertaities of the basic random variables is outlined, together with assumed distributions, in the Table 3.

Table 2: Geometrics and operational condition

r_i (mm)	t (mm)	p_b (MPa)
8.4327	1.0923	28.26

Table 3: Uncertainies of the basic variables

$Basic\ variable$	$Parameter$	$Unit$
$S_y + S_u$	$\mu_{S_y+S_u} = 1011, \sigma_{S_y+S_u} = 42.9$	MPa
ϕ	$\mu_\Phi = 1.104, \sigma_\Phi = 0.0705$	-
l	$\mu_L = l_{ECT}, \sigma_L = 2.54$	mm
h	$\mu_H = 1.01h_{ECT}, \sigma_H = 11.0$	%TW

The curve concerning the CM structural limit is depicted in the Figure 1, obtained by the $FOSM$ method and the Monte Carlo simulation. The CM structural limit is associated with the failure probability of 5%. We can note that the CM structural limit using the $FOSM$ method is in good agreement with the Monte Carlo simulation one.

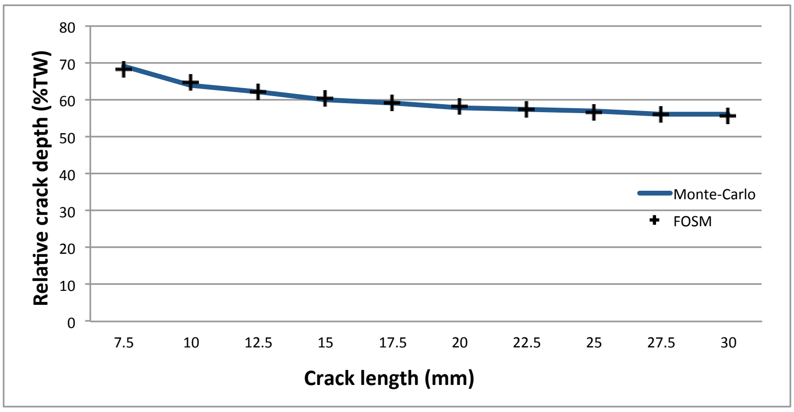

Figure 1: The CM structural limit

The Monte Carlo simulation usually provides accurate results of failure probability. In the SG tube problem, the $FOSM$ method has proved to be so accurate as the Monte Carlo simulation. By the other side, for estimating probabilities smaller than 5%, the Monte Carlo simulation is costly and lengthy, especially in performing parametric and sensitivity studies (Det Norske Veritas, 1992). The CM structural limit using the Monte Carlo simulation has required 10000 trials, while 6 iterations are required using the $FOSM$ method . The $FOSM$ method is able to improve the calculation times, so it is more efficient than the Monte Carlo simulation for the reliability assessment of this SG tube problem.

Now, consider that the last ECT inspection performed in tubes of a steam generator has found external axial cracks, located between the support plate and the tube sheet. As inspection informations, cracks are identified by position on the tube bundle (row and column) and their maximum measured sizes (length and relative depth). In the Table 4, the inspection informations are shown in addition to the results of failure probability according to the CM structural limit. We can note that just the crack with 13.9 mm of length and 67% TW of relative depth does not attend the CM structural limit of 5%-maximum failure probability.

Table 4: CM structural limit for cracks detected by ECT

Row	$Column$	l_{ECT} (mm)	h_{ECT} (%TW)	P_f
2	48	18.1	42	0.00190
4	71	17.8	37	0.00054
4	76	20.2	40	0.00140
5	76	13.6	56	0.02100
6	38	15.9	35	0.00026
6	44	18.9	44	0.00325
6	74	11.3	47	0.00247
7	47	11.6	53	0.00909
7	72	13.4	43	0.00141
9	39	22.3	38	0.00100
10	93	13.9	67	0.11110
19	91	10.5	41	0.00050

6 CONCLUSIONS

As relatively low failure probabilities are estimated, the Monte Carlo simulation proved to be costly and lengthy. Despite the Monte Carlo simulation may be accurate and widely applied, in the present work an attempt is made to improve the calculation times while retaining similar accurate results of failure probability.

We have directed the focus on quite efficient $FOSM$ method, which could be proved to be consistent with the results obtained by the Monte Carlo simulation, in addition to save computacional times. Therefore, the $FOSM$ method has demonstrated applicability for assessing the structural reliability of SG tubes containing cracks.

ACKNOWLEDGEMENTS

The authors thank the ELETRONUCLEAR/CAPES for the financial support that allows for the accomplishment of the results of this research.

REFERENCES

Bergant, M. A., Yawny, A. A., & Perez Ipina, J. E., 2015. Failure assessment diagram in structural integrity analysis of steam generator tubes. *Procedia Materials Science*, vol. 8, pp. 128-138.

Cizelj, L., Mavko, B., & Hiesch-Opperman, H., 1994. Application of first and second order reliability methods in the safety assessment of cracked steam generator tubing. *Nuclear Engineering and Design*, vol. 147, pp. 1-10.

Det Norske Veritas, 1992. Structural Reliability Analysis of Marine Structures. *Det Norske Veritas*, DNV Classification Notes no. 30.6.

Eletric Power Research Institute, 2001. Steam Generator Degradation Specific Management Flaw Handbook. *Eletric Power Research Institute*, Report 1001191.

Eletric Power Research Institute, 2006. Steam Generator Integrity Assessment Guidelines, Revision 2. *Eletric Power Research Institute*, Report 1012987.

Erdogan, F., 1976. Ductil fracture theories of pressurized pipes and containers. *International Journal of Pressure Vessels & Piping*, vol. 4, pp. 253-283.

Guoshoa, C., & Shuho, D., 1996. Study on the reliability assessment methodology for pressure vessels containing defects. *International Journal of Pressure Vessels & Piping*, vol. 69, pp. 273-277.

Hasofer, A. M., & Lind, N. C., 1974. An exact and invariant first order reliability format. *Proc. ASCE, J. Eng. Mech. Div.*, pp. 111-121.

Huh, N., Kim, J., Chang, Y., Kim, Y., Hwang, S., & Kim, J., 2006. Elastic-plastic fracture mechanics assessment for steam generator tubes with through-wall cracks. *Fatigue & Fracture of Engineering Materials & Structures*, vol. 30, pp. 131-142.

Maneschi, J. E., & Miranda, C. A. J., 2014.*Mecanica da Fratura na Industria Nuclear*. Rio de Janeiro: Lithos Edicao de Arte.

Melchers, R. E., 1987. *Structural Reliability: Analysis and Prediction*. Ellis Horwood Limited, 1st edition.

Nuclear Energy Institute, 2005. Steam Generator Program Guidelines, Revision 2. *Nuclear Energy Institute*, NEI 97-06.

Wang, X., & Reinhardt, W., 2003. On the assessment of through-wall circumferential cracks in steam generator tubes with tube supports. *Journal of Pressure Vessel Technology*, vol. 125, pp. 85-90.