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DYNAMIC STABILITY OF CIRCULAR THIN AND THICK PLATES ON ELASTIC FOUNDATION

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Abstract. *This paper studies the dynamic buckling of thin and thick circular and annular plates on elastic foundation subject to nonconservative forces. The initial elastic, geometric stiffness and mass matrices are obtained using the Rayleigh-Ritz method. The approximation functions for displacements are polynomials in the radial direction combined with trigonometric functions in the circumferential direction, enriched by higher order polynomials without inclusion of additional nodes. The effect of load displacement-dependence is introduced in the so-called load matrices, which are obtained from equilibrium considerations in the case of nonconservative behavior. The model was implemented in MAPLE. The examples present comparisons illustrating the effect of shear deformation, rotatory inertia and elastic foundation on flutter loads.*

Keywords: *Circular plate; elastic foundation; stability of circular plates; nonconservative tangential follower loads; Rayleigh-Ritz method*

INTRODUCTION

The dynamic stability of structures subjected to nonconservative forces is a subject of classical and present research in the areas of civil, mechanical and aerospace engineering..

The case of Euler-Bernouilli beams on elastic foundation was studied by Smith and Herrmann (1972). They investigated the phenomenon of the critical flutter load of a column supported entirely by an elastic foundation of varying constant and subjected to an axial load. They concluded that, surprisingly, the flutter load is independent of the rigidity of the elastic foundation in the case of follower loads. More recent research (Jae-On Kim et al., 2008) introduced a consideration of a mass on a cantilevered beam totally and partially attached to an elastic foundation and subjected to a distributed follower force.

This paper addresses the case of circular and annular plates subjected to in-plane forces which remain tangent to the deformed surface (a particular type of nonconservate loading). The presence of an elastic foundation of Winkler type is supposed to affect differently the flutter load and the static buckling load. Previous studies (Salas, 2016) considered the effect of shear deformation, which is also included in the present analysis. A Rayleigh-Ritz model is used to obtain results which illustrate the phenomena.

1 MATHEMATICAL MODEL

Figure 1 shows the mathematical model of a circular annular plate with clamped inner edge and outer edge free (C - F), supported by the elastic foundation of stiffness k, under a follower force P.

Herein an energy approach and the Rayleigth-Ritz method is adopted in order to investigate the frequencies and the critical loads.

1.1 Description of the geometrical parameters

Consider a homogeneous isotropic annular plate of thickness h, with inner and outer radius denoted by b and a respectively. The origin of the coordinate system is taken at the center of the plate in the middle plane, as show in Figure 2. Thus, the circular plate geometry and dimensions are defined by a cylindrical coordinate system(r, θ, z), the corresponding displacement components at a generic point are u_r, u_θ and u_z in the radial, circumferential and transversal directions, respectively.

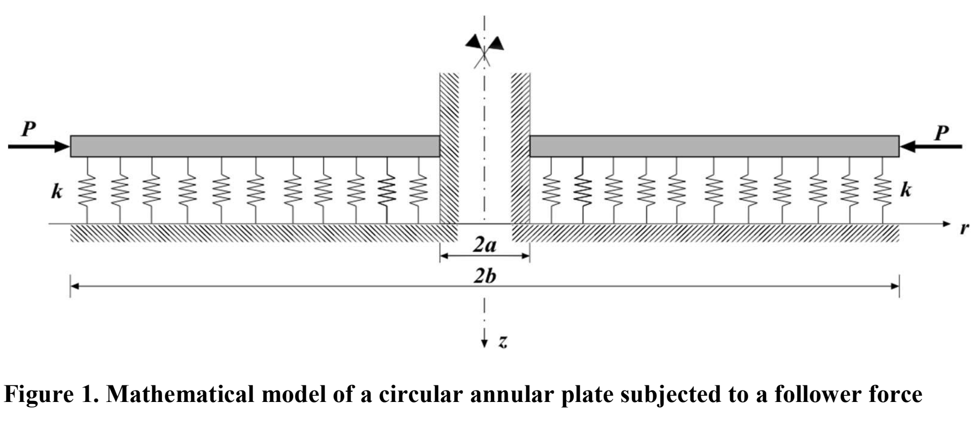

Figure 1. Mathematical model of a circular annular plate subjected to a follower force

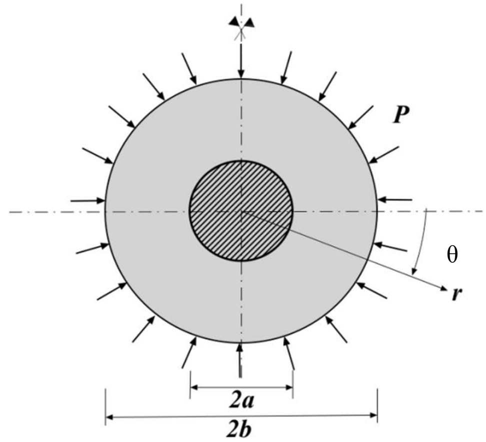

Figure 2. Geometry of a circular annular plate in cylindrical coordinate system

1.2 Equation of motion

Based on the assumptions of Mindlin-Reissener plate theory, the displacement field of the circular plate may be described as follows:

$$u = u_0 + z\psi_r$$
$$v = v_0 + z\psi_\theta \tag{1}$$
$$w = w_0$$

In the above equations, the terms u_0, v_0 and w_0 are the radial, circumferential and transverse displacement components of the mid-surface of the circular annular plate, respectively, and ψ_r and ψ_θ are rotations in the radial and circumferential directions, respectively. In bending analysis, u_0 and v_0 may be set equal to zero.

1.3 Strain displacement relations

For small deflections, the strain-displacement relation may be described as follows:

$$\varepsilon_{rr} = \frac{\partial u}{\partial r}, \quad \varepsilon_{\theta\theta} = \frac{u}{r} + \frac{\partial v}{r\,\partial\theta}, \quad \varepsilon_{zz} = 0,$$
$$\varepsilon_{r\theta} = \frac{\partial v}{\partial r} + \frac{\partial u}{r\,\partial\theta} - \frac{v}{r}, \quad \varepsilon_{rz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}, \quad \varepsilon_{\theta z} = \frac{\partial v}{\partial z} + \frac{\partial w}{r\,\partial\theta} \tag{2}$$

Where ε_{rr} and $\varepsilon_{\theta\theta}$ are the normal strains and ε_{rz}, $\varepsilon_{r\theta}$ and $\varepsilon_{\theta z}$ are the engineering shear strains.

1.4 Hooke's law

The stress-strain relations may be described as follows:

$$\sigma_{rr} = \frac{E}{(1-\nu^2)}\left(\varepsilon_{rr} + \nu\,\varepsilon_{\theta\theta}\right), \quad \sigma_{\theta\theta} = \frac{E}{(1-\nu^2)}\left(\varepsilon_{\theta\theta} + \nu\,\varepsilon_{rr}\right), \quad \sigma_{r\theta} = \frac{E}{2\,(1+\nu)}\,\varepsilon_{r\theta}$$

$$\sigma_{rz} = k^2\,\frac{E}{2\,(1+\nu)}\,\varepsilon_{rz}\,, \quad \sigma_{\theta z} = k^2\,\frac{E}{2\,(1+\nu)}\,\varepsilon_{\theta z} \tag{3}$$

where E and ν are the Young's modulus and Poisson's ratio, respectively, and k^2 denotes the transverse shear correction factor, adopted as $k^2 = \pi^2/12$ (according to Reddy, 2007).

1.5 Rayleigh-Ritz for analysis

The problem is to determine the buckling load of the thick circular plate under uniform in-plane radial compression.

The plate model contains three degrees of freedom for both end nodes. The accuracy of the Rayleigh-Ritz model is controlled by the number of functions (N) used to provide the description of the displacement field, in addition to the basic linear functions (for the thick plate model) or cubic functions (for the thin plate model). There is considerable freedom in the choice of the additional functions, except that they should be zero at the nodes (the thin plate functions must also have zero derivatives at the nodes).

$$\begin{aligned} w_n^{(1)} &= \sum_{i=1}^{N} c_i w_i(x) \psi r^{(1)} = \sum_{i=1}^{N} d_i \psi r_i \, \psi\theta^{(1)} = \sum_{i=1}^{N} e_i \psi\theta_i \\ w_n^{(2)} &= \sum_{i=1}^{N} f_i w_i(x) sen\theta \; \psi r^{(2)} = \sum_{i=1}^{N} j_i \psi r_i \cos\theta \; \psi\theta^{(2)} = \sum_{i=1}^{N} s_i \psi\theta_i sen\theta \end{aligned} \tag{4}$$

The coefficients (*c, d, e, f, j, s*) in Eq. (4) are weighting terms associated with their shape functions w_n, ψ_r and ψ_θ which correspond to the transverse displacement and the two rotations at each point along the radius in the mid surface. The indexes $^{(1),(2)}$ correspond to the polynomial functions in the radial directions and trigonometric functions in the circumferential directions for displacements.

The boundary conditions are imposed by adding adequate spring stiffness values at the diagonals of the stiffness submatrix associated with the nodal degrees of freedom, zero representing a free displacement and a very large value (penalty approach) corresponding to a restraint. Herein the approximation to be used for the displacement field is polynomial in the radial direction and trigonometric in the circumferential direction. The parameters are displacements and rotations in $r = a$ and $r = b$ ("nodal" displacements), plus the magnitudes of an arbitrary number of polynomial functions of increasing order which are zero at the ends. This approach satisfies exactly the essential boundary conditions (displacements and rotations) at $r = a$ and $r = b$, and is assured to converge for other boundary conditions.

1.6 Equation of motion

The governing equations of motion for the model, can be expressed as

$$T = \frac{1}{2}\int_b^a \int_0^{2\pi} \int_{-h/2}^{h/2} \rho\left(\dot{u}^2 + \dot{v}^2 + \dot{w}^2\right) r dr d\theta dz \tag{5}$$

$$U = \frac{1}{2}\int_b^a \int_0^{2\pi} \int_{-h/2}^{h/2} \left(\sigma_{rr}\varepsilon_{rr} + \sigma_{\theta\theta}\varepsilon_{\theta\theta} + \sigma_{rz}\varepsilon_{rz} + \sigma_{\theta z}\varepsilon_{\theta z} + \sigma_{r\theta}\varepsilon_{r\theta}\right) rdrd\theta dz + k\int_b^a \int_0^{2\pi} w\, r\, dr\, d\theta \quad (6)$$

and

$$W_c = \frac{1}{2}\int_b^a \int_0^{2\pi} \int_{-h/2}^{h/2} \left(N_r \phi_r^{\ 2} + N_\theta \phi_\theta^{\ 2}\right) rdrd\theta dz \quad (7)$$

In the above expressions, T is the kinetic energy, U is the elastic potential energy from both circular annular plate and foundation, k is the spring constant of the elastic foundation, ρ is the specific mass of the plate material,d the upper dot represents the differentiation with respect to the time variable t, W_c the work done by the conservative component of the distributed follower force, and (N_r, N_θ) are radial and axial components of forces applied along edge.

2 FORMULATION OF THE EIGENVALUE PROBLEM

The extended Hamilton's principles for a nonconservative system under consideration can be written in the form

$$\delta\int_{t1}^{t2} \left(T + W_c - U\right) dt = 0 \quad (8)$$

Substituting Eq. (5) - (7) into the Eq. (8)

$$\int_{t1}^{t2} \left[\int_b^a \int_0^{2\pi} \int_{-h/2}^{h/2} \rho\left(\dot{u}^2 + \dot{v}^2 + \dot{w}^2\right) r\, drd\theta dz\right] dt = 0 \quad (9)$$

For linear small-strain assuming harmonic motion, the global displacement vectors can be written as,

$$x = \bar{x}e^{i\omega t} \quad (10)$$

$$\bar{x} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad (11)$$

Here $\bar{x}$ is a nonzero vector of constants to be determined, ω is the natural circular frequency. Substituting Eq. (5), (6) and (7) into the expressions (8) leads to the governing eigenvalue equation of the form

$$\left(\overline{K} + \lambda_{cr}\left(\overline{G} + \overline{G}_L^{\ s}\right) - \omega^2\overline{M}\right)\bar{x} = 0 \quad (12)$$

In the above expression, $\overline{K}$ is the stiffness matrix, $\overline{G}$ is the geometric matrix, $\overline{G}_L^{\ s}$ is the symmetric load matrix, $\overline{M}$ is the mass matrix, λ_{cr} is the parameter load and ω is the frequency. Clearly, this matrix cannot be obtained from the energy functional, but results from consideration of equilibrium in the deformed configuration.

3 RESULTS AND DISCUSSION

The numerical analyses on the dynamic stability of a circular annular plate with clamped inner edge and outer edge free (C - F) attached to an elastic foundation under a follower force are performed by employing the Rayleigh-Ritz method. For the next examples the geometry $a = 9.0\ m$, $b = 2.7\ m$ and materials $E = 2.5\ GPa$, $\nu = 0.3$ is considered.

Shear deformation and inertial rotation for the thickness circular annular plate are considered. The inner edge of the plate is clamped and the outer edge is free. Figures 1-3 show the variation of the first and second eigenfrequencies non-dimensionally with the change of the elastic foundation (k from 4.15*10^7 to 1.38*10^8) for K = Kirchhoff and RM = Reissner-Mindlin theories, represented with the continuous and dashed line respectively.

In case of h = 0.20m Figure 1 shows that. as the spring constant k increases, the first and second eigenfrequencies also increase. The values of the first and second eigenfrequencies for K and RM theories do not differ much.

The value of the flutter load force is more affected than the static buckling load. For example, the relative load value for the K and RM circular annular plate without elastic foundation WEF is approximately 6.33 and with elastic foundation EF k=4.15*10^7 is approximately 0.33. As the value of k increases, the value of the critical flutter load decreases.

In the case of h = 0.60m Figure 2 shows that. as the spring constant k increases, the first and second eigenfrequencies also increase but do not increase the value in the same proportion that for h = 0.20m. The second frequency is particularly affected because of the greater shear deformation content in the second mode. The flutter load is changed little, with respect to the static buckling load considered, for example a circular annular plate with elastic foundation KCAP_EF for a k=4.15*10ˆ7 is approximately 1.44 times the static buckling load, and for the circular annular plate without elastic foundation KCAP_WEF 1.34 times the static buckling load. Moreover, the coalescence and consequently the flutter load are considerably affected when considering the elastic foundation, the value of the flutter load for KCAP_EF k= 1.38*10ˆ8 is 0.89, k=5.54*10ˆ7 is 1.30 times the static buckling and for KCAP_WEF is 6. 26 times approximately.

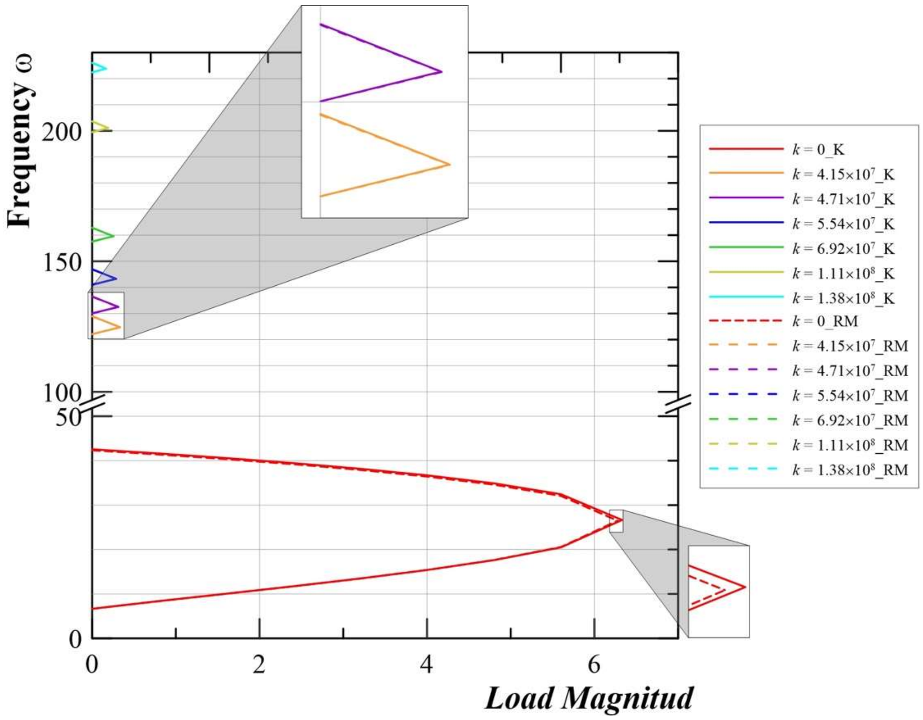

Figure 1. The first and second eigenfrequencies for the circular annular plate with h = 0.20m

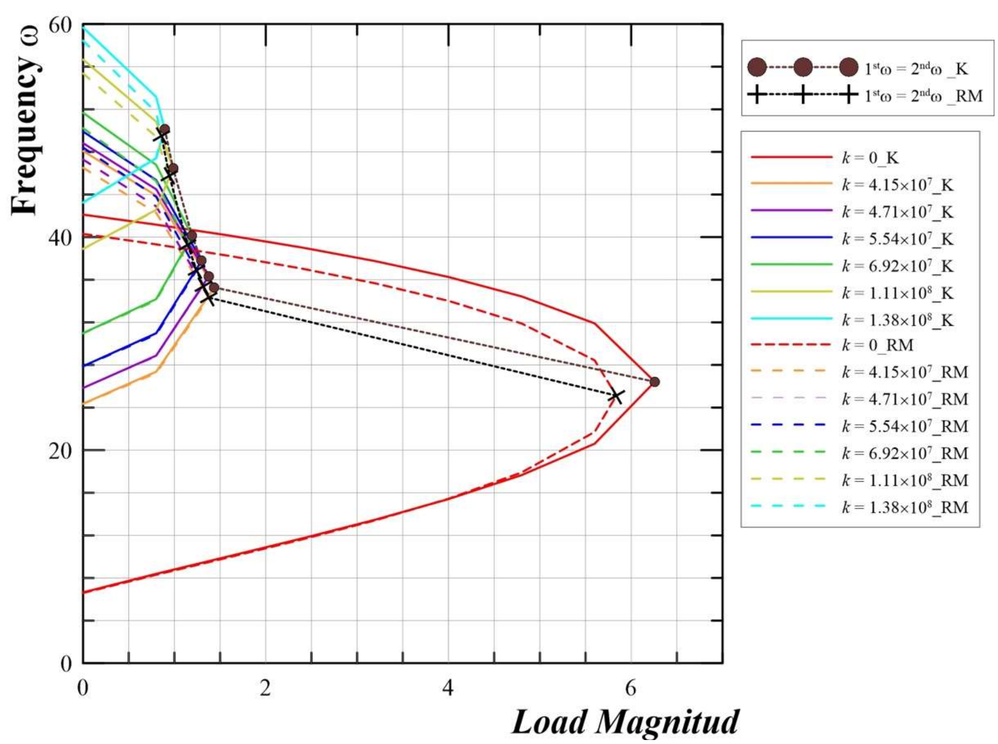

Figure 2. The first and second eigenfrequencies for the circular annular plate with h = 0.60 m

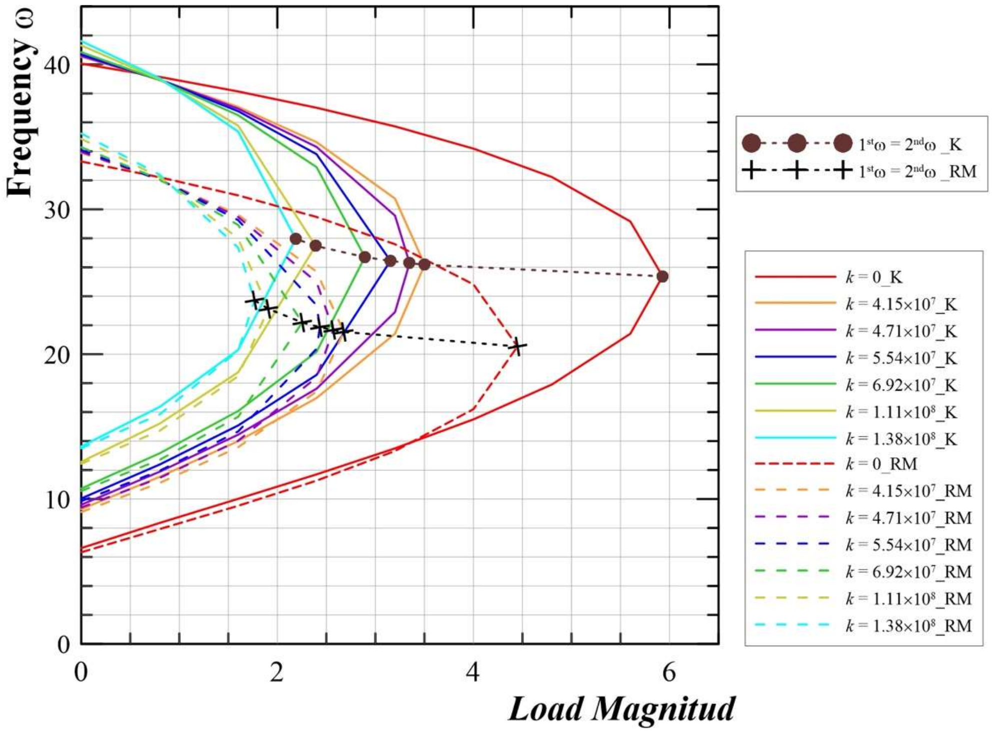

Figure 3. The first and second eigenfrequencies for the circular annular plate with h = 1.40 m

In the case of h = 1.40m, the second frequency is more affected by the thickness because of the higher shear deformation content in the second mode. When considering the RM theory. Additionally, increasing the elastic foundation constant k the second frequency increases. The ratio of the flutter load with respect to the static buckling load is reduced as the foundation constant is increased. For instance, a circular annular plate with elastic foundation k=1.38*10^8 and k=4.15*10^7 is approximately 3.50 and 2.19 times the static buckling load respectively, and for the circular annular plate without elastic foundation 5.93 times the static buckling load.

In Fig. 4 the continuous line indicates the results obtained considering the presence of an elastic foundation, while the dashed line corresponds to the results without elastic foundation. The vertical axis corresponds to the flutter load divided by the static critical load, considering RM and K theory.

When the value of the thickness is small for example h = 0.20m, in the absence of elastic foundation, the value of the ratio flutter load/static buckling load for the Kirchhoff plate is 6.33, just a little higher than the RM plate value of 6.28. It is possible to observe that the ratio is reduced with the increase of thickness (and, consequently, the effect of shear) and with the increase in the foundation constant.

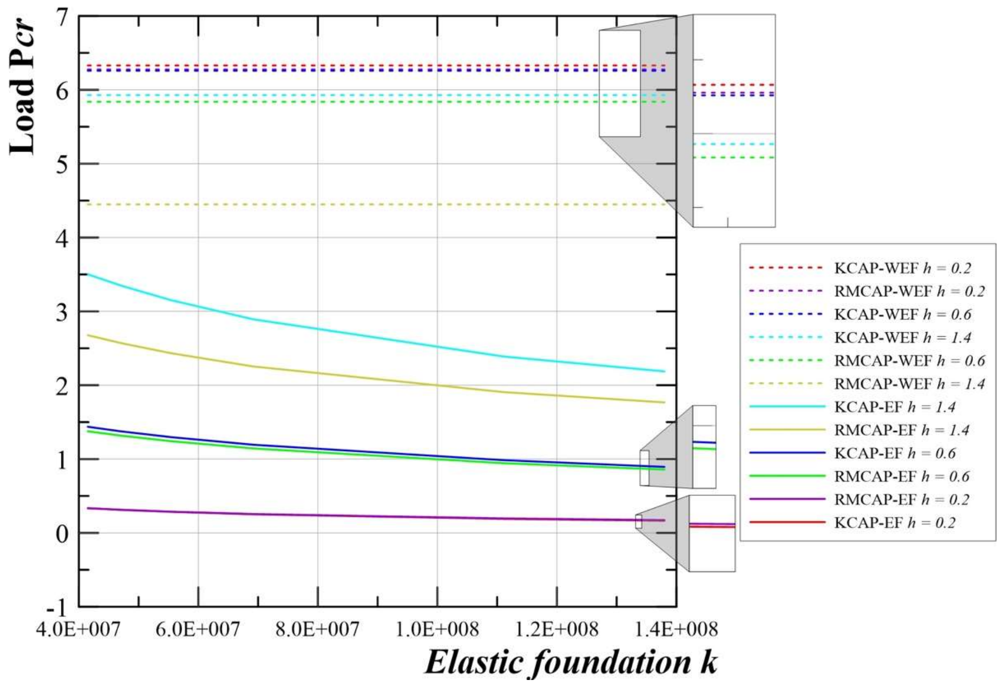

Figure 4. The ratio flutter/static buckling load for circular annular thin and thick plate, with different elastic foundation values

CONCLUSIONS

We have used a simple Rayleigh-Ritz model to study the stability behavior of a circular annular plate with and without shear deformation attached to an elastic foundation, under a distributed radial follower force. The implication of a number variable of the stiffness of the elastic foundation have the influence in the first and second eigenfrequencies.

The results show that the the presence of an elastic foundation tend to reduce the ratio between the flutter load and the static buckling load. This conclusion is not the same as expected from the results of Smith and Herrmann for beams, where the flutter load for Euler-Bernouilli beams on elastic foundation is found to be independent of the foundation constant. Further studies are required to completely clarify this subject.

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