



OPTIMAL PLACEMENT OF HYSTERETIC OR VISCOUS DAMPER BASED ON THE INCREMENTAL INVERSE PROBLEM

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Abstract. *Structural control system aims to improve the protection of buildings and civil structures, occupants and contents from the destructive forces of nature due to earthquakes, wind and waves. Control techniques can be classified according to how the system manipulates, absorbs and dissipates the imposed energy. Passive damping system absorbs or consumes a portion of the input energy, reducing energy dissipation on primary structural members and does not require an external power source. In this work the efficiency of four Matlab programmed routines in terms of time or computational cost and flexibility according to the type of damper will be assessed. Two techniques were evaluated: (a) An analytical procedure known as incremental inverse problem for redesign of structural system with a hysteretic damping system for target transfer functions and (b) to apply an efficient and systematic procedure for to find the optimal damper placement to minimize the sum of amplitudes of the transfer functions evaluated at the undamped fundamental natural frequency of a structural system subject to a constraint on the sum of the damping coefficients of added dampers.*

Keywords: *Optimal placement of dampers, Hysteretic and viscous devices, Incremental inverse problem, Transfer function, Passive control*

1 INTRODUCTION

One of the most important tasks for the designer is to define the optimal placement of devices to maximize its efficiency, taking advantage of the energy dissipation capacity and location of structural damage. After being subjected to destructive environmental forces, these devices are easily replaced without compromising the stability and functionality of the structure.

Optimal placement of passive devices and transfer function as a general dynamical property have been investigated in recent decades. Takewaki (1997a) presented a new analytical procedure for redesign of structural systems with an arbitrary damping system for target transfer functions, the design variables were the spring stiffnesses. Takewaki (1997b) proposed an efficient and systematic procedure for finding the optimal damper placement to minimize the sum of amplitudes of the transfer functions evaluated at the undamped fundamental natural frequency of a structural system subject to a constraint on the sum of the damping coefficients of added dampers and Aydin et al. (2007) used the coefficients of the dampers introduced as the design variables. The results of the numerical procedure show that the proposed procedure based on the transfer function of the base shear force can also be beneficial in the rehabilitation of seismic response of the structures. Aydin and Boduroglu (2008) presented a study to the optimal placement of X steel diagonal braces (SDBs) to upgrade the seismic response of a planar building frame. The objective functions were chosen as the amplitude of transfer function of the top displacement and the amplitude of transfer function of the base shear force evaluated at the undamped fundamental natural frequency of the structure. Aydin (2012) proposed a new damper optimization method for finding optimal size and location of the added viscous dampers based on the elastic base moment in planar steel building frames. The transfer function amplitude of the elastic base moment evaluated at the first natural circular frequency of the structure was chosen as a new objective function in the minimization problem. Martinez et al. (2013) proposed a procedure to optimally define the damping coefficients of added linear viscous dampers to meet an expected level of performance on buildings under seismic excitation. The performance criterion is expressed in terms of a maximum interstory drift, which is one of the most important limitations provided by the seismic design codes. Martínez et al. (2014) proposed an efficiently procedure to optimally define the energy dissipation capacity of added nonlinear hysteretic dampers, to meet an expected level of performance on planar structures under seismic excitation. Kandemir-Mazanoglu and Mazanoglu (2016) investigated optimum viscous damper capacity and number for prevention of one-sided structural pounding between two adjacent buildings under earthquake motion. Other researchers have studied the topic (Murakami et al. 2015; Orlandi, 2010; Takewaki and Uetani, 1999; Takewaki, 2000b).

In this paper two mathematical formulations developed by Izuru Takewaki for viscous systems were adapted for hysteretic systems and subsequently the computational cost and efficiency of each of the types of dampers were measured.

2 REDESIGN FOR TARGET TRANSFER FUNCTIONS (RTTF)

When small perturbations are given for the lowest eigenvalue and the lowest-mode deformation ratio for the model with (k) , the corresponding stiffnesses $(k + \Delta k)$ can be found. In this sense, Izuru Takewaki calls the present problem an incremental inverse problem in damped vibration (Takewaki, 2000a).

As shown in Figure 1, a three-story mass–spring–dashpot model with hysteric dampers is considered. Masses, spring stiffnesses, and damping coefficients are denoted by $\{\bar{m}_1, \bar{m}_2, \bar{m}_3\}$, $\{k_1, k_2, k_3\}$ and $\{\bar{\beta}_1, \bar{\beta}_2, \bar{\beta}_3\}$. The design variables are the spring stiffnesses $\{k_1, k_2, k_3\}$.

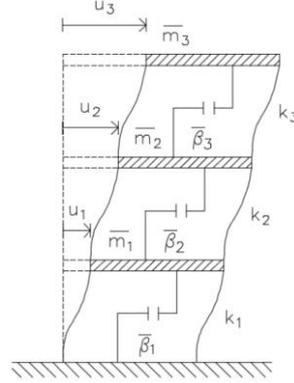


Figure 1: Three-story shear building model with added hysteretic damper

The Fourier transforms of the equation of motion for hysteretic damper model may be written as:

$$\begin{aligned} & \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} + 2i \begin{bmatrix} \bar{\beta}_1 k_1 + \bar{\beta}_2 k_2 & -\bar{\beta}_2 k_2 & 0 \\ -\bar{\beta}_2 k_2 & \bar{\beta}_2 k_2 + \bar{\beta}_3 k_3 & -\bar{\beta}_3 k_3 \\ 0 & -\bar{\beta}_3 k_3 & \bar{\beta}_3 k_3 \end{bmatrix} - \omega^2 \begin{bmatrix} \bar{m}_1 & 0 & 0 \\ 0 & \bar{m}_2 & 0 \\ 0 & 0 & \bar{m}_3 \end{bmatrix} \begin{Bmatrix} U_1(\omega) \\ U_2(\omega) \\ U_3(\omega) \end{Bmatrix} \\ & = - \begin{bmatrix} \bar{m}_1 & 0 & 0 \\ 0 & \bar{m}_2 & 0 \\ 0 & 0 & \bar{m}_3 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} \ddot{U}_g \end{aligned} \quad (1)$$

Where $U_1(\omega)$, $U_2(\omega)$, $U_3(\omega)$, $\ddot{U}_g(\omega)$ denote the Fourier transforms of u_1 , u_2 , u_3 , \ddot{u}_g , where i is the imaginary unit, u_i denote the displacements of masses m_i and \ddot{u}_g the base acceleration.

The Fourier transforms $\delta_1(\omega)$, $\delta_2(\omega)$, $\delta_3(\omega)$ of relative nodal displacements $d_1 = u_1$, $d_2 = u_2 - u_1$, $d_3 = u_3 - u_2$ can be expressed in terms of $U_1(\omega)$, $U_2(\omega)$, $U_3(\omega)$ by

$$\begin{Bmatrix} \delta_1(\omega) \\ \delta_2(\omega) \\ \delta_3(\omega) \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} U_1(\omega) \\ U_2(\omega) \\ U_3(\omega) \end{Bmatrix} \quad (2)$$

ω_1 denote the undamped fundamental natural circular frequency of the model and \hat{U}_1 , \hat{U}_2 , \hat{U}_3 new complex-value quantities.

$$\hat{U}_1 = \frac{U_1(\omega_1)}{\ddot{U}_g(\omega_1)} \quad \hat{U}_2 = \frac{U_2(\omega_1)}{\ddot{U}_g(\omega_1)} \quad \hat{U}_3 = \frac{U_3(\omega_1)}{\ddot{U}_g(\omega_1)} \quad (3)$$

\hat{U}_i represents the value such that ω_1 is substituted in the frequency response function obtained as $U_i(\omega)$ after substituting $\ddot{U}_g(\omega) = 1$ in Eq. (1). Furthermore, new complex-value quantities are defined by $\hat{\delta}_1 = \hat{U}_1$, $\hat{\delta}_2 = \hat{U}_2 - \hat{U}_1$ and $\hat{\delta}_3 = \hat{U}_3 - \hat{U}_2$.

Substitution of Eq. (3) into (1) with $\omega = \omega_1$ provides

$$A_H \begin{Bmatrix} \hat{U}_1 \\ \hat{U}_2 \\ \hat{U}_3 \end{Bmatrix} = - \begin{Bmatrix} \bar{m}_1 \\ \bar{m}_2 \\ \bar{m}_3 \end{Bmatrix} \quad (4)$$

Where A_H is the following matrix

$$A_H = \begin{bmatrix} k_1(1+2\bar{\beta}_1 i) + k_2(1+2\bar{\beta}_2 i) - \omega_1^2 \bar{m}_1 & -k_2(1+2\bar{\beta}_2 i) & 0 \\ -k_2(1+2\bar{\beta}_2 i) & k_2(1+2\bar{\beta}_2 i) + k_3(1+2\bar{\beta}_3 i) - \omega_1^2 \bar{m}_2 & -k_3(1+2\bar{\beta}_3 i) \\ 0 & -k_3(1+2\bar{\beta}_3 i) & k_3(1+2\bar{\beta}_3 i) - \omega_1^2 \bar{m}_3 \end{bmatrix} \quad (5)$$

Partial differentiation of Eq. (4) with respect to a design variable k_j provides

$$A_{H,j} \begin{Bmatrix} \hat{U}_1 \\ \hat{U}_2 \\ \hat{U}_3 \end{Bmatrix} + A_H \begin{Bmatrix} \hat{U}_{1,j} \\ \hat{U}_{2,j} \\ \hat{U}_{3,j} \end{Bmatrix} = 0 \quad (6)$$

$A_{H,j}$ in Eq. (6) can be expressed as

$$\begin{aligned} A_{H,1} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + i \begin{bmatrix} 2\bar{\beta}_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \Omega_{1,1} \begin{bmatrix} \bar{m}_1 & 0 & 0 \\ 0 & \bar{m}_2 & 0 \\ 0 & 0 & \bar{m}_3 \end{bmatrix} \\ A_{H,2} &= \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + i \begin{bmatrix} 2\bar{\beta}_2 & -2\bar{\beta}_2 & 0 \\ -2\bar{\beta}_2 & 2\bar{\beta}_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \Omega_{1,2} \begin{bmatrix} \bar{m}_1 & 0 & 0 \\ 0 & \bar{m}_2 & 0 \\ 0 & 0 & \bar{m}_3 \end{bmatrix} \\ A_{H,3} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} + i \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2\bar{\beta}_3 & -2\bar{\beta}_3 \\ 0 & -2\bar{\beta}_3 & 2\bar{\beta}_3 \end{bmatrix} - \Omega_{1,3} \begin{bmatrix} \bar{m}_1 & 0 & 0 \\ 0 & \bar{m}_2 & 0 \\ 0 & 0 & \bar{m}_3 \end{bmatrix} \end{aligned} \quad (7)$$

In the following, the partial differentiation with respect to k_j is denoted by $(\cdot)_{,j}$. Since the matrix A_H is regular, the first-order sensitivities of the complex-value quantities $\hat{U}_1, \hat{U}_2, \hat{U}_3$ are derived from Eq. (6) as

$$\begin{Bmatrix} \hat{U}_{1,j} \\ \hat{U}_{2,j} \\ \hat{U}_{3,j} \end{Bmatrix} = -A_H^{-1} A_{H,j} \begin{Bmatrix} \hat{U}_1 \\ \hat{U}_2 \\ \hat{U}_3 \end{Bmatrix} \quad (8)$$

From Eqs. (2), (8), the first derivatives of $\hat{\delta}_1 = \hat{U}_1$, $\hat{\delta}_2 = \hat{U}_2 - \hat{U}_1$ and $\hat{\delta}_3 = \hat{U}_3 - \hat{U}_2$ with respect to k_j can be computed as

$$\begin{Bmatrix} \hat{\delta}_{1,j} \\ \hat{\delta}_{2,j} \\ \hat{\delta}_{3,j} \end{Bmatrix} = - \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} A_H^{-1} A_{H,j} \begin{Bmatrix} \hat{U}_1 \\ \hat{U}_2 \\ \hat{U}_3 \end{Bmatrix}, \quad T = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad (9)$$

Where T indicates the deformation–displacement transformation matrix. The complex-value quantity $\hat{\delta}_i$ may be rewritten symbolically as

$$\hat{\delta}_i = Re[\hat{\delta}_i] + iIm[\hat{\delta}_i] \quad (10)$$

Where $Re[]$, $Im[]$ denote the real and imaginary parts respectively of a complex number. The first-order sensitivity of $\hat{\delta}_i$ may be formally expressed as

$$\hat{\delta}_{i,j} = \left(Re[\hat{\delta}_i] \right)_{,j} + i \left(Im[\hat{\delta}_i] \right)_{,j} \quad (11)$$

The absolute value $|\hat{\delta}_i|$ of $\hat{\delta}_i$ is defined by

$$|\hat{\delta}_i| = \sqrt{\left(Re[\hat{\delta}_i] \right)^2 + \left(Im[\hat{\delta}_i] \right)^2} \quad (12)$$

The first-order sensitivity of $|\hat{\delta}_i|$ with respect to k_j may be expressed as

$$|\hat{\delta}_i|_{,j} = \frac{1}{|\hat{\delta}_i|} \left\{ Re[\hat{\delta}_i] \left(Re[\hat{\delta}_i] \right)_{,j} + Im[\hat{\delta}_i] \left(Im[\hat{\delta}_i] \right)_{,j} \right\} \quad (13)$$

Where $\left(Re[\hat{\delta}_i] \right)_{,j}$ and $\left(Im[\hat{\delta}_i] \right)_{,j}$ can be calculated from Eq. (9).

New quantity is defined as the ratio between two relative nodal displacements by

$$\alpha_1(k) = \frac{|\hat{\delta}_2(k)|}{|\hat{\delta}_1(k)|} \quad \alpha_2(k) = \frac{|\hat{\delta}_3(k)|}{|\hat{\delta}_1(k)|} \quad (14)$$

The variation of the lowest eigenvalue due to Δk is evaluated by the linear approximation

$$\Delta\Omega_1(k) = \frac{\partial\Omega_1(k)}{\partial k} \Delta k \quad (15)$$

Similarly, the variations of the deformation ratios defined in Eq. (14) due to Δk may be evaluated by the following linear approximation

$$\begin{aligned} \Delta\alpha_1(k) &= \left(\frac{\partial|\hat{\delta}_2(k)|}{\partial k} \frac{1}{|\hat{\delta}_1(k)|} - \frac{\partial|\hat{\delta}_1(k)|}{\partial k} \frac{|\hat{\delta}_2(k)|}{|\hat{\delta}_1(k)|^2} \right) \Delta k = \frac{1}{|\hat{\delta}_1(k)|} \left(\frac{\partial|\hat{\delta}_2(k)|}{\partial k} - \frac{\partial|\hat{\delta}_1(k)|}{\partial k} \alpha_1(k) \right) \Delta k \\ \Delta\alpha_2(k) &= \left(\frac{\partial|\hat{\delta}_3(k)|}{\partial k} \frac{1}{|\hat{\delta}_1(k)|} - \frac{\partial|\hat{\delta}_1(k)|}{\partial k} \frac{|\hat{\delta}_3(k)|}{|\hat{\delta}_1(k)|^2} \right) \Delta k = \frac{1}{|\hat{\delta}_1(k)|} \left(\frac{\partial|\hat{\delta}_3(k)|}{\partial k} - \frac{\partial|\hat{\delta}_1(k)|}{\partial k} \alpha_2(k) \right) \Delta k \end{aligned} \quad (16)$$

Equations. (15), (16) may be arranged to the following set of simultaneous linear equations with respect to Δk

$$\begin{aligned} & \begin{bmatrix} \frac{\partial\Omega_1(k)}{\partial k_1} & \frac{\partial\Omega_1(k)}{\partial k_2} & \frac{\partial\Omega_1(k)}{\partial k_3} \\ \frac{\partial|\hat{\delta}_2(k)|}{\partial k_1} - \frac{\partial|\hat{\delta}_1(k)|}{\partial k_1} \alpha_1(k) & \frac{\partial|\hat{\delta}_2(k)|}{\partial k_2} - \frac{\partial|\hat{\delta}_1(k)|}{\partial k_2} \alpha_1(k) & \frac{\partial|\hat{\delta}_2(k)|}{\partial k_3} - \frac{\partial|\hat{\delta}_1(k)|}{\partial k_3} \alpha_1(k) \\ \frac{\partial|\hat{\delta}_3(k)|}{\partial k_1} - \frac{\partial|\hat{\delta}_1(k)|}{\partial k_1} \alpha_2(k) & \frac{\partial|\hat{\delta}_3(k)|}{\partial k_2} - \frac{\partial|\hat{\delta}_1(k)|}{\partial k_2} \alpha_2(k) & \frac{\partial|\hat{\delta}_3(k)|}{\partial k_3} - \frac{\partial|\hat{\delta}_1(k)|}{\partial k_3} \alpha_2(k) \end{bmatrix} \begin{Bmatrix} \Delta k_1 \\ \Delta k_2 \\ \Delta k_3 \end{Bmatrix} \\ & = \begin{Bmatrix} \Delta\Omega_1(k) \\ |\hat{\delta}_1(k)| \Delta\alpha_1(k) \\ |\hat{\delta}_1(k)| \Delta\alpha_2(k) \end{Bmatrix} \end{aligned} \quad (17)$$

Equation (17) indicates that, once $\Delta\Omega_1(k)$ and $\Delta\alpha_1(k)$ are given, Δk to be determined can be found.

The first-order sensitivity of the lowest eigenvalue $\Omega_1(k)$ is well-known in the field of structural optimization and maybe expressed as (Fox and Kapoor, 1968):

$$\Omega_1(k)_{,j} = V^{(1)T} K_{,j} V^{(1)} \quad (18)$$

Where $V^{(1)}$ indicates the undamped lowest eigenvector and satisfies the following normalization condition

$$V^{(1)T} M V^{(1)} = 1 \quad (19)$$

K and M are the stiffness and mass matrices. Then the design sensitivity of undamped fundamental natural circular frequency may be given by

$$\omega_1(k)_{,j} = \frac{1}{2\omega_1(k)} V^{(1)T} K_{,j} V^{(1)} \quad (20)$$

The linear increment of α and Ω_1 to be specified in this formulations are given as

$$\Delta\alpha = \frac{1}{N} (\alpha_F - \alpha_0) \quad \Delta\Omega_1 = \frac{1}{N} (\Omega_{1F} - \Omega_{1(0)}) \quad (21)$$

3 OPTIMAL DAMPER PLACEMENT (ODP)

As shown in Figure 1 , a three-story mass-spring-dashpot model with hysteretic dampers is considered. The design variables are the damping coefficient $\beta = \{\beta_1, \beta_2, \beta_3\}$ of added hysteretic dampers and $K = \{\bar{k}_1, \bar{k}_2, \bar{k}_3\}$ are fixed values. It is also assumed here that the original structural damping is negligible compared with the damping of the added dampers.

The Fourier transforms of the equation of motion for hysteretic damper model may be written as:

$$\begin{aligned} & \left(\begin{bmatrix} \bar{k}_1 + \bar{k}_2 & -\bar{k}_2 & 0 \\ -\bar{k}_2 & \bar{k}_2 + \bar{k}_3 & -\bar{k}_3 \\ 0 & -\bar{k}_3 & \bar{k}_3 \end{bmatrix} + 2i \begin{bmatrix} \beta_1 \bar{k}_1 + \beta_2 \bar{k}_2 & -\beta_2 \bar{k}_2 & 0 \\ -\beta_2 \bar{k}_2 & \beta_2 \bar{k}_2 + \beta_3 \bar{k}_3 & -\beta_3 \bar{k}_3 \\ 0 & -\beta_3 \bar{k}_3 & \beta_3 \bar{k}_3 \end{bmatrix} - \omega^2 \begin{bmatrix} \bar{m}_1 & 0 & 0 \\ 0 & \bar{m}_2 & 0 \\ 0 & 0 & \bar{m}_3 \end{bmatrix} \right) \begin{Bmatrix} U_1(\omega) \\ U_2(\omega) \\ U_3(\omega) \end{Bmatrix} \\ & = - \begin{bmatrix} \bar{m}_1 & 0 & 0 \\ 0 & \bar{m}_2 & 0 \\ 0 & 0 & \bar{m}_3 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} \ddot{U}_g \end{aligned} \quad (22)$$

This procedure considered the Eqs. (2), (3), (4), (5).

The problem of optimal damper placement for a fixed shear building model may be described as

$$V = \sum_{i=1}^3 |\hat{\delta}_i(\beta)| \quad (23)$$

The quantity V represents the global flexibility and its minimization is preferred from the view of performance-based design.

Subject to a constraint on the sum of the damping coefficients of added dampers

$$\sum_{i=1}^3 \beta_i = \bar{W} \quad (\bar{W} : \text{specified value}) \quad (24)$$

The Lagrangian for this problem may be expressed as

$$L(\beta, \lambda) = \sum_{i=1}^3 |\hat{\delta}_i(\beta)| + \lambda \left(\sum_{i=1}^3 \beta_i - \bar{W} \right) \quad (25)$$

From the stationarity condition of this Lagrangian with respect to $L(\beta, \lambda)$, the following optimality conditions can be derived

$$\left(\sum_{i=1}^3 |\hat{\delta}_i| \right)_{,j} + \lambda = 0 \quad (j=1, 2, 3) \quad (26)$$

$$\sum_{i=1}^3 \beta_i - \bar{W} = 0 \quad (27)$$

Where $(\cdot)_{,j}$ indicates the partial differentiation with respect to $\beta_{,j}$. If $\beta_{,j} = 0$, then Eq. (26) must be modified into:

$$\left(\sum_{i=1}^3 |\hat{\delta}_i| \right)_{,j} + \lambda \geq 0 \quad (28)$$

The optimality criteria, Eqs. (25), (28) include an undetermined parameter λ . Another expression without the parameter λ can be obtained by defining the following quantity

$$\gamma_1 = \frac{\left(\sum_{i=1}^3 |\hat{\delta}_i| \right)_{,2}}{\left(\sum_{i=1}^3 |\hat{\delta}_i| \right)_{,1}}, \quad \gamma_2 = \frac{\left(\sum_{i=1}^3 |\hat{\delta}_i| \right)_{,3}}{\left(\sum_{i=1}^3 |\hat{\delta}_i| \right)_{,1}} \quad (29)$$

The optimality conditions, Eqs. (26), (28) can be rewritten as $\gamma_1 = 1$ for $\beta_1 > 0$, $\beta_2 > 0$ and $\gamma_1 \geq 1$ for $\beta_1 = 0$ with the quantity defined by Eq. (29).

Differentiation of Eq. (4) with respect to a design variable $\beta_{,j}$ provides

$$A_{H,j} \begin{Bmatrix} \hat{U}_1 \\ \hat{U}_2 \end{Bmatrix} + A_H \begin{Bmatrix} \hat{U}_{1,j} \\ \hat{U}_{2,j} \end{Bmatrix} = 0 \quad (30)$$

$A_{H,j}$ in Eq. (30) can be expressed as

$$A_{H,1} = 2i \begin{bmatrix} \bar{k}_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad A_{H,2} = 2i \begin{bmatrix} \bar{k}_2 & -\bar{k}_2 & 0 \\ -\bar{k}_2 & \bar{k}_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad A_{H,3} = 2i \begin{bmatrix} 0 & 0 & 0 \\ 0 & \bar{k}_3 & -\bar{k}_3 \\ 0 & -\bar{k}_3 & \bar{k}_3 \end{bmatrix} \quad (31)$$

Since A_H is regular, the first derivatives of \hat{U}_1 , \hat{U}_2 and \hat{U}_3 can be written as

$$\begin{Bmatrix} \hat{U}_{1,j} \\ \hat{U}_{2,j} \\ \hat{U}_{3,j} \end{Bmatrix} = -A_H^{-1} A_{H,j} \begin{Bmatrix} \hat{U}_1 \\ \hat{U}_2 \\ \hat{U}_3 \end{Bmatrix} \quad (32)$$

Furthermore the first derivate of the interstory drift can be expressed as

$$\begin{Bmatrix} \hat{\delta}_{1,j} \\ \hat{\delta}_{2,j} \\ \hat{\delta}_{3,j} \end{Bmatrix} = -T A_H^{-1} A_{H,j} T^{-1} \begin{Bmatrix} \hat{\delta}_1 \\ \hat{\delta}_2 \\ \hat{\delta}_3 \end{Bmatrix} \quad (33)$$

$\hat{\delta}_i$ may be expressed formally as

$$\hat{\delta}_i = Re[\hat{\delta}_i] + iIm[\hat{\delta}_i] \quad (34)$$

The first derivative of $\hat{\delta}_i$ may be formally written as

$$\hat{\delta}_{i,j} = \left(Re[\hat{\delta}_i] \right)_{,j} + i \left(Im[\hat{\delta}_i] \right)_{,j} \quad (35)$$

The absolute value of $\hat{\delta}_i$ is defined by

$$|\hat{\delta}_i| = \sqrt{\left(Re[\hat{\delta}_i] \right)^2 + \left(Im[\hat{\delta}_i] \right)^2} \quad (36)$$

The first derivative of $|\hat{\delta}_i|$ may then be written as

$$|\hat{\delta}_i|_{,j} = \frac{1}{|\hat{\delta}_i|} \left\{ Re[\hat{\delta}_i] \left(Re[\hat{\delta}_i] \right)_{,j} + Im[\hat{\delta}_i] \left(Im[\hat{\delta}_i] \right)_{,j} \right\} \quad (37)$$

Where $\left(Re[\hat{\delta}_i] \right)_{,j}$, $\left(Im[\hat{\delta}_i] \right)_{,j}$ are calculated from Eq. (33).

The linear increment $\Delta\gamma_1$, $\Delta\gamma_2$ of γ_1 , γ_2 may be expressed as

$$\begin{aligned} \Delta\gamma_1 &= \left(\frac{1}{B_1} \frac{\partial B_2}{\partial \beta} - \frac{B_2}{B_1^2} \frac{\partial B_1}{\partial \beta} \right) \Delta\beta = \frac{1}{B_1} \left(\frac{\partial B_2}{\partial \beta} - \frac{\partial B_1}{\partial \beta} \gamma_1 \right) \Delta\beta \\ \Delta\gamma_2 &= \left(\frac{1}{B_1} \frac{\partial B_3}{\partial \beta} - \frac{B_3}{B_1^2} \frac{\partial B_1}{\partial \beta} \right) \Delta\beta = \frac{1}{B_1} \left(\frac{\partial B_3}{\partial \beta} - \frac{\partial B_1}{\partial \beta} \gamma_2 \right) \Delta\beta \end{aligned} \quad (38)$$

Where B_1 , B_2 , B_3 are the following quantities

$$B_1 = \left(\sum_{i=1}^3 |\hat{\delta}_i| \right)_{,1} \quad B_2 = \left(\sum_{i=1}^3 |\hat{\delta}_i| \right)_{,2} \quad B_3 = \left(\sum_{i=1}^3 |\hat{\delta}_i| \right)_{,3} \quad (39)$$

The increments $\Delta\beta$ must satisfy the following relation due to the constraint Eq. (27)

$$\sum_{i=1}^3 \Delta\beta_i = 0 \quad (40)$$

Arrangement of Eqs. (38), (40) leads to the following set of simultaneous linear equations with respect to $\Delta\beta$

$$\begin{bmatrix} \frac{1}{B_1} \left(\frac{\partial B_2}{\partial \beta_1} - \frac{\partial B_1}{\partial \beta_1} \gamma_1 \right) & \frac{1}{B_1} \left(\frac{\partial B_2}{\partial \beta_2} - \frac{\partial B_1}{\partial \beta_2} \gamma_1 \right) & \frac{1}{B_1} \left(\frac{\partial B_2}{\partial \beta_3} - \frac{\partial B_1}{\partial \beta_3} \gamma_1 \right) \\ \frac{1}{B_1} \left(\frac{\partial B_3}{\partial \beta_1} - \frac{\partial B_1}{\partial \beta_1} \gamma_2 \right) & \frac{1}{B_1} \left(\frac{\partial B_3}{\partial \beta_2} - \frac{\partial B_1}{\partial \beta_2} \gamma_2 \right) & \frac{1}{B_1} \left(\frac{\partial B_3}{\partial \beta_3} - \frac{\partial B_1}{\partial \beta_3} \gamma_2 \right) \\ 1 & 1 & 1 \end{bmatrix} \begin{Bmatrix} \Delta\beta_1 \\ \Delta\beta_2 \\ \Delta\beta_3 \end{Bmatrix} = \begin{Bmatrix} \Delta\gamma_1 \\ \Delta\gamma_2 \\ 0 \end{Bmatrix} \quad (41)$$

The increments $\Delta\gamma_1$ and $\Delta\gamma_2$ are given here as $\Delta\gamma_1 = (1 - \gamma_{01})/N$ and $\Delta\gamma_2 = (1 - \gamma_{02})/N$ where N is the step number and γ_{01}, γ_{02} denotes the initial value of γ_1 and γ_2 . It should be remarked that, if either one of β_1, β_2 or β_3 vanishes, the following relation must be satisfied. If $\beta_1 = 0, \gamma_1 \geq 1$. If $\beta_2 = 0, \gamma_1 \leq 1$. Similarly for γ_2 .

$\partial\beta_1/\partial\beta_i, \partial\beta_2/\partial\beta_i$ and $\partial\beta_3/\partial\beta_i$ can be evaluated in the following manner. Partial differentiation of Eq. (37) with respect to β_k leads to

$$\frac{\partial |\hat{\delta}_i|_{,jk}}{\partial \beta_i} = \frac{1}{|\hat{\delta}_i|^2} \left\{ \begin{array}{l} \left[\left(\text{Re}[\hat{\delta}_i] \right)_{,k} \left(\text{Re}[\hat{\delta}_i] \right)_{,j} + \text{Re}[\hat{\delta}_i] \left(\text{Re}[\hat{\delta}_i] \right)_{,jk} \right] \\ + \left[\left(\text{Im}[\hat{\delta}_i] \right)_{,k} \left(\text{Im}[\hat{\delta}_i] \right)_{,j} + \text{Im}[\hat{\delta}_i] \left(\text{Im}[\hat{\delta}_i] \right)_{,jk} \right] \end{array} \right\} \frac{\partial |\hat{\delta}_i|_{,k}}{\partial \beta_i} \left\{ \begin{array}{l} \text{Re}[\hat{\delta}_i] \left(\text{Re}[\hat{\delta}_i] \right)_{,j} \\ + \text{Im}[\hat{\delta}_i] \left(\text{Im}[\hat{\delta}_i] \right)_{,j} \end{array} \right\} \quad (42)$$

Then $\partial\beta_1/\partial\beta_i, \partial\beta_2/\partial\beta_i$ and $\partial\beta_3/\partial\beta_i$ may be expressed as follows

$$\frac{\partial B_1}{\partial \beta_1} = \left\{ |\hat{\delta}_1|_{,11} + |\hat{\delta}_2|_{,11} + |\hat{\delta}_3|_{,11} \right\} \quad \frac{\partial B_1}{\partial \beta_2} = \left\{ |\hat{\delta}_1|_{,12} + |\hat{\delta}_2|_{,12} + |\hat{\delta}_3|_{,12} \right\} \quad \frac{\partial B_1}{\partial \beta_3} = \left\{ |\hat{\delta}_1|_{,13} + |\hat{\delta}_2|_{,13} + |\hat{\delta}_3|_{,13} \right\} \quad (43)$$

$$\frac{\partial B_2}{\partial \beta_1} = \left\{ |\hat{\delta}_1|_{,21} + |\hat{\delta}_2|_{,21} + |\hat{\delta}_3|_{,21} \right\} \quad \frac{\partial B_2}{\partial \beta_2} = \left\{ |\hat{\delta}_1|_{,22} + |\hat{\delta}_2|_{,22} + |\hat{\delta}_3|_{,22} \right\} \quad \frac{\partial B_2}{\partial \beta_3} = \left\{ |\hat{\delta}_1|_{,23} + |\hat{\delta}_2|_{,23} + |\hat{\delta}_3|_{,23} \right\} \quad (44)$$

$$\frac{\partial B_3}{\partial \beta_1} = \left\{ \left| \hat{\delta}_1 \right|_{,31} + \left| \hat{\delta}_2 \right|_{,31} + \left| \hat{\delta}_3 \right|_{,31} \right\} \quad \frac{\partial B_3}{\partial \beta_2} = \left\{ \left| \hat{\delta}_1 \right|_{,32} + \left| \hat{\delta}_2 \right|_{,32} + \left| \hat{\delta}_3 \right|_{,32} \right\} \quad \frac{\partial B_3}{\partial \beta_3} = \left\{ \left| \hat{\delta}_1 \right|_{,33} + \left| \hat{\delta}_2 \right|_{,33} + \left| \hat{\delta}_3 \right|_{,33} \right\} \quad (45)$$

$\left(\text{Re} \left[\hat{\delta}_i \right] \right)_{,jk}$ and $\left(\text{Im} \left[\hat{\delta}_i \right] \right)_{,jk}$ in Eq. (42) can be obtained from

$$\begin{Bmatrix} \hat{\delta}_{1,jk} \\ \hat{\delta}_{2,jk} \\ \hat{\delta}_{3,jk} \end{Bmatrix} = T A_H^{-1} A_{H,k} A_H^{-1} A_{H,j} T^{-1} \begin{Bmatrix} \hat{\delta}_1 \\ \hat{\delta}_2 \\ \hat{\delta}_3 \end{Bmatrix} - T A_H^{-1} A_{H,j} T^{-1} \begin{Bmatrix} \hat{\delta}_{1,k} \\ \hat{\delta}_{2,k} \\ \hat{\delta}_{3,k} \end{Bmatrix} \quad (46)$$

Which is derived by differentiating Eq. (33) with respect to β_k and using the relation $A_{H,k}^{-1} = -A_H^{-1} A_{H,k} A_H^{-1}$. Since A_H is a linear function of β , $A_{H,jk}$ becomes a null matrix for all j and k .

For a detail explanation of the mathematical formulation for a system with viscous damping, see Takewaki (1997a, b); Takewaki (2000a) and Takewaki (2009).

4 NUMERICAL EXAMPLE

4.1 Example 1

A hysteretic and viscous damping model mass-spring-dashpot, with two to nine floors will be considered. The objective of this exercise is to determine the computational cost (time) and to know which of the two types of dampers is more efficient.

For RTTF the design variables are the spring stiffnesses $\{k_1 \dots k_n\}$ and for ODP the design variables are the damping coefficients $\{\beta_1 \dots \beta_n\}$ or $\{c_1 \dots c_n\}$.

The model properties are given in Table 1: $\{m_1 \dots m_n\}$ are the masses, $\{k_1 \dots k_n\}$ the spring stiffnesses, $\{c_1 \dots c_n\}$ the viscous damping and $\{\beta_1 \dots \beta_n\}$ the hysteretic damping.

Table 1. Model properties

| Viscous System | | | Hysteretic System | | |
|-----------------|----------|---------|-------------------------|----------|-------|
| $m_1 \dots m_n$ | 8.00E+04 | [kg] | $m_1 \dots m_n$ | 8.00E+04 | [kg] |
| $c_1 \dots c_n$ | 3.05E+06 | [N·s/m] | $\beta_1 \dots \beta_n$ | 0.20 | |
| $k_1 \dots k_n$ | 4.00E+07 | [N/m] | $k_1 \dots k_n$ | 4.00E+07 | [N/m] |

The Table 2 shows the initial lowest eigenvalue of the undamped model ($\Omega_{1(0)}$), the target value of the lowest eigenvalue (Ω_{1F}), the undamped fundamental natural circular

frequency (ω_1), the target value of the transfer function ratio (α_{Fj}), the final values of γ (γ_{Fj}) and the number of steps in the redesign process (N).

Table 2. Initial and target values for RTTF and ODP

| | 2 DOF | 3 DOF | 4 DOF | 5 DOF | 6 DOF | 7 DOF | 8 DOF | 9 DOF |
|-----------------------------|---------|--------|--------|--------|--------|--------|--------|--------|
| $\Omega_{1(0)} (rad^2/s^2)$ | 190.983 | 99.031 | 60.307 | 40.500 | 29.058 | 21.852 | 17.027 | 13.639 |
| $\Omega_{1F} (rad^2/s^2)$ | 190.980 | 99.031 | 60.307 | 40.500 | 29.058 | 21.852 | 17.027 | 13.639 |
| $\omega_1 (rad/s)$ | 13.820 | 9.951 | 7.766 | 6.365 | 5.391 | 4.675 | 4.126 | 3.693 |
| α_{Fj} | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| γ_{Fj} | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| N | 60 | 60 | 55 | 60 | 55 | 60 | 60 | 55 |

Other model properties are given in Table 3 and Table 4.

Table 3. Initial values of transfer function ratios $\{\alpha_j\}$

| | DOF | α_1 | α_2 | α_3 | α_4 | α_5 | α_6 | α_7 | α_8 |
|------------|---------|------------|------------|------------|------------|------------|------------|------------|------------|
| Hysteretic | 2 | 0.597 | - | - | - | - | - | - | - |
| | 3 | 0.777 | 0.425 | - | - | - | - | - | - |
| | 4 | 0.856 | 0.625 | 0.329 | - | - | - | - | - |
| | 5 | 0.897 | 0.733 | 0.519 | 0.269 | - | - | - | - |
| | 6 | 0.922 | 0.799 | 0.637 | 0.443 | 0.227 | - | - | - |
| | 7 | 0.938 | 0.842 | 0.714 | 0.561 | 0.386 | 0.197 | - | - |
| | 8 | 0.949 | 0.871 | 0.768 | 0.643 | 0.500 | 0.341 | 0.173 | - |
| | 9 | 0.957 | 0.892 | 0.807 | 0.704 | 0.584 | 0.450 | 0.306 | 0.155 |
| | Viscous | 2 | 0.547 | - | - | - | - | - | - |
| 3 | | 0.742 | 0.395 | - | - | - | - | - | - |
| 4 | | 0.837 | 0.602 | 0.315 | - | - | - | - | - |
| 5 | | 0.889 | 0.723 | 0.509 | 0.263 | - | - | - | - |
| 6 | | 0.921 | 0.797 | 0.635 | 0.442 | 0.227 | - | - | - |
| 7 | | 0.941 | 0.847 | 0.720 | 0.566 | 0.390 | 0.199 | - | - |
| 8 | | 0.955 | 0.880 | 0.779 | 0.654 | 0.509 | 0.349 | 0.177 | - |
| 9 | | 0.964 | 0.904 | 0.822 | 0.719 | 0.598 | 0.463 | 0.315 | 0.160 |

Table 4. Initial values of $\{\gamma_j\}$

| | DOF | γ_1 | γ_2 | γ_3 | γ_4 | γ_5 | γ_6 | γ_7 | γ_8 |
|------------|---------|------------|------------|------------|------------|------------|------------|------------|------------|
| Hysteretic | 2 | 0.393 | - | - | - | - | - | - | - |
| | 3 | 0.632 | 0.215 | - | - | - | - | - | - |
| | 4 | 0.753 | 0.425 | 0.138 | - | - | - | - | - |
| | 5 | 0.821 | 0.568 | 0.305 | 0.098 | - | - | - | - |
| | 6 | 0.863 | 0.664 | 0.440 | 0.230 | 0.074 | - | - | - |
| | 7 | 0.891 | 0.731 | 0.542 | 0.350 | 0.181 | 0.058 | - | - |
| | 8 | 0.910 | 0.779 | 0.619 | 0.448 | 0.285 | 0.146 | 0.048 | - |
| | 9 | 0.925 | 0.814 | 0.678 | 0.528 | 0.377 | 0.237 | 0.121 | 0.040 |
| | Viscous | 2 | 0.435 | - | - | - | - | - | - |
| 3 | | 0.626 | 0.243 | - | - | - | - | - | - |
| 4 | | 0.741 | 0.428 | 0.153 | - | - | - | - | - |
| 5 | | 0.814 | 0.565 | 0.309 | 0.104 | - | - | - | - |
| 6 | | 0.862 | 0.663 | 0.440 | 0.231 | 0.075 | - | - | - |
| 7 | | 0.895 | 0.735 | 0.544 | 0.350 | 0.178 | 0.056 | - | - |
| 8 | | 0.918 | 0.788 | 0.626 | 0.451 | 0.283 | 0.141 | 0.043 | - |
| 9 | | 0.934 | 0.827 | 0.689 | 0.535 | 0.377 | 0.233 | 0.114 | 0.034 |

Results. The Table 5 shows the distribution of the spring stiffnesses when the first technique is applied (RTTF) on hysteretic system, also the initial and final flexibility. This technique is not appropriate for use in hysteretic systems because the final flexibility was higher than the initial.

Table 5: Result for RTTF – Hysteretic system

| DOF | Flexibility | | Spring stiffnesses $[\times 10^7 N/m]$ | | | | | | | | |
|-----|-------------|-------|--|-------|-------|-------|-------|-------|-------|-------|-------|
| | V_I | V_F | k_1 | k_2 | k_3 | k_4 | k_5 | k_6 | k_7 | k_8 | k_9 |
| 2 | 0.015 | 0.016 | 4.465 | 3.180 | - | - | - | - | - | - | - |
| 3 | 0.031 | 0.032 | 4.615 | 3.936 | 2.549 | - | - | - | - | - | - |
| 4 | 0.051 | 0.054 | 4.653 | 4.236 | 3.461 | 2.146 | - | - | - | - | - |
| 5 | 0.077 | 0.083 | 4.723 | 4.408 | 3.871 | 3.038 | 1.795 | - | - | - | - |
| 6 | 0.108 | 0.117 | 4.713 | 4.484 | 4.109 | 3.549 | 2.739 | 1.592 | - | - | - |
| 7 | 0.144 | 0.157 | 4.763 | 4.569 | 4.267 | 3.834 | 3.239 | 2.439 | 1.378 | - | - |
| 8 | 0.185 | 0.203 | 4.775 | 4.614 | 4.371 | 4.032 | 3.579 | 2.984 | 2.216 | 1.234 | - |
| 9 | 0.231 | 0.254 | 4.748 | 4.618 | 4.427 | 4.164 | 3.817 | 3.368 | 2.793 | 2.063 | 1.143 |

The Table 6 shows the distribution of the spring stiffnesses when the first technique is applied (RTTF) on viscous system, also the initial and final flexibility. This technique is appropriate for use in viscous systems because the final flexibility was lower than the initial.

Table 6: Result for RTTF – Viscous system

| Flexibility | | | Spring stiffnesses $[\times 10^7 N/m]$ | | | | | | | | |
|-------------|-------|-------|--|-------|-------|-------|-------|-------|-------|-------|-------|
| DOF | V_I | V_F | k_1 | k_2 | k_3 | k_4 | k_5 | k_6 | k_7 | k_8 | k_9 |
| 2 | 0.007 | 0.007 | 6.433 | 2.196 | - | - | - | - | - | - | - |
| 3 | 0.017 | 0.016 | 6.485 | 4.787 | 1.386 | - | - | - | - | - | - |
| 4 | 0.036 | 0.033 | 6.085 | 5.179 | 3.753 | 1.006 | - | - | - | - | - |
| 5 | 0.065 | 0.059 | 6.008 | 5.384 | 4.477 | 3.148 | 0.726 | - | - | - | - |
| 6 | 0.107 | 0.098 | 5.607 | 5.211 | 4.641 | 3.836 | 2.681 | 0.620 | - | - | - |
| 7 | 0.164 | 0.151 | 5.588 | 5.277 | 4.840 | 4.245 | 3.446 | 2.355 | 0.475 | - | - |
| 8 | 0.238 | 0.223 | 5.427 | 5.195 | 4.868 | 4.429 | 3.852 | 3.099 | 2.097 | 0.407 | - |
| 9 | 0.332 | 0.319 | 5.148 | 4.988 | 4.757 | 4.445 | 4.034 | 3.502 | 2.813 | 1.907 | 0.415 |

The Table 7 and Table 8 shows the distribution of the damping coefficient when the second technique is applied (ODP), also the initial and final flexibility. This technique is appropriate for use in hysteretic or viscous systems because the final flexibility was lower than the initial.

Table 7: Result for ODP – Hysteretic system

| Flexibility | | | Damping coefficients $[\times 10^6 N \cdot s/m]$ | | | | | | | | |
|-------------|-------|-------|--|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| DOF | V_I | V_F | β_1 | β_2 | β_3 | β_4 | β_5 | β_6 | β_7 | β_8 | β_9 |
| 2 | 0.015 | 0.013 | 0.400 | 0.000 | - | - | - | - | - | - | - |
| 3 | 0.031 | 0.023 | 0.385 | 0.215 | 0.000 | - | - | - | - | - | - |
| 4 | 0.051 | 0.038 | 0.443 | 0.357 | 0.000 | 0.000 | - | - | - | - | - |
| 5 | 0.077 | 0.055 | 0.422 | 0.359 | 0.219 | 0.000 | 0.000 | - | - | - | - |
| 6 | 0.108 | 0.078 | 0.379 | 0.325 | 0.275 | 0.222 | 0.000 | 0.000 | - | - | - |
| 7 | 0.144 | 0.100 | 0.443 | 0.402 | 0.341 | 0.214 | 0.000 | 0.000 | 0.000 | - | - |
| 8 | 0.185 | 0.141 | 0.362 | 0.326 | 0.279 | 0.198 | 0.195 | 0.240 | 0.000 | 0.000 | - |
| 9 | 0.231 | 0.159 | 0.450 | 0.420 | 0.381 | 0.325 | 0.224 | 0.000 | 0.000 | 0.000 | 0.000 |

Table 8: Result for ODP – Viscous system

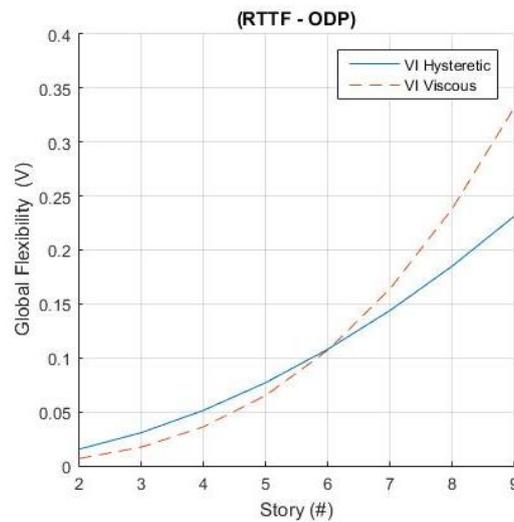
| Flexibility | | | Damping coefficients | | | | | | | | |
|-------------|-------|-------|----------------------|-------|-------|-------|-------|-------|-------|-------|-------|
| DOF | V_I | V_F | c_1 | c_2 | c_3 | c_4 | c_5 | c_6 | c_7 | c_8 | c_9 |
| 2 | 0.007 | 0.007 | 4.040 | 2.060 | - | - | - | - | - | - | - |
| 3 | 0.017 | 0.014 | 5.204 | 3.946 | 0.000 | - | - | - | - | - | - |
| 4 | 0.036 | 0.028 | 5.423 | 4.408 | 2.368 | 0.000 | - | - | - | - | - |
| 5 | 0.065 | 0.047 | 6.088 | 5.234 | 3.928 | 0.000 | 0.000 | - | - | - | - |
| 6 | 0.107 | 0.075 | 6.565 | 5.815 | 4.601 | 1.319 | 0.000 | 0.000 | - | - | - |
| 7 | 0.164 | 0.115 | 6.985 | 6.327 | 5.348 | 1.106 | 1.584 | 0.000 | 0.000 | - | - |
| 8 | 0.238 | 0.156 | 7.442 | 6.839 | 5.919 | 4.199 | 0.000 | 0.000 | 0.000 | 0.000 | - |
| 9 | 0.332 | 0.212 | 8.149 | 7.600 | 6.741 | 4.960 | 0.000 | 0.000 | 0.000 | 0.000 | - |

The computational cost is shown in Table 9. The technique RTTF had lower computational cost than the ODP technique, but the technique ODP it is more practical and efficient, because the final sensitivity values were lower compared with the values obtained from the RTTF technique (See Figure 3, Figure 4, Figure 5 and Figure 6).

Table 9: Computational cost (seconds)

| | | 2 DOF | 3 DOF | 4 DOF | 5 DOF | 6 DOF | 7 DOF | 8 DOF | 9 DOF |
|-------------|------------|-------|-------|-------|--------|--------|--------|--------|--------|
| RTTF | Hysteretic | 1.578 | 2.039 | 2.616 | 3.230 | 4.263 | 4.642 | 5.400 | 6.142 |
| | Viscous | 1.549 | 2.100 | 2.621 | 3.253 | 3.756 | 4.583 | 5.751 | 5.821 |
| ODP | Hysteretic | 1.816 | 3.365 | 6.001 | 11.304 | 11.451 | 23.892 | 20.384 | 46.169 |
| | Viscous | 2.055 | 3.776 | 5.656 | 10.259 | 10.762 | 13.430 | 37.337 | 68.249 |

According to Figure 2 is more efficient to use hysteretic damping for a lower number than 6 DOF the difference between the two dampers it is low, but for a higher number of DOF the difference turns out to be higher, being more efficient the hysteretic damper type.

**Figure 2. Initial flexibility – Hysteretic vs. Viscous system**

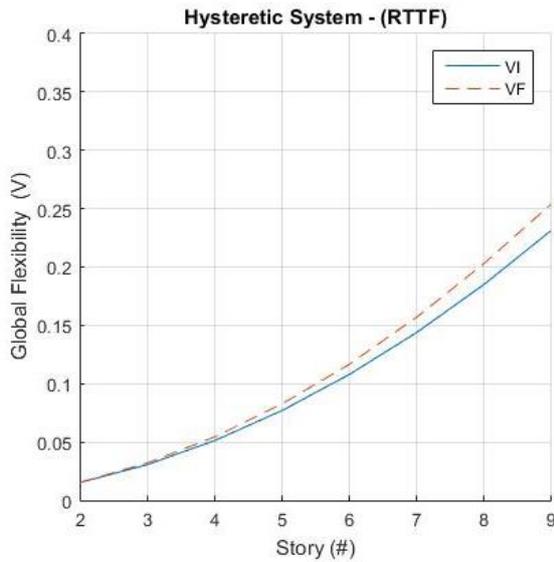


Figure 3. Initial and final flexibility – Hysteretic system (RTTF)

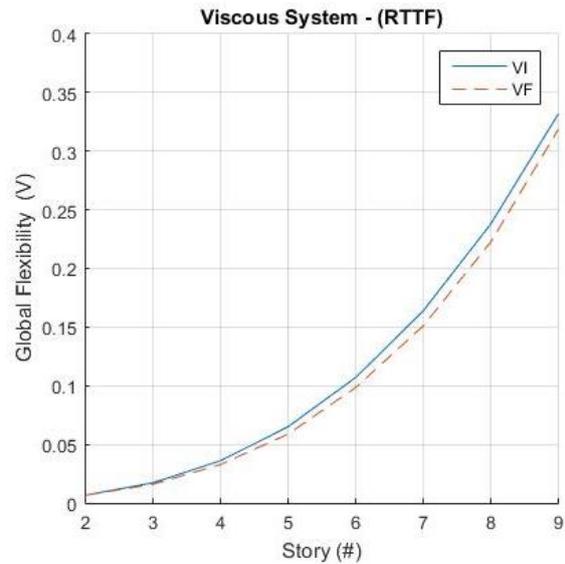


Figure 4. Initial and final flexibility – Viscous system (RTTF)

The initial sensibility for RTTF and ODP techniques are plotted in Figure 2. The idea is to show that for 6 DOF, both hysteretic or viscous systems have the same value of initial sensibility, that is, both dampers dissipate the same amount of energy. Then, if it is increased or decreased the number of floors it is possible to know which damping is more efficient.

Figure 3 and Figure 4 show the initial and final flexibility when the RTTF technique is applied in a system with hysteretic or viscous damping (see Table 5 and Table 6). It is noted that the amplitude of the hysteretic system was lower. It is therefore more efficient hysteretic system rather than viscous system.

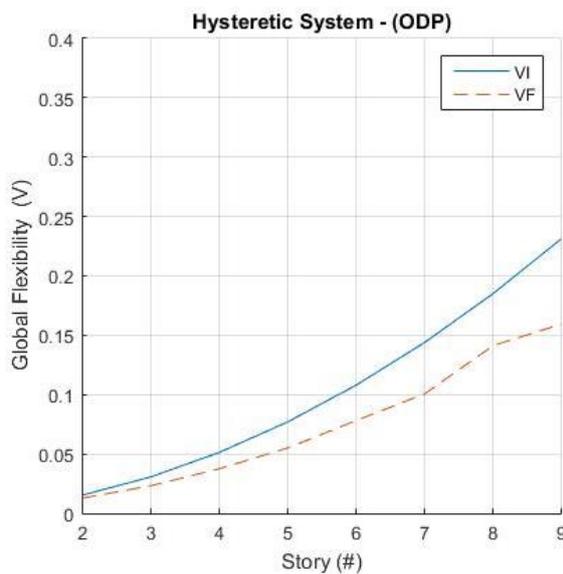


Figure 5: Initial and final flexibility – Hysteretic system (ODP)

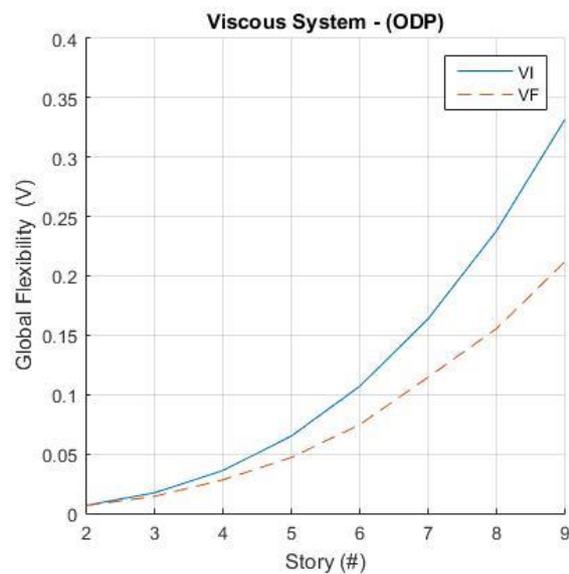


Figure 6: Initial and final flexibility – Viscous system (ODP)

Figure 5 and Figure 6 show the initial and final flexibility when the ODP technique is applied in a system with hysteretic or viscous damping (See Table 7 and Table 8). It is noted that the amplitude of the hysteretic system was lower. It is therefore more efficient hysteretic system rather than viscous system.

5 CONCLUSIONS

In this paper has been adapted two procedures proposed by Izuru Takewaki, facilitating the understanding of mathematical formulation for the hysteretic system. The equations have been expressed for a three-story shear building model with hysteretic damper.

In in this work were evaluated four routines with support MATLAB program. the computational cost was measured for the technique RTTF in a hysteretic and viscous system; then the same was done when it was applied the ODP technique. The results showed that the RTTF technique consumes less computational cost, the recorded minimum was 1.550 seconds to 2 DOF and maximum 6.142 seconds to 9 DOF, while the ODP technique yielded minimum values of 1.816 for 2 DOF and maximum of 68.249 to 9 DOF (see Table 9).

The results of the calculation process were represented by the global flexibility, which consists in the sum of the amplitudes of the transfer function of the floors that compose the model evaluated at the undamped fundamental natural frequency. The results show that it is more efficient use of the ODP technique because the amplitude was lower compared with that obtained from the RTTF technique. For example, to 6 DOF with viscous system using the technique RTTF an initial and final flexibility obtained were 0.107 and 0.098 respectively representing a reduction of 8% (see Table 6) while from the ODP technique the initial and final flexibility obtained were 0.107 and 0.075 respectively, representing a reduction of 30% (see Table 8).

Finally, the results for each of the types of damping evaluated (hysteretic and viscous) were compared and found that using both techniques RTTF or ODP, the damped system with device hysteretic had lower values of flexibility that systems with viscous devices. Therefore, it is concluded that it is more efficient use of hysteretic type devices in the structural control (see Figure 3 vs Figure 4 and Figure 5 vs Figure 6).

ACKNOWLEDGEMENTS

The financial support of *Decanato de Pesquisa e Pós-graduação* (DPP) and University of Brasilia is gratefully acknowledged.

REFERENCES

- Aydin, E., 2012. Optimal damper placement based on base moment in steel building frames. *Journal of Constructional Steel Research*, vol. 79, pp. 216–225.
- Aydin, E., & Boduroglu, M. H., 2008. Optimal placement of steel diagonal braces for upgrading the seismic capacity of existing structures and its comparison with optimal dampers. *Journal of Constructional Steel Research*, vol. 64, n. 1, pp. 72–86.
- Aydin, E., Boduroglu, M. H., & Guney, D., 2007. Optimal damper distribution for seismic rehabilitation of planar building structures. *Engineering Structures*, vol. 29, n. 2, pp. 176–185.

- Fox, R. L., & Kapoor, M. P., 1968. Rates of change of eigenvalues and eigenvectors. *AIAA Journal*, vol. 6, n. 12, pp. 2426–2429.
- Kandemir-Mazanoglu, E. C., & Mazanoglu, K., 2016. An optimization study for viscous dampers between adjacent buildings. *Mechanical Systems and Signal Processing*, pp. 1–9.
- Martinez, C. A., Curadelli, O., & Compagnoni, M. E., 2013. Optimal design of passive viscous damping systems for buildings under seismic excitation. *Journal of Constructional Steel Research*, vol. 90, pp. 253–264.
- Martínez, C. A., Curadelli, O., & Compagnoni, M. E., 2014. Optimal placement of nonlinear hysteretic dampers on planar structures under seismic excitation. *Engineering Structures*, vol. 65, pp. 89–98.
- Murakami, Y., Noshi, K., Fujita, K., Tsuji, M., & Takewaki, I., 2015. Optimal Placement of Hysteretic Dampers via Adaptive Sensitivity-Smoothing Algorithm. *Engineering and Applied Sciences Optimization*, vol. 38, pp. 233–247.
- Orlandi, S., 2010. *Meccanica delle Strutture Procedure di Progettazione per Sistemi di Dissipazione Passiva per Costruzioni in Zona Sismica*. PhD thesis, Univesity of Bologna.
- Takewaki, I., 1997a. Efficient redesign of damped structural systems for target transfer functions. *Computer Methods in Applied Mechanics and Engineering*, vol. 147, pp. 275–286.
- Takewaki, I., 1997b. Optimal damper placement for minimum transfer functions. *Earthquake Engineering & Structural Dynamics*, vol. 26, pp. 1113–1124.
- Takewaki, I., 2000a. *DYNAMIC STRUCTURAL DESING, Inverse Problem Approach*. Southampton, Boston: WIT Press.
- Takewaki, I., 2000b. Optimal damper placement for critical excitation. *Probabilistic Engineering Mechanics*, vol. 15, n. 4, pp. 317–325.
- Takewaki, I., 2009. *Building Control with Passive Dampers: Optimal Performance-based Design for Earthquakes*. *Building Control with Passive Dampers: Optimal Performance-based Design for Earthquakes*. Singapore: John Wiley & Sons (Asia)
- Takewaki, I., & Uetani, K., 1999. Optimal damper placement for building structures including surface ground amplification. *Soil Dynamics and Earthquake Engineering*, vol. 18, n. 5, pp. 363–371.