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NONLINEAR DYNAMIC DAMAGE EVOLUTION OF A HIGHWAY BRIDGE DUE TO ITS DYNAMIC INTERACTION WITH RANDOM FORMS OF IRREGULARITIES AND MOVING VEHICLES

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Abstract. *The effects of vehicle-structure interaction have a significant importance on the dynamic responses of both systems with several applications within the highways field. If the vehicle-irregularities-bridge dynamic interaction is capable to produce accumulated strains that cause damage to the structure, the dynamic responses are affected. By crossing a highway bridge with any speed, the vehicle is subjected to the highway irregularities. The movement of a vehicle on a bridge is already a dynamic action on the structure. However, the highway irregularities tend to excite the vehicle dynamically which in turns triggers additional vibrations in the highway bridge structure apart from those produced by their own movement, increasing the bridge's damage evolution. This modifies the dynamic responses of the structure, increasing the magnitude and the oscillations particularly at critical speeds of the vehicle, capable to provoke some resonance. Apart from changing the displacements, velocities and accelerations responses, the damage alters the structural natural frequencies of vibration. Such effects are not possible to be analysed with linear dynamic models. The nonlinearities occur by the fact that the forces no longer linearly depend from the displacements when damage occurs. This work aims to evaluate the nonlinear dynamic damage evolution of a reinforced concrete highway bridge through the Finite Element Method, on which the degree of damage is altered over time by the dynamic interaction with random irregularities and moving vehicles due to the stiffness loss of the structure by Damage Mechanics. The highway irregularities are represented by random functions. Euler-Bernoulli beam elements with Hermite cubic interpolation functions are used for the bridge model. The Mazars Damage Constitutive Model is implemented with the condition of stress inversion due to vibration. The continuum damage mechanics is considered dynamically. Therefore, the damage is evaluated in each layer of the structure cross section for each iteration within each time step. The structural damping is defined by the Rayleigh method with updated coefficients due to damage. The equations of motion are obtained by nonlinear dynamic equilibrium and numerically integrated in time using the Newmark Method together with the Newton-Raphson iterative Method. This proposal seeks to contribute to the study of the health monitoring and the structural integrity of damaged highway bridges structures.*

Keywords: *Damage Mechanics, Dynamic Interaction, Finite Element Method, Nonlinear Dynamics, Computational Modeling*

1 INTRODUCTION

The problem of dynamic interaction between vehicles and bridges structures has been studied by researchers in the last 150 years. This problem is among the oldest problems of structural dynamics. Earlier on, analytical methods were applied to simple models varying the boundary conditions of problem. The development of computers and numerical methods such as the finite element method allows for complex and refined models to produce accurate results, which can be verified through experimental measurements (Beghetto & Abdalla Filho, 2010).

The increase in road and railroad transport cargo, especially in Brazil, with increased intensity values of loads on the roads, has been producing degradation in many bridges throughout the countries. This is an issue of paramount importance, as is related to the structural health of bridges and with the matter of the conservation of highways and railways structures.

To contribute with this theme, the problems of dynamic interaction between vehicle and bridge considering the track irregularities are being studied. There are numerous references in the literature dealing with this topic and the modeling of vehicle-irregularities-bridge systems has been studied by researchers around the world in the last thirty years.

The emergence of new and complex structural systems, subjected to preponderantly dynamic actions is another important factor. This is the case, for example, for offshore structures, oil exploration and exploitation in the continental offshore. The dynamic analysis of these structures is fundamental because one of the most important loads to be considered in the project is due to wave action (Machado, 1983).

This work seeks to contribute to these studies by analyzing the nonlinear dynamic damage evolution of a highway bridge due to its dynamic interaction with vehicles coupled with random forms of irregularities.

2 MATHEMATICAL MODELS FOR MATERIALS SCIENCE

This session presents the structural steel constitutive model, the Mazars' damage constitutive model and the equivalent stiffness model that couple both materials.

2.1 Structural Steel Constitutive Model

A simple bilinear elastoplastic model with strain hardening is adopted for the structural steel as this material has the same behavior when subjected to traction and compression.

As in reinforced concrete structures the steel bars resist fundamentally the axial forces, it is used an uniaxial model to describe the behavior of the reinforcement.

Although very simplistic, the structural steel model is more reasonable than the linear elastic normative calculation models used in the majority of structural design offices for considering the hardening by plastic deformation. The Fig. 1 represents this model.

Thus, in this model, if the structural steel is unloaded at the second section of the diagram shown in Fig. 1, the model has a permanent strain associated.

The stress acting on the structural steel is determined by (Tiago *et al.*, 2002)

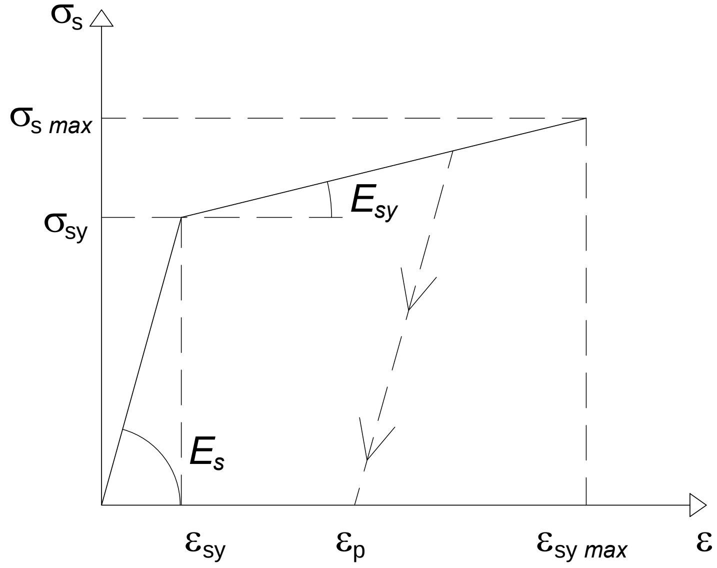

Figure 1: Bilinear elastoplastic model with strain hardening for the structural steel

$$\sigma = \begin{cases} E_s\varepsilon & , -\varepsilon_{sy} \leq \varepsilon \leq \varepsilon_{sy} \\ \sigma_{sy} + E_{sy}(\varepsilon - \varepsilon_{sy}) & , \text{otherwise} \end{cases} \tag{1}$$

where E_s is the initial elastic modulus of the structural steel, σ_{sy} is the yield stress, ε_{sy} is the yield extension and E_{sy} is the longitudinal elastic modulus after the yield of the steel defined by

$$E_{sy} = k_s E_s \tag{2}$$

where k_s is the relation between the longitudinal elastic modulus after the yield of the steel E_{sy} and the longitudinal elastic modulus of the steel E_s.

2.2 Mazars' Damage Constitutive Model

Rabotnov *et al.* (1970) proposed to consider the loss of material stiffness as a result of cracking. Posteriorly, the continuum damage mechanics was formalized based on thermodynamics of irreversible processes by Lemaitre & Chaboche (1985).

The damage evolution in the concrete is simulated with the damage constitutive model proposed by Mazars (1984). This model is based on some experimental evidences observed in uniaxial experiments in concrete, having the fundamental hypotheses (Pituba, 1998):

– the damage is represented by a scalar variable D $(0 \leq D \leq 1)$ whose evolution occurs when a reference value for the 'equivalent stretching' is exceeded;

– locally the damage comes from the existence of stretching deformations;

– its considered, that the damage is isotropic, although experimental tests show that the damage leads, in general, to an anisotropy of concrete, which may initially be considered as isotropic; and

– the damaged concrete behaves as an elastic medium, therefore the permanent deformation evidenced experimentally in a situation of unloading is neglected.

The square of the equivalent strain is equal to the sum of the squares of the main components of the positive principal strain

$$\tilde{\varepsilon}^2 = \sum_i \langle \varepsilon_i^2 \rangle_+ \tag{3}$$

where ε_i are the principal strain components and its positive parts defined by

$$\langle \varepsilon_i \rangle_+ = \frac{1}{2}[\varepsilon_i + |\varepsilon_i|] \tag{4}$$

The extension state is locally characterized by a stretching or an equivalent strain, expressed as (Pituba, 1998)

$$\tilde{\varepsilon} = \sqrt{\langle \varepsilon_1 \rangle_+^2 + \langle \varepsilon_2 \rangle_+^2 + \langle \varepsilon_3 \rangle_+^2} \tag{5}$$

It is adopted that the damage starts when the equivalent strain $\tilde{\varepsilon}$ reaches a value of the reference strain ε_{d0} determined in uniaxial traction tests in correspondence to the maximum stress, as shown in Fig. 2.

The constitutive relation, for the particular case of one-dimensional stress state, is given by (Tiago *et al.*, 2002)

$$\sigma = (1 - D(\varepsilon))\, E_{c0}\varepsilon \tag{6}$$

The damage variable D is defined by a linear combination of the basic damage variables D_T and D_C through the combination coefficients α_T and α_C by

$$D(\varepsilon) = \alpha_T D_T + \alpha_C D_C, \alpha_T + \alpha_C = 1 \tag{7}$$

in which the value of the coefficients α_T and α_C are contained in the closed interval $[0, 1]$, and seek to represent the contribution of the mechanical requests to traction and compression for the extension local state, respectively (Pituba & Proença, 2005).

The basic damage variables are given by

$$D_T(\tilde{\varepsilon}) = 1 - \frac{\varepsilon_{d0}\,(1 - A_T)}{\tilde{\varepsilon}} - \frac{A_T}{e^{B_T(\tilde{\varepsilon} - \varepsilon_{d0})}} \tag{8}$$

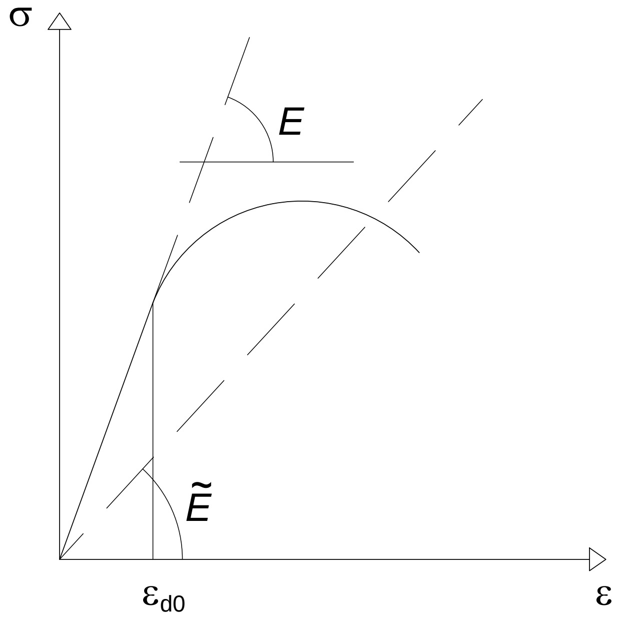

Figure 2: Stress-strain diagram for the Mazars' damage model

and

$$D_C(\tilde{\varepsilon}) = 1 - \frac{\varepsilon_{d0}\,(1 - A_C)}{\tilde{\varepsilon}} - \frac{A_C}{e^{B_C(\tilde{\varepsilon}-\varepsilon_{d0})}} \tag{9}$$

where A_T, B_T, A_C and B_T are the characteristic parameters of the material in uniaxial traction and uniaxial compression, respectively, $\tilde{\varepsilon}$ the equivalent strain below which no damage occurs and ε_{d0} the parameter of the limit elastic deformation. The subscripts T and C refer to traction and compression, respectively. Therefore, if the equivalent strain is lesser than the reference strain $(\tilde{\varepsilon} \leq \varepsilon_{d0})$, then there is no damage at all $(D = 0)$.

In order to consider the Poisson effect in the concrete, the equivalent strain is given by

$$\tilde{\varepsilon} = \begin{cases} \varepsilon & \text{, if } \varepsilon \geq 0 \\ -\upsilon\sqrt{2\varepsilon} & \text{, otherwise} \end{cases} \tag{10}$$

where υ is the Poisson's ratio of the concrete.

Mazars (1984) proposed the following ranges of variation for the parameters A_T, B_T, A_C and B_T, obtained from the calibration with experimental results as (Pituba, 1998)

$$\begin{array}{ll} 0.7 \leq A_T \leq 1 & 10^4 \leq B_T \leq 10^5 \\ 1 \leq A_C \leq 1.5 & 10^3 \leq B_C \leq 2.10^3 \\ 10^{-5} \leq \varepsilon_{d0} \leq 10^{-4} & \end{array} \tag{11}$$

The behavior of the Mazars' damage model when the material is subjected to traction and compression is shown, respectively, in the Fig. 3.

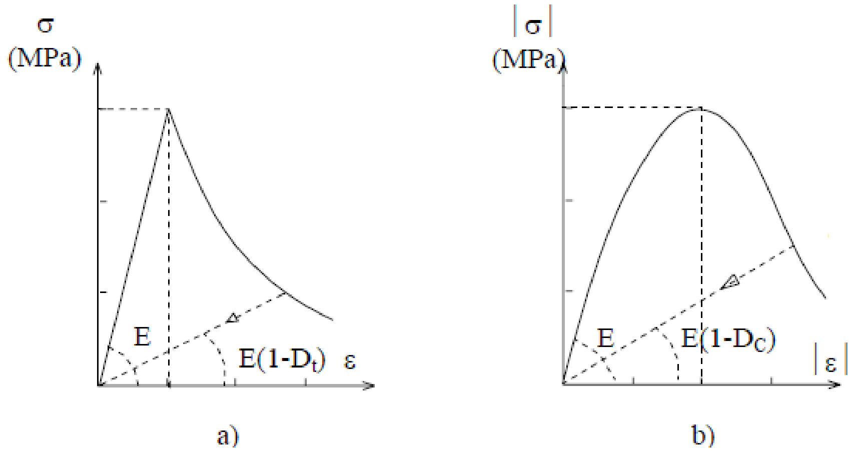

Figure 3: Uniaxial response: (a) traction, (b) compression, adapted from (Pituba, 1998)

A more rigorous way to define the damage is through the fourth order damage tensor D_ijrs considering the operator that transforms the initial elasticity tensor of the intact material E_ijkl in the current elasticity tensor of the material softened by damage $\tilde{E}_{ijkl}$ by

$$\tilde{E}_{ijkl} = (I_{ijrs} - D_{ijrs})\, E_{rskl} \tag{12}$$

From a purely theoretical point of view, the relationship shown above in Eq. (12) does not produce a true state variable because it requires knowledge of a particular behavior of the material, such as elasticity (Lemaitre & Desmorat, 2005). However, this relationship allows to indirectly determining the damage variable for elastic materials from Young's modulus measurements performed in tests with loading and unloading cycles.

In the case of uniaxial isotropic damage without the effect of micro cracks closure in compression, the average value of micro stresses is obtained from the equilibrium of forces (Rabotnov *et al.*, 1970). Thus, the effective stress can be written as

$$\tilde{\sigma} = \frac{F}{\tilde{S}} = \frac{F}{S(1-D)} = \frac{\sigma}{(1-D)} \tag{13}$$

where F is the applied force in the representative volume element, $\tilde{S}$ is the effective area which is the intact area S subtracted by the defects area S_D , and σ is the stress acting on the intact area.

The state of deformation one-dimensional or three-dimensional of a damaged material is obtained by the behaviour of the intact material where the stress is replaced by the effective stress (Lemaitre & Chaboche, 1985). Thus the equivalent deformation ε_e can be defined by

$$\varepsilon_e = \frac{\tilde{\sigma}}{E} = \frac{\sigma}{(1-D)E} \tag{14}$$

where E is the Young's modulus of the intact material.

Consequently, the Young's modulus of the damaged material $\tilde{E}$ for a continuous medium with equivalent response to the material with imperfections is obtained by

$$\tilde{E} = (1 - D)\, E \tag{15}$$

Albeit this model neglects the permanent deformation in unloading situations, the damage is irreversible and cumulative. Therefore, if a damaged material has its load removed, the Young's modulus E is updated to the damaged Young's modulus $\tilde{E}$ and does not return to its initial state. It is only possible to cause more damage to the material, reducing the Young's modulus even more.

2.3 Equivalent Stiffness Model

The rectangular cross section of the bridge's elements are divided in n layers as laminated composite beams in order to be able to determine the equivalent stiffness of each element. The Fig. 4 shows this division.

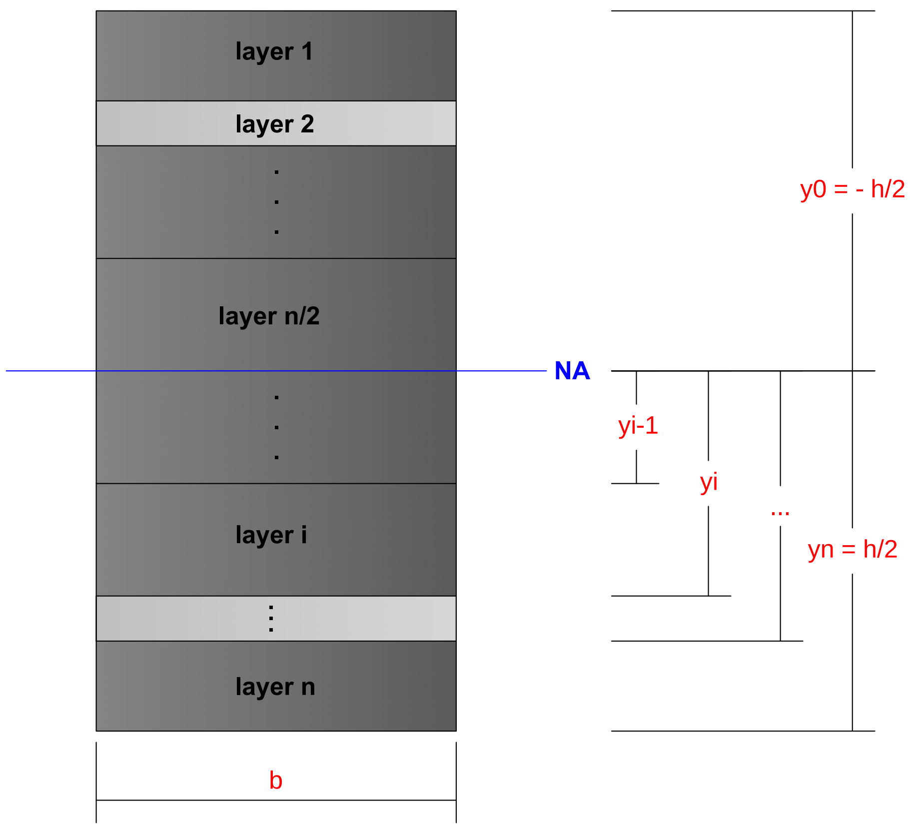

Figure 4: Bridge's cross section divided in n layers

For the particular case of symmetric laminated composite beam with b width, the equivalent stiffness EI_{eqv} is determined by

$$EI_{eqv} = \frac{1}{3}\sum_{i=1}^{n} E_i(y_i^3 - y_{i-1}^3) \tag{16}$$

where n is the number of layers, b is the constant width of the rectangular cross section, E_i is the elastic modulus of the i^{th} layer, in the case E_c for the concrete layers and E_s for the structural steel layers, y_i and y_{i-1} are the y axis coordinate values of the division points of the i^{th} which subtracted result in the height of the layer.

When the concrete or structural steel, present nonlinear physical behavior, the position of the neutral axis varies and is recalculated at each numerical iteration according to the deterioration of any layer by

$$y_{NA} = \frac{\sum\left(\frac{y_i - y_{i-1}}{2}\right)E_iA_i}{\sum E_iA_i} \tag{17}$$

in which A_i is the area of the layer and y_{NA} is the recalculated position of the neutral axis.

As the continuum damage mechanics is calculated dynamically, in order to consider these effects, it is required to evaluate the damage in each cross section of the bridge's elements for each iteration within each time steps.

3 MATHEMATICAL MODELS OF VEHICLES COUPLED WITH IRREGULARITIES

The computational model implemented considers the passage of the vehicle with one degree of freedom in the different forms of track irregularities and transmits the stresses generated between the vehicle and irregularities to the railway bridge, in uncoupled way, for its dynamic linear or nonlinear analysis. In other words, the dynamic responses of the bridge do not affect the forces caused by the coupling between the passing vehicle and irregularities. Thus, this computational routine firstly analyzes the forces generated by the coupling between vehicle and irregularities and transmits them to the bridge for the desired analysis. Therefore, it is a model coupled between vehicle and irregularities and uncoupled between the coupled vehicle-irregularities and the bridge.

The model with 1 degree of freedom consists of a wheel mass m_1 in contact with the irregularities, a suspended vehicle mass m_2, a spring with stiffness coefficient k and a damper with damping coefficient c, as showed in Fig. 5.

When crossing a bridge with speed v, the vehicle is subjected to the effects of track irregularities, represented by $y(t)$. Considering the wheel mass undeformable, this is similar to a case of base excitation. It is merely necessary to consider the vehicle stationary subjected to a harmonic base excitation $y(t)$. The spring and the dumper masses are neglected throughout the analysis. Assuming a linear elastic behavior for the spring and a linear viscous damping for the damper throughout the analysis, the elastic force acting on the spring and the damping force are

$$f_M = k[u_v(t) - y(t)] \tag{18}$$

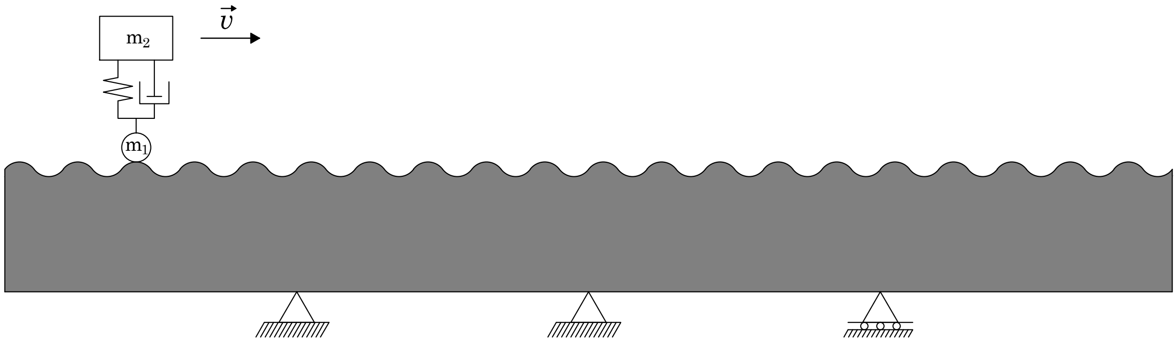

Figure 5: Vehicle-irregularities theoretical model

$$f_A = k[\dot{u}_v(t) - \dot{y}(t)] \tag{19}$$

Thus, the governing equation of motion of the problem can be written as

$$m_2\ddot{u}_v(t) + c[\dot{u}_v(t) - \dot{y}(t)] + k[u_v(t) - y(t)] = 0 \tag{20}$$

3.1 Forms of Track Irregularities

In this paper random shapes of irregularities are used, which may have portions of rectangular, triangular, sinusoidal, as periodic or pulses, or any functions form representing the track irregularities of the highway bridge. Although the irregularities are dealt randomly, the linear or nonlinear dynamic solutions of the problem are not solved by probabilistic but deterministic models. This is the case of mapping the unknown irregularities of a road through mechanical instrumentations and to determine the dynamic responses of the structure.

For each random forms of irregularity obtained are determined the different vehicle-irregularities coupled models. In each base excitation function $y(t)$ that represents each type of track irregularities, the function is derivate with respect to time to obtain the portion $\dot{y}(t)$ related to the resistive force due to damping as

$$\dot{y}(t) = \frac{dy(t)}{dt} \tag{21}$$

The numerical differentiation is used analyzing the accuracy.

Considering v as the vehicle speed, l the wave length of the irregularities, t the time and A the amplitude, in order to exemplify a case of irregularities represented by sinusoidal harmonic functions which may be partially or fully obtained from a mechanical instrumentation measure, the irregularities functions can be expressed as

$$y(t) = A \sin\left(\frac{2\pi v}{l}t\right) \tag{22}$$

Equation (22) represents the irregularities of the track in the form of a sinusoidal harmonic function correlated with the vehicle speed. If deriving Eq. (22) with respect to time, one has

$$\dot{y} = \frac{2\pi A v}{l}\cos\left(\frac{2\pi v}{l}t\right) \tag{23}$$

By replacing the equations Eq. (22) and Eq. (23) in Eq. (20), one obtains

$$m_2\ddot{u}_v + c\dot{u}_v + ku_v = c\left[\frac{2\pi A v}{l}\cos\left(\frac{2\pi v}{l}t\right)\right] \tag{24}$$

The dynamic responses of displacements, velocities and accelerations of the vehicle are obtained by numerically integrating the Eq. (24) in time using the *Newmark* method and solving the system using the *Gaussian* elimination method.

Finally, the force produced by the base excitation generated by the coupling between vehicle and track irregularities is defined by

$$F_E B(t) = (m_1 + m_2)g + c[\dot{u}_v(t) - \dot{y}_v(t)] + k[u_v(t) - y(t)] \tag{25}$$

where g is the gravitational acceleration.

4 BRIDGE'S MATHEMATICAL MODELS

This section and subsequent subsections will treat about the dynamic mathematical models of the bridge. The focus of this paper is to study the damage evolution of a highway bridge's structure. In this sense, this work focus on the nonlinear dynamic mathematical model which, differently from the linear dynamic model, it can analyze physical nonlinearities, as the dynamic damage evolution case.

4.1 Linear Dynamic Mathematical Model

Just to describe the linear case in order to understand the nonlinear dynamic mathematical model, this subsection will simply describes a few particularities of the linear models and its governing equation.

Differently from the nonlinear dynamic model, on the linear no distinction is made for the geometry of the bridge cross section and its change of the material behaviors, analyzing the problem with only the moment of inertia and Young's modulus, both constants through time.

The governing equation of this problem is shown in Eq. (26)

$$[m]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F(t)\} \tag{26}$$

where $[m]$, $[C]$ and $[K]$ are the global matrices of mass, damping and stiffness, $\{\ddot{u}\}$, $\{\ddot{u}\}$ and $\{u\}$ are the global vectors of displacements, velocities and accelerations and, finally, the $\{F(t)\}$ represents the external force vector that varies over time.

To solve the Eq. (26), one may use a time integration method, as the Newmark method. The system, after that, can be solved by the Gaussian elimination method in each time step.

4.2 Nonlinear Dynamic Mathematical Model by Damage Mechanics

In a case of deterioration of the material, the forces are no longer linearly dependent of the displacements. This model considers that the damage directly affects the stiffness of the system and consequently the structural damping but not the mass of the system, as a case of a rocket launch, thus causing the previously fixed stiffness matrix to become instantaneous, characterizing physical nonlinearity, as shown below (Abeche *et al.*, 2015)

$$[K_B] = [K_B(\{u_B(\vec{x},t)\})] \tag{27}$$

It can be noted that the previously fixed stiffness matrix is now instantaneous. It is dependent of the displacements which are a functional of time and position vector.

The predictive values, or estimates, of velocities and displacements in time t with respect to time $t + \Delta t$ are respectively defined as (Machado, 1983)

$$\{\tilde{\dot{u}}_B\}_{t+\Delta t} = \{\dot{u}_B\}_t + (1-\gamma)\{\ddot{u}_B\}_t\Delta t \tag{28}$$

and

$$\{\tilde{u}_B\}_{t+\Delta t} = \{u_B\}_t + \{\dot{u}_B\}_t\Delta t + \left(\frac{1}{2} - \beta\right)\{\ddot{u}_B\}_t\Delta t^2 \tag{29}$$

where γ and β are the Newmark method parameters determined for the numerical integration accuracy and stability.

Due to physical nonlinearities, the previous linear global equation of motion presented in Eq. (30) becomes nonlinear dynamic and must be solved iteratively and incrementally by combining the iterative Newton-Raphson technique with the implicit time integration operator of the Newmark method. In this sense, the nonlinear dynamic global equation of motion for the damaged highway bridge can be written as (Jacob & Ebecken, 1994)

$$[M_B]^{(i-1)}_{t+\Delta t}\{\ddot{u}_B\}^{(i)}_{t+\Delta t} + [C_B]^{(i-1)}_{t+\Delta t}\{\dot{u}_B\}^{(i)}_{t+\Delta t} + [K_B]^{(i-1)}_{t+\Delta t}\{\Delta u\}^{(i)} = \{\Delta F_B\} \tag{30}$$

where

$$\{\Delta F_B\} = \{F^{ext}_B\}_{t+\Delta t} - \{F^{int}_B\}^{(i-1)}_{t+\Delta t} \tag{31}$$

where i is the present iteration, $\{F_B^{ext}\}_{t+\Delta t}$, $\{F_B^{int}\}_{t+\Delta t}$ and $\{\Delta F_B\}_{t+\Delta t}$ are, respectively, the external forces vector, the internal forces vector and the unbalanced forces vector in time $t + \Delta t$ and $\{\Delta u\}$ is the incremental displacements vector.

It can be observed that by combining the Newton-Raphson iterative technique with the Newmark method and the predictive values, the internal forces seeks the nonlinear dynamic equilibrium in each iteration within each time step in order to make the unbalanced forces vector tend to null vector with respect to a convergence criterion.

Applying Eq. (28) and Eq. (29) in Eq. (30), one obtains (Abeche *et al.*, 2016)

$$\begin{aligned} &\left(\frac{[M]_{t+\Delta t}^{(i-1)}}{\beta \Delta t^2}\right)\{u_B\}_{t+\Delta t}^{(i)} + [K_B]_{t+\Delta t}^{(i-1)}\{\Delta u\}^{(i)} = \{F_B^{ext}\}_{t+\Delta t} \\ &+ \frac{[M]_{t+\Delta t}^{(i-1)}}{\beta \Delta t^2}\left(\{u_B\}_t + \{\dot{u}_B\}_t \Delta t + (0.5-\beta)\{\ddot{u}_B\}_t \Delta t^2\right)^{(i)} \\ &+ [C]_{t+\Delta t}^{(i-1)}\left(\frac{\gamma}{\beta \Delta t}\{u_B\}_t + \left(\frac{\gamma}{\beta}-1\right)\{\dot{u}_B\}_t + \left(\frac{\gamma}{2\beta}-1\right)\{\ddot{u}_B\}_t \Delta t\right) \\ &- \{F_b^{int}\}_{t+\Delta t}^{(i-1)} \end{aligned} \tag{32}$$

The internal global force seeks the nonlinear dynamic equilibrium configuration. Its components are the forces parcels from the stresses acting in each of the layers of the cross section for each element. These are obtained at each iteration within each time step as (Bathe, 1996)

$$\{F_B^{int}\}_{t+\Delta t}^{(i-1)} = \sum_m \int_{V_{t+\Delta t}^{(m)}} [B]_{t+\Delta t}^{(m)^T} [\sigma]_{t+\Delta t}^{(m)} \mathrm{d}V_{t+\Delta t}^{(m)} \tag{33}$$

Making use of the Gauss quadrature technique, the internal global force that seeks the nonlinear dynamic equilibrium configuration in each iteration of the iterative Newton-Raphson technique, within each time step of the Newmark method is defined as

$$\{F_B^{int}\}_{t+\Delta t}^{(i-1)} = \sum_{m=1}^{nel} \sum_{p=1}^{npint} [B]_{t+\Delta t}^{(p,m)^T} [\sigma]_{t+\Delta t}^{(p,m)} J^{(m)} W_{Gauss}^p \tag{34}$$

where nel is the number of finite elements, np int is the number of Gaussian integration points, $[B]$ is the strains matrix, $[\sigma]$ is stress tensor, W_{Gauss} is the weight associated with Gaussian points, and J is value of the integration that in the case is defined as

$$J = \frac{L}{2} \tag{35}$$

The convergence criteria utilized is related to an energy criteria that includes both forces and displacements criteria, defined as (Bathe, 1996)

$$\{\Delta u\}^{(i)^T} \left(\{F_B^{ext}\}_{t+\Delta t} - \{F_B^{int}\}_{t+\Delta t}^{(i-1)} \right) \leq tol_E \left(\{\Delta u\}^{(1)^T} \left(\{F_B^{ext}\}_{t+\Delta t} - \{F_B^{int}\}_t \right) \right) \quad (36)$$

where tol_E is the energy convergence tolerance.

When the convergence is achieved, it reaches the end of the iterative process of Newton-Raphson and the dynamic responses for a given time step correspond to the values obtained in the last iteration, i.e.:

$$\{u_B\}_{t+\Delta t} = \{u_B\}_{t+\Delta t}^{(i+1)} \quad (37)$$

$$\{\dot{u}_B\}_{t+\Delta t} = \{\dot{u}_B\}_{t+\Delta t}^{(i+1)} \quad (38)$$

$$\{\ddot{u}_B\}_{t+\Delta t} = \{\ddot{u}_B\}_{t+\Delta t}^{(i+1)} \quad (39)$$

After obtaining the nonlinear dynamic responses at the end of the iteration within the required time step, a further time step is started whenever wished to calculate the displacements, velocities, accelerations, etc., for a new time until the last time of the analysis is reached, in which the process is, therefore, terminated.

5 NUMERICAL ANALYSIS

In order to compare the dynamic responses of the bridge's structure for random but deterministic forms of irregularities, such as the case of a mechanical mapping of a highway, this section analyze the problem for two similar types y_1 and y_2. Both are very alike, but the second one have greater amplitude than the first. This was made with the purpose to compare the nonlinear dynamic damage evolution of the highway bridge perspective due its dynamic interaction with different forms of irregularities and moving vehicles.

The Table 1 presents the input data for the vehicle-irregularities coupled model.

Table 1: Vehicle-irregularities coupled model input data

Vehicle	Irregularities
$m_1 = 4400$ kgf	$y_{1and2} =$ random forms
$m_2 = 15000$ kgf	$l =$ variable
$k = 9120$ kN/m	$A^1_{max} = 5.2$ mm
$c = 86$ kNs/m	$A^1_{min} = -5.5$ mm
$v = 50$ km/h	$A^2_{max} = 20.9$ mm
$a = 1$ m/s^2	$A^2_{min} = -21.99$ mm
6203 Time Steps	6203 Time Steps

The bridge's parameters, such as physical, finite element discretization, materials and geometric parameters are illustrated in Table 2.

Table 2: Bridge's parameters input data

Physical and FE Discretization	Materials	Geometric
20 Elements	$E_c = 29.43$ GPa	$b = 0.5$ m
2 dof / element	$\nu_c = 0.2$	$h = 2$ m
$tol_u = 0.00001$	40 Concrete's Layers	$I_b = 0.2667$ m^4
$tol_F = 0.00001$	$E_s = 210$ GPa	42 Layers
$tol_E = 0.00001$	$\nu_s = 0.3$	$A_c = 0.7857792$ m^2
$\zeta = 0.025$	2 Steel's Layers	$A_s = 0.0142208$ m^2
$\gamma = 0.5$	$k_s = 0.85$	$d = 1.81$ m
$\beta = 0.25$	$\rho = 0.0180977\%$	$d' = 0.19$ m
$dt = 7.2\text{e}^{-4}$ s	$m = 1996.792205$ kg/m	$L_b = 72$ m
7200 Time Steps		

Lastly, it is necessary to present the calibration parameters input for the Mazars' damage model, observed in Table 3.

Table 3: Mazars' damage model calibration parameters input

Damage Parameters				
$A_T = 0.995$	$B_T = 30000$	$A_C = 1.2$	$B_C = 1050$	$\varepsilon_{d0} = 5\text{e}^{-5}$

The boundary conditions and the length of the spans can be found in Fig. 6, which illustrates the problem in analysis.

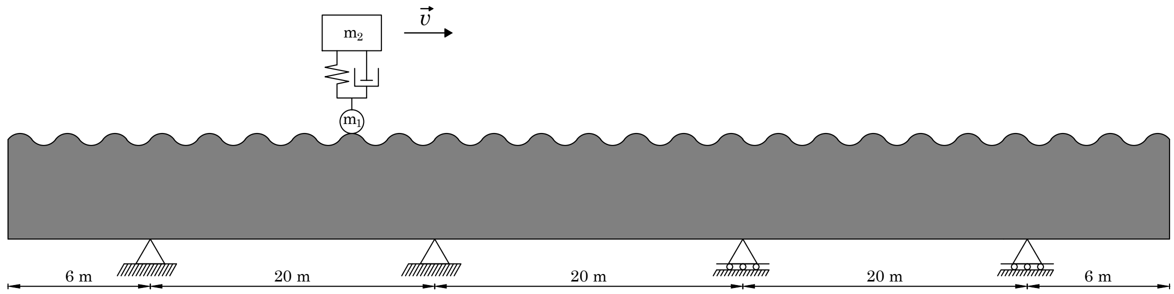

Figure 6: Vehicle-irregularities theoretical model

Both forms of highway random irregularities can be found in Fig. 7. It can be noted that the random form y_2 have an amplitude four times greater than the random form y_1.

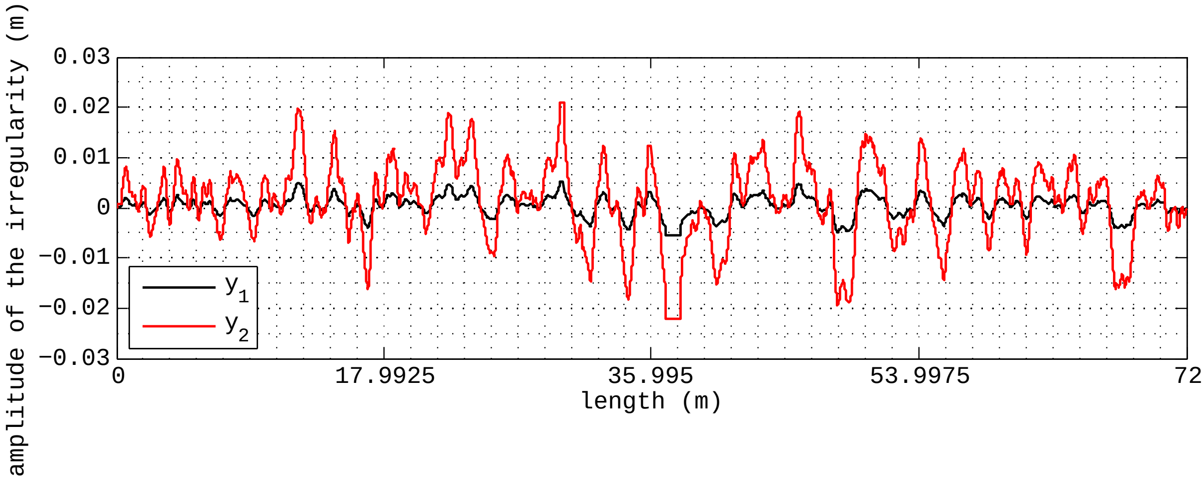

Figure 7: Random forms of highway irregularities (y_1 and y_2)

The dynamic responses of displacement for the midspan and right extremity are shown in Fig. 8.

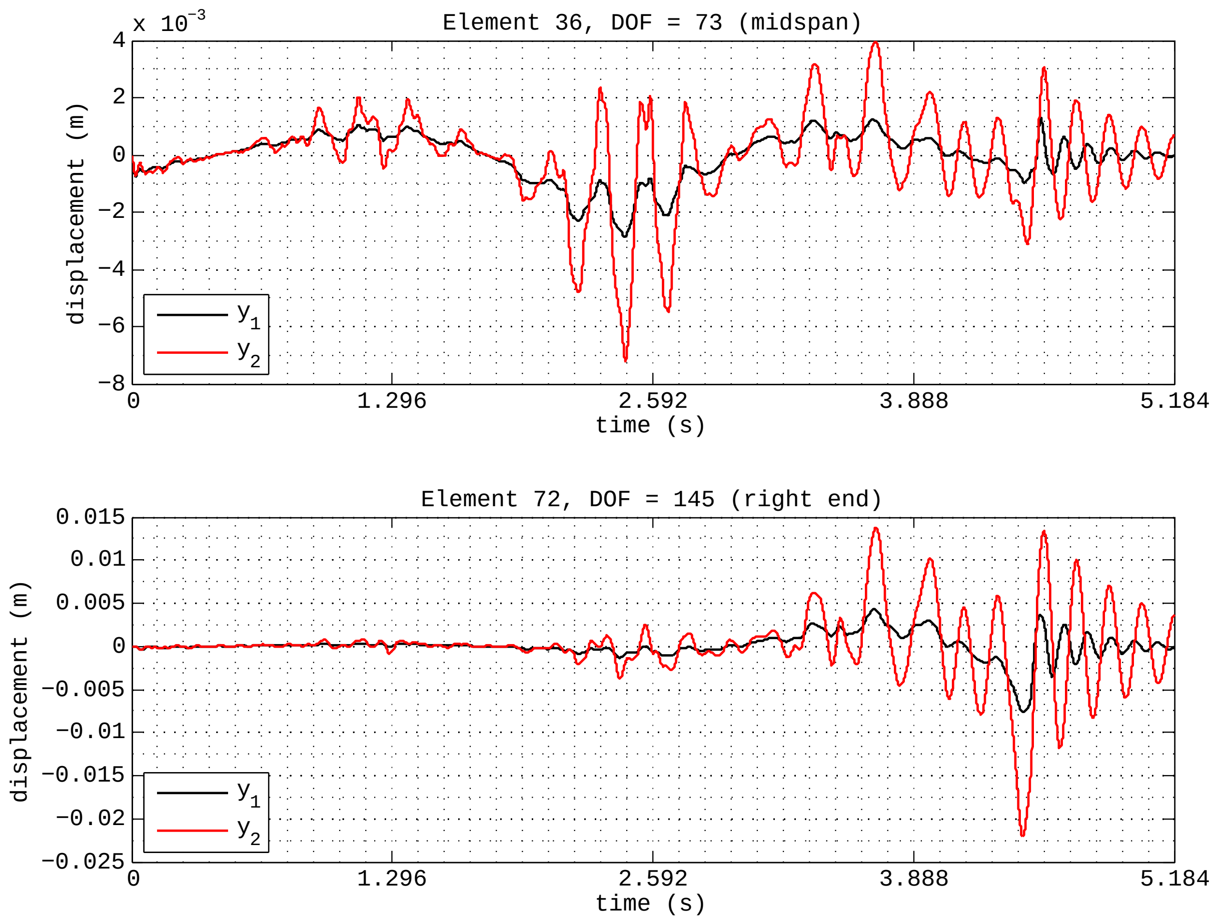

Figure 8: Dynamic responses of displacement (a) midspan (b) right end

The dynamic responses of velocity for both the midspan and right are presented in Fig. 9.

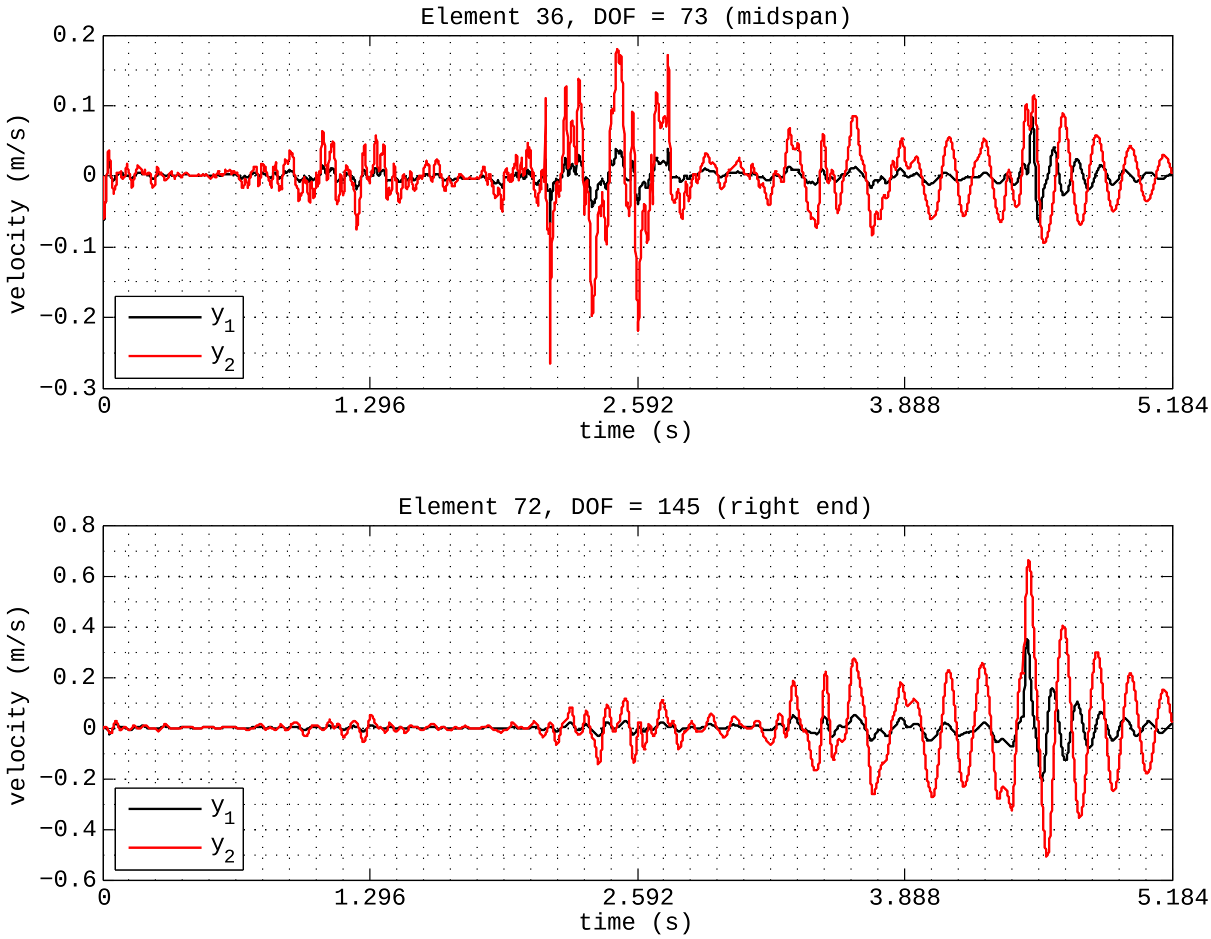

Figure 9: Dynamic responses of velocity (a) midspan (b) right end

Both nonlinear dynamic responses of displacement and velocity for the different form of random, but deterministic, forms of irregularities have a similar behavior. In all responses, there were major amplifications for the second case of random form of irregularities, y_2.

As explained before, as both forms of irregularities have a similar form, just varying the amplitude of each irregularity in about four times, being y_1 the minor one, this directly affects all dynamic responses of the structure. In addition to this, as both forms were computationally simulated with the nonlinear dynamic mathematical model, described in section 4.2, there is a mathematical and physical consideration for the physical nonlinearity of the material. In this sense, the higher amplitudes made the Mazar's damage model to amplify the damage on the concrete and even start a damaging process in the surrounding regions of the higher damages.

The damaged configuration of the bridge for both random forms of irregularities, y_1 and y_2 are illustrated in Fig. 10.

Obviously, there were higher damages on the second form of random irregularities. It is also very important to notice the interesting Mazar's damage model different behavior for traction and compression on the concrete. As traction effect is more detrimental then compression, some upper regions have suffered a damage by traction due to the strains inversion due to overhanging segments and vibrations.

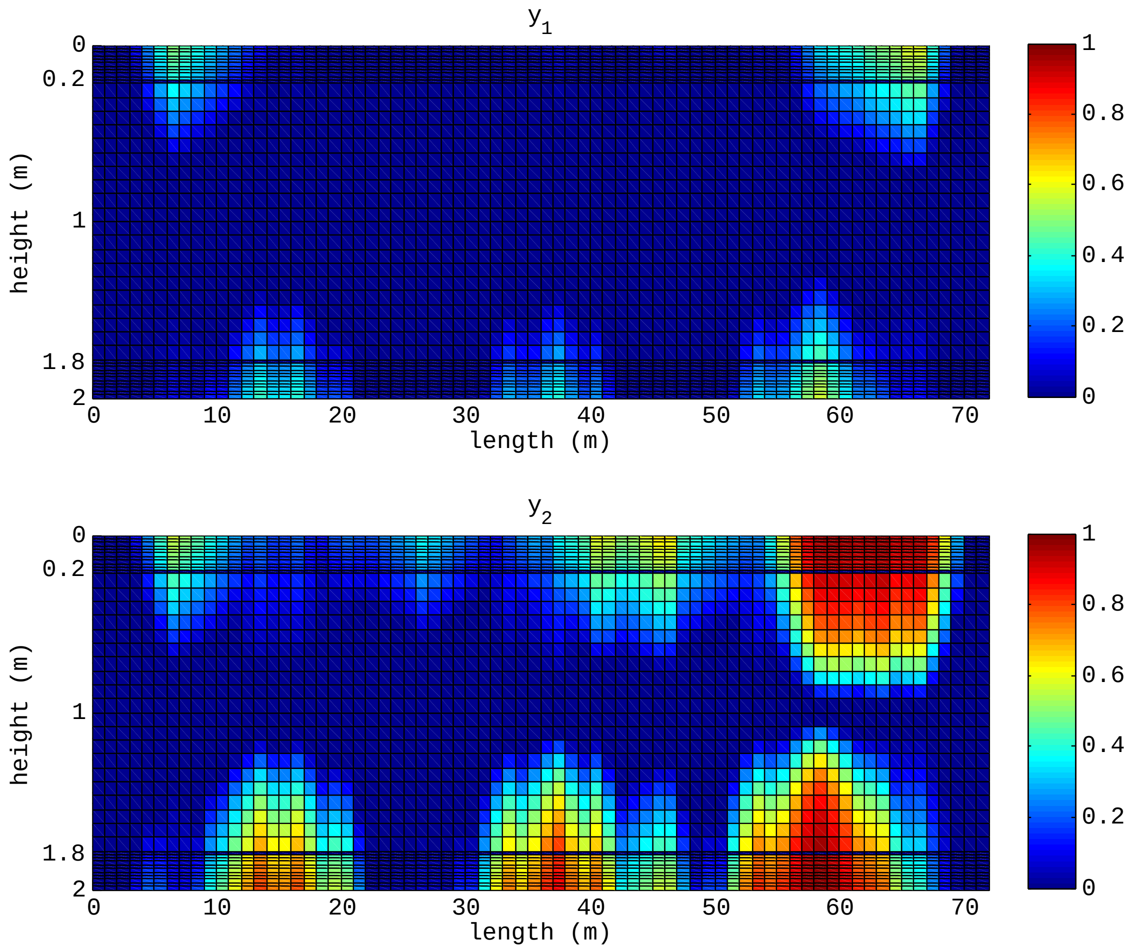

Figure 10: Damaged configuration (a) y_1 **(b)** y_2

The Fig. 11 shows the damage evolution on the cross section of the most affected element from the bridge with the y_1 form of irregularities over time. Each color represents one of the 42 layers.

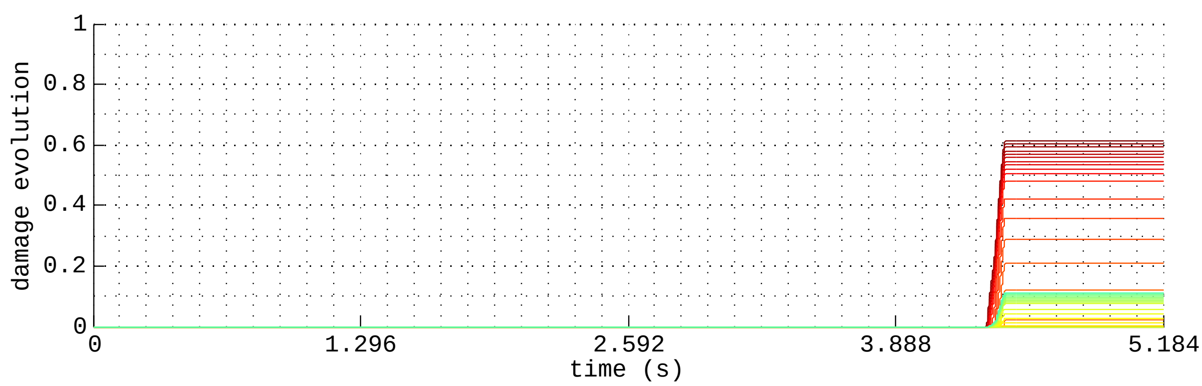

Figure 11: Damage evolution of the layers of the 66th element over time (y_1)

Similarly, the Fig. 12 shows the damage evolution on the cross section of the most affected element from the bridge with the y_2 form of irregularities over time. Each color also represent one of the 42 layers.

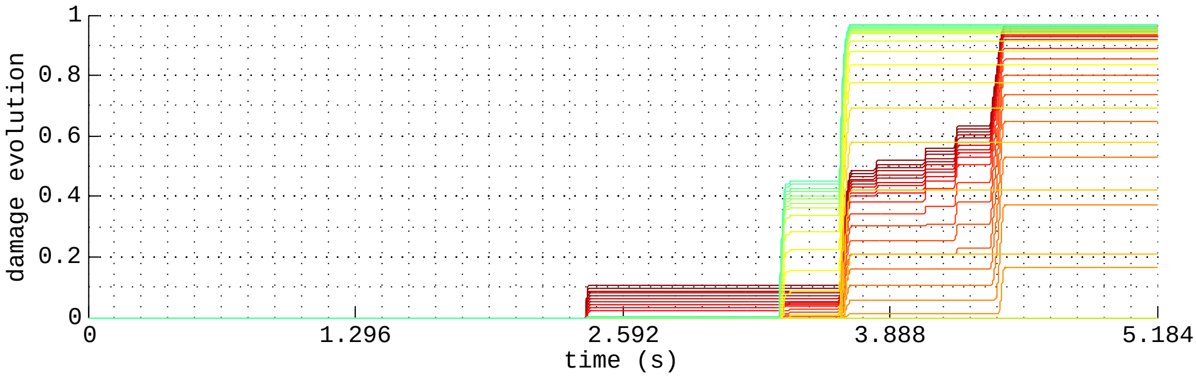

Figure 12: Damage evolution of the layers of the 59th element over time (y_2)

6 CONCLUSION

The dynamic responses of displacements and velocities were greater for the y_2 case of random irregularities of the highway bridge. As both random forms were geometrically alike but the second one have greater amplitude, it can be noted that the dynamic responses are directly related not only to the form of the irregularity, but also the its amplitude.

It must be noted that linear dynamic models cannot detect the presence of damages and its effect on the structural responses. Also, the damage evolution can only be detected through nonlinear dynamic analysis. In nonlinear dynamic analysis of highway structures, alteration occurs on the obtained responses due to the damage effect. This reduces its stiffness and indirectly modify its structural damping. These alterations can cause further damages to the structure and in some cases, such as the resonant condition, this process can become cyclical and progressive, having the risk to cause the structural collapse.

Furthermore, the nonlinear dynamic damage evolution of the bridge, due to this random forms, were also greater for the second case of irregularities. That noted, it can be observed that the amplitude of irregularities, not only its form, can specially affect how the damage evolves in dynamic perspective. Even the second form of irregularities having four times the amplitudes of the first one, it is still so small that it can surprise structural engineers that does not consider the irregularities' influence directly, only following its normative precepts. In a country that is world renowned for the lack of maintenance and terrible highways, such as Brazil, the bad planning allied with the disastrous execution of constructions risks safety and does not guarantee structural health.

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