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THE FINITE DIFFERENCE METHOD APPLIED TO THE 1D THERMOELASTIC PROBLEM

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Abstract. *In this paper the analytical and numerical solution are obtained for the problem of heating a metal bar by heat conduction in steady state at one of it's ends. Initially the analytical solution for each variable of interest (displacements, strains and stresses) is demonstrated, as well as the temperature distribution. Then, numerical approximations are performed using the Finite Difference Method for uniform grids, which obtain the numerical solutions of the variables of interest with their respective error of discretization. It is also performed an experiment in the laboratory where the displacements, strains and temperature distribution is verified. Finally, the error generated by the Finite Difference Method is verified comparing the numerical solutions with the analytical and the experimental results.*

Keywords: *Finite Difference Method, Heat Conduction, Thermoelasticity, Experiment.*

1 INTRODUCTION

The main goal of this paper is to analyze the thermoelactic behavior of a metal bar numerically and experimentally and to test efficiency of Finite Difference Method based in experimental data.

2 THERMOELASTICITY

When thermal energy is added to an elastic material it expands. For the simple unidimensional case of a bar of length L, initially at uniform temperature T_0 which is then heated to a nonuniform temperature T and thus grows in length by an amount ΔL(Tipler, 2008), the relative uni-axial stretching due to thermal expansion is

$$\frac{\Delta L}{L} = \epsilon = \alpha(T - T_0) \tag{1}$$

Where ϵ is the strain and α is the thermal expansion coefficient. For an isotropic cube of side L the (normal) thermoelastic strains are

$$\epsilon_x = \epsilon_y = \epsilon_z = \alpha(T - T_0) \tag{2}$$

Since the heated region is joined to, and constrained by rigid surroundings, it can not expand freely but becomes subjected to compressive stresses. At the same time the colder portion is subjected to the pull exerted by of the adjacent hot portion and it is thus under tension. Although Hooke's law is still applicable, due account must be taken of the additional stresses created by thermal expansion.

According Hacke (2006), the analytical solutions for the variables of interest (displacements, strains, stresses and forces) for the unidimensional thermoelastic problem at steady state are obtained through the following equations:

$$\frac{d^2u}{dx^2} - \alpha\frac{dT}{dx} = 0 \tag{3}$$

With u(0) = 0 and u(L) = 0, and, α is the thermal expansion coefficient.

$$\varepsilon(x) = \frac{du}{dx} \tag{4}$$

$$\sigma_x = E(\varepsilon_x - \alpha\theta) \tag{5}$$

Where E is the Young's Modulus of the material and $\theta = T - T_0$.

$$F = \sigma_x\, A_x \tag{6}$$

3 FINITE DIFFERENCE METHOD

The finite difference method is a method of solving differential equations, which is based on the approximation of the derivatives by finite differences. The approximation formula is obtained from the Taylor series of the derivative function. From approximations difference - quotient for derivatives of any order it's possible transform differential equations in linear problems. For this, we must ignore the error term and make h a very small number, but large enough not to cause instabilities in approximations of the derivatives.

The approximate equation by finite difference of Eq.(3) can be achieved by the central diferences scheme (Hacke, 2006; Incropera, 1981; Strickwerda, 1989).

$$\frac{u_W + u_E - 2.u_P}{h^2} - \frac{\propto(T_E - T_W)}{2h} = 0 \qquad (7)$$

4 EXPERIMENTAL RESULTS

The experiment was carried out using a dilatometer, an aluminum bar and 4 multimeters with thermocouples. The aluminum bar was set at the dilatometer and the thermocouples were fixed on the bar.

The data obtained in the experiment are listed in the following table:

Table 1: Nodes Temperatures in Function of Bar Length.

Aluminum	Node 1 (x = 0)	Node 2 (x = L/3)	Node 3 (x = 2L/3)	Node 4 (x = L)
Initial Temp. (°C)	22	18	18	19
Final Temp. (°C)	100	86	89	89
Θ (°C)	78	68	71	70

Where L is the length of the aluminum bar, which for this experiment is equal to 0.79 meters, and θ is the temperature difference at the point.

From these data, it was possible to determine the displacements, strains, stresses and the forces acting on the bar in each of its nodes.

Table 2: Displacements; Strains; Stresses and Forces in Function of Bar Length.

Position (m)	Displacements (m)	Strains (m/m)	Stresses (Pa)	Forces (N)
0	0	$1{,}35\text{x}10^{-4}$	9.450.000,0	70,686
L/3	$2{,}375\text{x}10^{-5}$	$4{,}51\text{x}10^{-5}$	10.045.000,0	75,136
2L/3	$2{,}375\text{x}10^{-5}$	$-4{,}51\text{x}10^{-5}$	10.619.000,0	79,430
L	0	$-1{,}35\text{x}10^{-4}$	9.492.000,0	71,000

For these calculations, the following values were adopted: Aluminum Elastic Modulus E = 70 Gpa, obtained in literature (Hibbeler, 2010); Linear expansion coefficient $\alpha = 24.6 \times 10^{-6}$ $°C^{-1}$ and cross-sectional area of the bar A = 7.48 mm² obtained in the experiment.

5 NUMERICAL RESULTS

Using the finite difference method, a computer simulation was performed to determine the numerical values of displacements, strains, stresses and forces, where it was possible to verify that due to linearity of the problem, numerical error was zero, i.e., the solution found by the simulation is the analytic solution of the problem.

The following are the graphs of the displacements and strains, respectively, in function of the length of the bar, using 9 nodes:

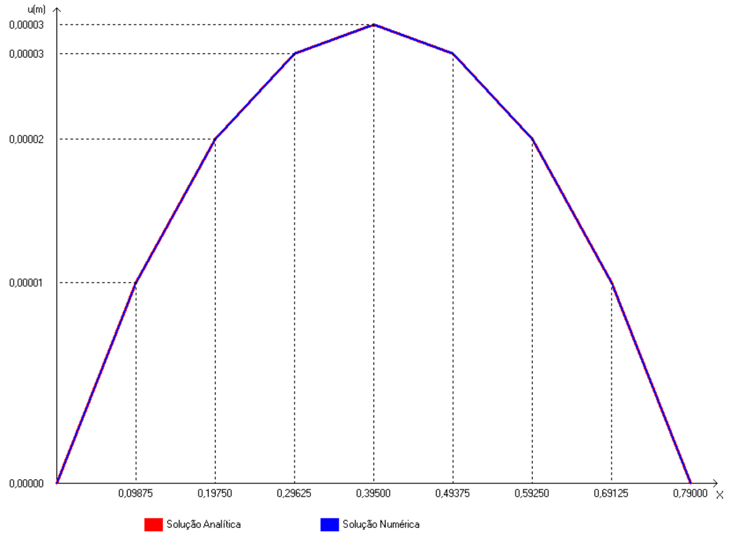

Figure 1: Displacements in Function of Bar Lenght.

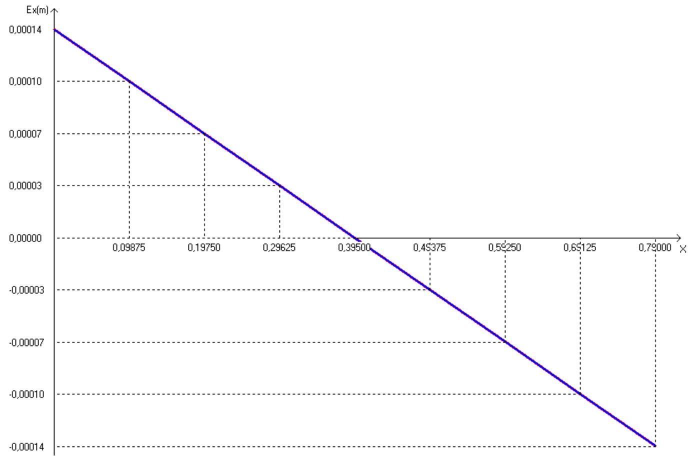

Figure 2: Strains in Function of Bar Lenght.

Figures 1 and 2 show the curves of the numerical solutions overlaps the curves of analytical solutions , indicating that the numerical error was zero.

6 CONCLUSIONS

Because of the simplicity and linearity of the problem, the computer simulation was able to reproduce the analytical solutions calculated to the problem, indicating that the only considerable source of errors to the problem are experimental errors and errors in mathematical modeling

The finite difference method was successful in determining the solutions of the variables of interest to the thermoelastic problem, proving that it is an effective tool in solving problems governed by differential equations in science and engineering. Since such problems doesn't always have analytical solutions, the use of numerical methods becomes a reliable and efficient alternative.

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