WAVE MODELLING OF A LIGHTWEIGHT AEROSPACE PANEL USING A FINITE ELEMENT APPROACH

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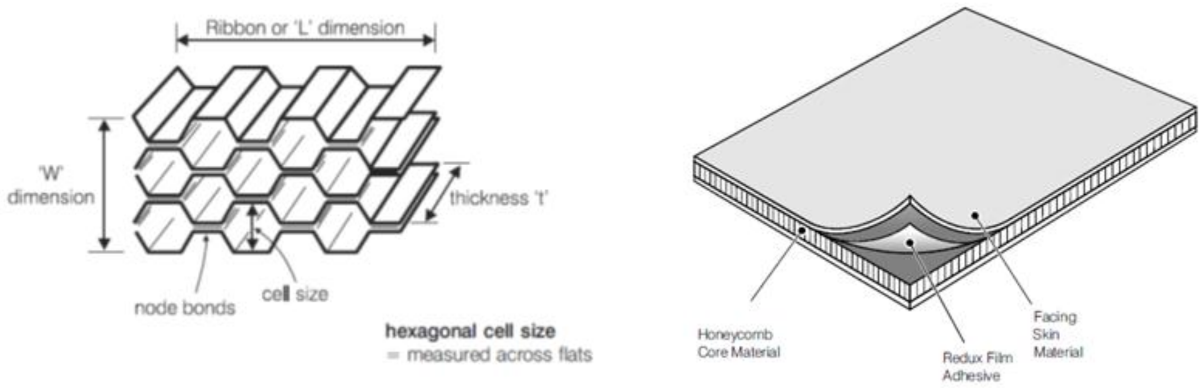
Abstract. *Aerospace structures require high strength and low mass, which has led to an increase in the use of composite materials by its industry. These materials result from the combination of two or more base materials in a way that one or more of the composite's properties are superior to either of the individual ones. A type of material that presents those desired attributes is the honeycomb sandwich panels and, as the industry is relying more and more on them, their accurate characterization for the given application is of extreme importance. In this paper, the wave and finite element (WFE) approach is applied and the wave parameters of a homogenized honeycomb sandwich panel model for aerospace applications are presented and numerical details discussed. The wave approach differs from the more usual modal analysis (MA) by focusing in properties such as the dispersion relations, wave modes, phase and group velocities, and energy transmission. Although MA with the aid of finite element modelling (FEM) is a widely used technique, as the frequencies of interest increase, the computational cost also increases. Moreover, the size of the elements also limits the maximum frequency that can be accurately characterized. On the other hand, the WFE method requires the model of a single period of a periodic structure, which can be obtained from any commercial FE software, benefiting from the available element libraries, reducing the computational cost when applied to a wider frequency range. The numerically obtained parameters are compared to an analytical model and show agreement with the theory.*

Keywords: *Wave and finite element, Wave propagation, Sandwich panel*

1 INTRODUCTION

The aerospace industry depends heavily on the use of strong and rigid yet lightweight materials for its components whenever possible. This naturally led the industry to use composites of various kinds, including honeycomb sandwich structures. A sandwich composite is a layered construct with at least three distinct phases: two external thin faces of high strength, and a thick but light core enclosed in the middle. One alternative found to make the core weight as little as possible and still maintain its strength is by constructing it in a way that it has hollow spaces, called cells, while keeping a bi-dimensional structure to support itself. It is found that the most efficient structure possible in terms of mass to strength is a hexagonal honeycomb, so named for its similarity to the structures found in beehives as seen in Fig. (1).



(a) (b)

FIGURE 1. Geometry of sandwich honeycomb panels: (a) core cell shape and (b) layered construction (*HexWeb Honeycomb Attributes and Properties*)

The design of aerospace structures have a great emphasis on efficiency, therefore the use of numerical and computational models is very important that during the initial phase of the project, to allow for some adjustments and optimizations in the design. However, predicting the dynamic behaviour of these materials can be a difficult task, especially when one attempts to do so at mid to high frequencies. Furthermore, such structures tend to possess orthotropic properties, increasing the difficulty in predicting its dynamic response (Schwingshackl et al., 2006).

The finite element analysis (FEA) computational cost becomes too expensive for mid and high frequencies. A different approach with great potential is the wave method, that focuses in properties such as the dispersion relations, wave modes, phase and group velocities, and energy transmission. Unfortunately, wave methods require analytical models that can become unviable in most non-trivial cases.

The wave and finite element (WFE) approach combines the use of FEA, and its extensive library of conventional finite elements, and the wave approach, that allows the propagation of waves to a single section of a periodic structure to be expanded and obtain the dynamic behavior of the entire structure.

In this work, the WFE approach is used to obtain the wave parameters of a satellite structural sandwich honeycomb panel for frequencies of up to 10kHz to demonstrate the efficiency of the method in characterizing the dynamic properties of a structure in mid and high frequencies.

This paper is presented in seven parts, first an introduction to present a general idea of the WFE method and this paper objectives. On chapter 2 the general description as given by the manufacturer of the panel is provided. Chapter 3 gives a general idea of the wave parameters that are used to characterize a structure with the wave approach. In chapter 4 the theoretical and mathematical concepts that allow the use of the WFE are briefly presented. Chapter 5 describes the finite element modeling used to numerically model the panel. In chapter 6 the results are discussed and in chapter 7 some conclusions are presented as well as some future work.

2 DESCRIPTION OF THE PANEL

The structure under analysis is a rectangular sandwich panel of dimensions 670 mm length by 300 mm wide and 10 mm thick, made with 2024 T3 aluminium face sheets and HexWeb CRIII 5056 hexagonal aluminium honeycomb core with ¼’’ cell size and 0.001’’ perforated foil thickness. Along the thickness the panel is layered with a 0.3 mm face sheet, 9.4 mm core and a 0.3 mm face sheet.

3 WAVE PARAMETERS

A brief description of some wave parameters used to designate some elements of the structural dynamics is presented in this section. A travelling wave can be characterized by a displacement field of the form

$$\varphi(x,t) = \tilde{A}e^{j(\omega t - kx)}, \tag{1}$$

where $\tilde{A}$ is the wave complex amplitude, which tells us both the maximum displacement and the phase, ω is the phase change over time and k is the wavenumber, that describe the behaviour of the wave over distance. While the frequency must be a real value, the wavenumber k can be described with a complex value, of the form $k = -\alpha j + \beta$, where α is called the *attenuation constant*, and defines how the amplitude decreases over distance, while β is the *phase constant* and defines the phase change over distance.

According to its wavenumber a wave can be classified as *propagating*, for a purely real wavenumber; *attenuating*, for a complex wavenumber; or *evanescent*, for a purely imaginary wavenumber. The special behaviour of these waves can be visualized in Fig. 2.

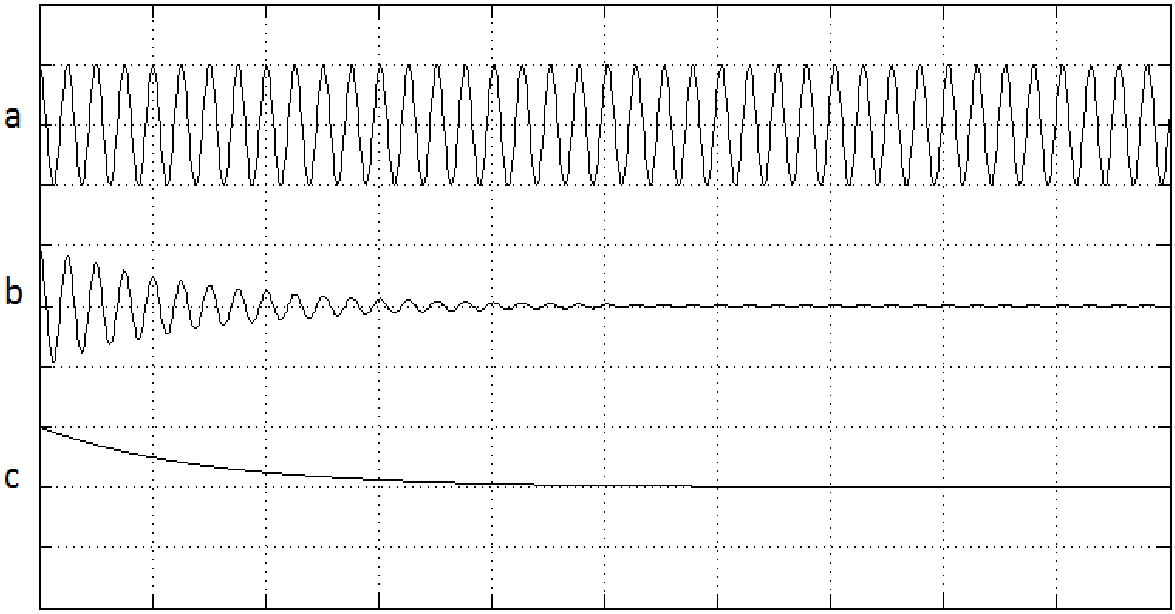


Figure 2. Types of waves by wavenumber: (a) propagating; (b) attenuating; and (c) evanescent.

The relation between k and ω is termed the dispersion relation (Fahy and Gardonio, 2007) and it depends on both the type of wave and the medium through which the wave propagates. If such relation is linear, the spatial form of the wave will be conserved throughout its propagation and the wave mode are called non-dispersive. On the other hand, the wave will be distorted throughout its propagation and be called dispersive.

From these basic properties, it is possible to define phase and group velocity. The former is the relation between frequency and wave number and describes the velocity which a wave moves in relation to a given referential. It is given by

$$c_f = \frac{\omega}{k}. \tag{2}$$

It can be seen that if the wavenumber is purely imaginary, so it will be the phase velocity, therefore the evanescent waves will not be able to transport energy. Due to their immobility, some authors may prefer to not call them waves and instead refer to them as 'near fields' (Fahy and Gardonio, 2007).

The latter, group velocity c_g, is the speed at which energy is transported by a wave. It can be obtained from the dispersion curve by

$$c_g = \frac{\partial\omega}{\partial k}. \tag{3}$$

For non-dispersive waves, the phase velocity will be equal to the group speed.

4 FINITE ELEMENT ANALYSIS

4.1 Dynamic Stiffness Matrix and the transfer matrix

Considering a one-dimensional periodic structure with an infinite number of cell, numbered n, with the relation between displacement and forces as given by Fig. 3.

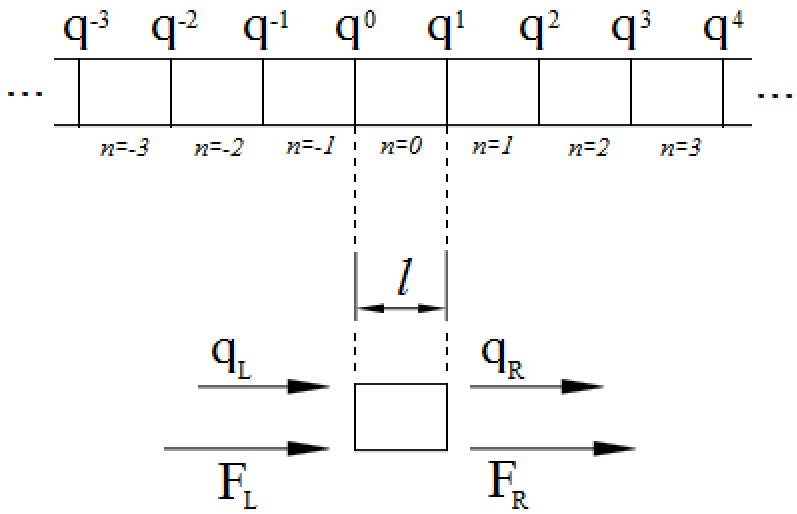


Figure 3. Forces and displacements in a cell

Assuming harmonic movement, the equation of motion is

$$(\mathbf{K} - j\omega\mathbf{C} - \omega^2\mathbf{M})\mathbf{q} = \mathbf{f}, \tag{4}$$

where **K**, **M** e **C** are respectively the stiffness, inertia and damping matrices obtained from a Finite Element model, **f** the nodal forces vector and **q** the nodal displacement vector. Defining the dynamic stiffness matrix $\widetilde{\mathbf{D}} = (\mathbf{K} - j\omega\mathbf{C} - \omega^2\mathbf{M})$, and rewriting the equation in terms of left (L), right (R) and interior (I) degrees of freedom, it is possible to obtain

$$\begin{bmatrix} \widetilde{\boldsymbol{D}}_{II} & \widetilde{\boldsymbol{D}}_{IL} & \widetilde{\boldsymbol{D}}_{IR} \\ \widetilde{\boldsymbol{D}}_{LI} & \widetilde{\boldsymbol{D}}_{LL} & \widetilde{\boldsymbol{D}}_{LR} \\ \widetilde{\boldsymbol{D}}_{RI} & \widetilde{\boldsymbol{D}}_{RL} & \widetilde{\boldsymbol{D}}_{RR} \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_I \\ \boldsymbol{q}_L \\ \boldsymbol{q}_R \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{f}_L \\ \boldsymbol{f}_R \end{bmatrix}. \tag{5}$$

Appling a dynamic condensation procedure (Duhamel et al., 2006), the internal DOF's can be eliminated, resulting in

$$\begin{bmatrix} \boldsymbol{D}_{LL} & \boldsymbol{D}_{LR} \\ \boldsymbol{D}_{RL} & \boldsymbol{D}_{RR} \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_L \\ \boldsymbol{q}_R \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_L \\ \boldsymbol{f}_R \end{bmatrix}. \tag{6}$$

It is possible to relate the displacement and forces of any cell to its neighbours through the transfer matrix **T** and equilibrium and continuity conditions such that

$$\boldsymbol{T}\begin{bmatrix}\boldsymbol{q}_L^n\\ \boldsymbol{f}_L^n\end{bmatrix} = \begin{bmatrix}\boldsymbol{q}_R^n\\ -\boldsymbol{f}_R^n\end{bmatrix} = \begin{bmatrix}\boldsymbol{q}_L^{n+1}\\ \boldsymbol{f}_L^{n+1}\end{bmatrix}, \tag{7}$$

and applying periodicity conditions and assuming that free waves propagates as Bloch waves,

$$e^{-jkl}\begin{bmatrix}\boldsymbol{q}_L^n\\ \boldsymbol{f}_L^n\end{bmatrix} = \lambda\begin{bmatrix}\boldsymbol{q}_L^n\\ \boldsymbol{f}_L^n\end{bmatrix} = \begin{bmatrix}\boldsymbol{q}_L^{n+1}\\ \boldsymbol{f}_L^{n+1}\end{bmatrix}, \tag{8}$$

it follows that

$$\boldsymbol{T}\begin{bmatrix}\boldsymbol{q}_L\\ \boldsymbol{f}_L\end{bmatrix} = \lambda\begin{bmatrix}\boldsymbol{q}_L\\ \boldsymbol{f}_L\end{bmatrix}, \tag{9}$$

which is an eigenproblem of the type

$$[\,\boldsymbol{T} - \boldsymbol{I}\lambda]\begin{bmatrix}\boldsymbol{q}_L\\ \boldsymbol{f}_L\end{bmatrix} = 0. \tag{10}$$

The propagation constants and free waves will be respectively the eigenvalues and eigenvectors of the eigenproblem above.

The eigenproblem can be formulated in a polynomial form as (Hinke et al., 2004)

$$\left[\boldsymbol{D}_{LL} + \boldsymbol{D}_{RR} + \lambda\boldsymbol{D}_{LR} + \frac{1}{\lambda}\boldsymbol{D}_{RL}\right]\boldsymbol{q} = 0, \tag{11}$$

or

$$\left[\boldsymbol{D}_{LL} + \boldsymbol{D}_{RR} + \lambda\boldsymbol{D}_{RL} + \frac{1}{\lambda}\boldsymbol{D}_{LR}\right]\boldsymbol{q} = 0, \tag{12}$$

which can also be written as:

$$[\boldsymbol{A_0} + \boldsymbol{A_1}\lambda + \boldsymbol{A_2}\lambda^2]\boldsymbol{q} = 0, \tag{13}$$

where $\boldsymbol{A_0} = \boldsymbol{D}_{RL}\ or\ \boldsymbol{D}_{LR}$, $\boldsymbol{A_1} = \boldsymbol{D}_{LL} + \boldsymbol{D}_{RR}$ and $\boldsymbol{A_2} = \boldsymbol{D}_{LR}\ or\ \boldsymbol{D}_{RL}$

For the two-dimensional problem, the equivalent equation polynomial problem is written as (Manconi, 2008)

$$\begin{aligned}\big[&(\boldsymbol{D}_{11} + \boldsymbol{D}_{33})\lambda_y\lambda_x + (\boldsymbol{D}_{22} + \boldsymbol{D}_{44})\lambda_y\lambda_x + (\boldsymbol{D}_{12} + \boldsymbol{D}_{34})\lambda_y\lambda_x^2 + (\boldsymbol{D}_{13} +\\ &\boldsymbol{D}_{24})\lambda_y^2\lambda_x + \boldsymbol{D}_{32}\lambda_x^2 + \boldsymbol{D}_{23}\lambda_y^2 + (\boldsymbol{D}_{21} + \boldsymbol{D}_{43})\lambda_y + (\boldsymbol{D}_{31} + \boldsymbol{D}_{42})\lambda_x +\\ &\boldsymbol{D}_{14}\lambda_y^2\lambda_x^2 + \boldsymbol{D}_{41}\big]\boldsymbol{q} = 0,\end{aligned} \tag{14}$$

for which there are propagation constants λ_x and λ_y in the x and y directions. Each propagation constant is related to its respective wavenumber k_x and k_y. They are restrained by the following relation

$$k^2 = k_x^2 + k_y^2. \quad (15)$$

It follows that

$$k_y = f(k_x) = k_x \tan\theta, \quad (16)$$

where θ is the direction of propagation. Therefore Eq. (4) can be rewritten as a function of k_x, analogously to Eq. (13), i.e., the two-dimensional problem can be effectively written as one-dimensional one, in which case

$$\begin{aligned} \boldsymbol{A_0} &= \boldsymbol{D}_{21} + \boldsymbol{D}_{23} + \boldsymbol{D}_{41} + \boldsymbol{D}_{43} \\ \boldsymbol{A_1} &= (\boldsymbol{D}_{11} + \boldsymbol{D}_{13} + \boldsymbol{D}_{31} + \boldsymbol{D}_{33}) + (\boldsymbol{D}_{22} + \boldsymbol{D}_{24} + \boldsymbol{D}_{42} + \boldsymbol{D}_{44}) \\ \boldsymbol{A_2} &= \boldsymbol{D}_{12} + \boldsymbol{D}_{14} + \boldsymbol{D}_{32} + \boldsymbol{D}_{34} \end{aligned} \quad (17)$$

4.2 Zhong's Method

Numerical issues can appear if the transfer matrix $\mathbf{T}$ is ill-conditioned, which usually is the case when a large number of wave modes is calculated. Zhong applies the sympletic properties of the transfer matrix to find the solution to the eigen equation and find the wavenumbers in a better conditioned way(Zhong and Williams, 1995). Rather than using the transfer matrix that relates the displacement and forces through the equation

$$\boldsymbol{T}\begin{bmatrix}\boldsymbol{q}_L \\ \boldsymbol{f}_L\end{bmatrix} = \lambda\begin{bmatrix}\boldsymbol{q}_L \\ \boldsymbol{f}_L\end{bmatrix}, \quad (18)$$

it uses two other matrices **L** and **N**, such that $\mathbf{T} = \mathbf{L}\mathbf{N}^{-1}$ and obtaining

$$\boldsymbol{N}\begin{bmatrix}\boldsymbol{q}_L \\ \boldsymbol{q}_R\end{bmatrix} = \lambda\boldsymbol{L}\begin{bmatrix}\boldsymbol{q}_L \\ \boldsymbol{q}_R\end{bmatrix}. \quad (19)$$

Since Eq. (19) only uses displacements, the formulation is numerically more robust. Through the sympletic properties of the matrix, it is possible to show that

$$(\mathbf{L}^{\mathbf{T}}\mathbf{J}\mathbf{N} + \mathbf{N}^{\mathbf{T}}\mathbf{J}\mathbf{L})\mathbf{q} = \mu\mathbf{L}^{\mathbf{T}}\mathbf{J}\mathbf{L}\mathbf{q}, \quad (20)$$

where $\mu = (\lambda + \frac{1}{\lambda})$. Since finding the results of Eq. (20) means finding the eigenvalues and its inverses, solving for μ is a further step to a better-conditioned problem as the absolute values will be closer of each other.

The linearization of Eq. (13), can be done by the following possibilities

$$\left(\lambda\begin{bmatrix}\boldsymbol{A_2} & \boldsymbol{0} \\ \boldsymbol{A_1} & \boldsymbol{A_2}\end{bmatrix} - \begin{bmatrix}\boldsymbol{0} & \boldsymbol{A_2} \\ -\boldsymbol{A_0} & \boldsymbol{0}\end{bmatrix}\right)\begin{pmatrix}\boldsymbol{q} \\ \lambda\boldsymbol{q}\end{pmatrix} = \boldsymbol{0}, \quad (21)$$

$$\left(\lambda\begin{bmatrix}\mathbf{0} & \boldsymbol{A}_2\\ -\boldsymbol{A}_0 & \mathbf{0}\end{bmatrix}-\begin{bmatrix}-\boldsymbol{A}_0 & -\boldsymbol{A}_1\\ \mathbf{0} & -\boldsymbol{A}_0\end{bmatrix}\right)\begin{pmatrix}\boldsymbol{q}\\ \lambda\boldsymbol{q}\end{pmatrix}=\mathbf{0}. \tag{22}$$

From all of the possible linearizations, these are chosen because we can define

$$\boldsymbol{N}=\begin{bmatrix}\mathbf{0} & \boldsymbol{I}\\ -\boldsymbol{A}_0 & -\boldsymbol{A}_{12}\end{bmatrix} \text{ and} \tag{23}$$

$$\boldsymbol{L}=\begin{bmatrix}\boldsymbol{I} & \mathbf{0}\\ \boldsymbol{A}_{11} & \boldsymbol{A}_2\end{bmatrix}, \tag{24}$$

where

$$\begin{aligned}\boldsymbol{A}_{\mathbf{11}} &= (\boldsymbol{D}_{11}+\boldsymbol{D}_{13}+\boldsymbol{D}_{31}+\boldsymbol{D}_{33})\\ \boldsymbol{A}_{\mathbf{12}} &= (\boldsymbol{D}_{22}+\boldsymbol{D}_{24}+\boldsymbol{D}_{42}+\boldsymbol{D}_{44})\end{aligned} \tag{25}$$

Summing Eqs. (21) and (22), a form equivalent to Eq. (20) can be found such

$$\begin{bmatrix}\mathbf{A}_2-\mathbf{A}_0 & -\mathbf{A}_1\\ \mathbf{A}_1 & \mathbf{A}_2-\mathbf{A}_0\end{bmatrix}\mathbf{q}=\mu\begin{bmatrix}\mathbf{0} & \mathbf{A}_2\\ -\mathbf{A}_0 & \mathbf{0}\end{bmatrix}\mathbf{q}. \tag{26}$$

Therefore, the Zhong's method can be applied to a 2D problem.

5 HONEYCOMB SANDWICH PANEL MODEL

The sandwich honeycomb panel under analysis made of aluminum faces and honeycomb core, Fig 4. The finite element model of the plate section was built using the ANSYS element library, element type SOLID185, a 3D solid with eight nodes, and three DOF's, translations in the x, y and z directions, per node. The periodic section is 0.1 mm wide in both x and y directions. The sandwich faces are 3 mm thick isotropic aluminum sheets.

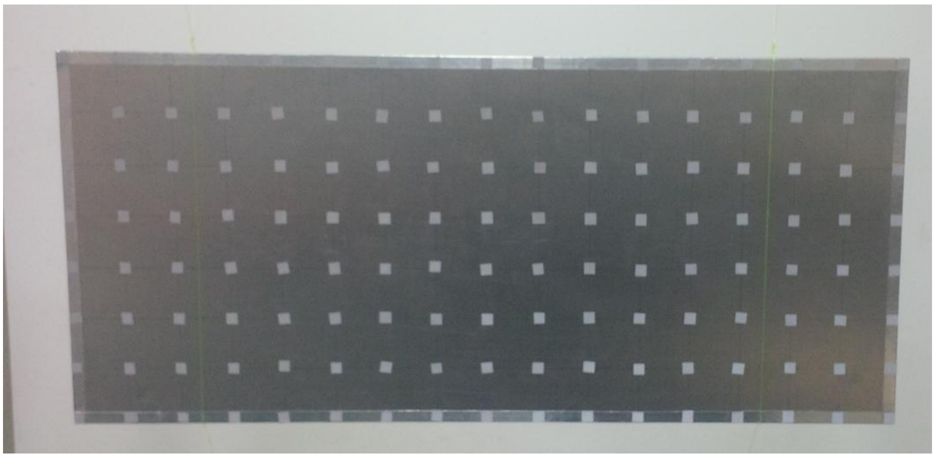


Figure 4. Picture of the sandwich panel hung on nylon treads with attached reflective tapes

Twenty six elements are used to mesh the section, three for each face and twenty for the core, as shown in Fig. 5. Only a small fraction of the panel needs to be taken into account in this model, which contrast with usual modal approach that requires a FE model hundreds of times larger than this one. For lower frequencies, it is expected that the internal structure of the core will not greatly affect the wave modes and so the core was homogenized into an equivalent 94 mm thick orthotropic solid. The panel material properties were found from an Experimental Modal Analysis (de Sousa et al., 2016) and are presented in Tabs. 1 and 2. The element implementation at the ANSYS software requires some amendments such that the Honeycomb core's shear modulus in the xy plane should be set to zero (*HexWeb Honeycomb Attributes and Properties*) and the poison ratio ν_{xy} should be close to 1 for small displacements (Gibson and Ashby, 2001).

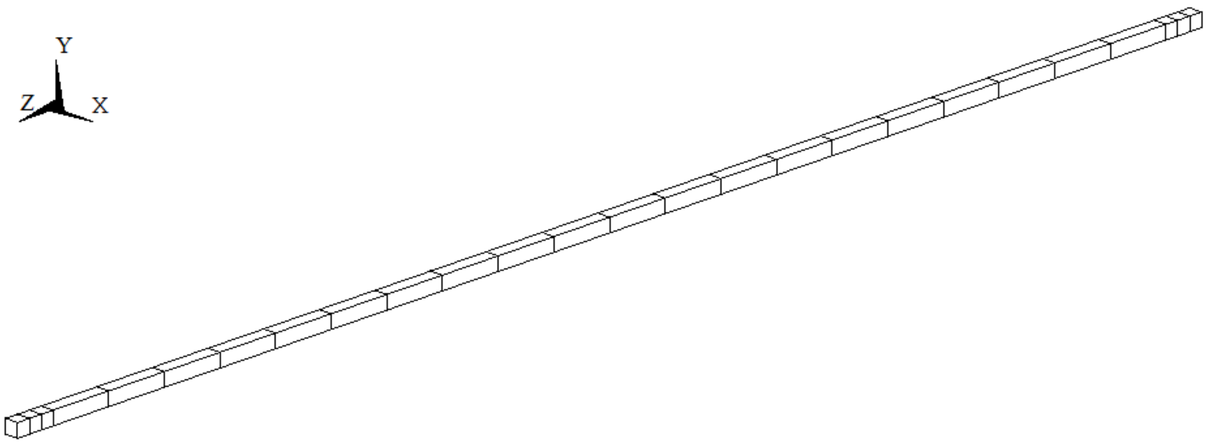


Figure 5. Plate section mesh.

Table 1. Face sheet material properties.

Property	Nomenclature	Value	Dimension
Young modulus	E	68e9	N/m²
Poisson ratio	ν	0.35	dimensionless
Density	ρ	2780	kg/m³

Table 2. Homogenized core material properties.

Property	Nomenclature	Value	Dimension
Young modulus	E_x	1.24e6	N/m²
	E_y	0.7e6	
	E_z	1.8e6	
Shear modulus	G_{xy}	1e-6	N/m²
	G_{yz}	220e6	
	G_{xz}	103e6	
Poisson ratio	ν_{xy}	0.6	dimensionless
	ν_{yz}	0.35	
	ν_{xz}	0.35	
Density	ρ	0.08	kg/m³

6 RESULTS AND DISCUSSION

In this section, the wavenumbers and wavemodes are calculated using the stiffness and mass matrices obtained from the FE model of a small segment of panel, as shown in the previous section, and results are obtained from a Matlab routine. Figures 6 and 7 shows the panel's dispersion relations up to a frequency of 10 kHz for both x and y directions, respectively. From these results, it can be seen that for lower frequencies, the panel behaves as a simple orthotropic plate, with one bending mode, dispersive curve, and two perpendicular longitudinal modes, non-dispersive curve.

From Figs. 6 and 7, it can also be seen that the wave propagation on both directions are very similarly, as it is expected due to the panel symmetry. The longitudinal modes are mainly influenced by the face properties and are nearly identical for both cases. The bending modes rely mostly on the orthotropic core's properties and results from x and y directions differ significantly after 1 kHz, but retaining a similar dispersion curve shape throughout.

For the x direction, at the frequency of 4,395 Hz a complex wave mode becomes fully real. This frequency is called the cut on frequency and means that a mode that was non-propagating starts to propagate, i.e. to transport energy. This particular case further creates two branches with different wavemodes. Surprisingly, as the dispersion relation of the branch with smaller wavenumber has negative inclination, its group velocity is also negative, meaning that the energy will travel in the direction opposite to that of the phase velocity. This phenomenon happens until the cut off frequency of 5,325 Hz, after which this branch of the wave mode ceases to transport energy, i.e. becomes evanescent. For the y direction, the cut on frequency of the complex mode happens later at 4,740 Hz, while the cut off frequency remains the same.

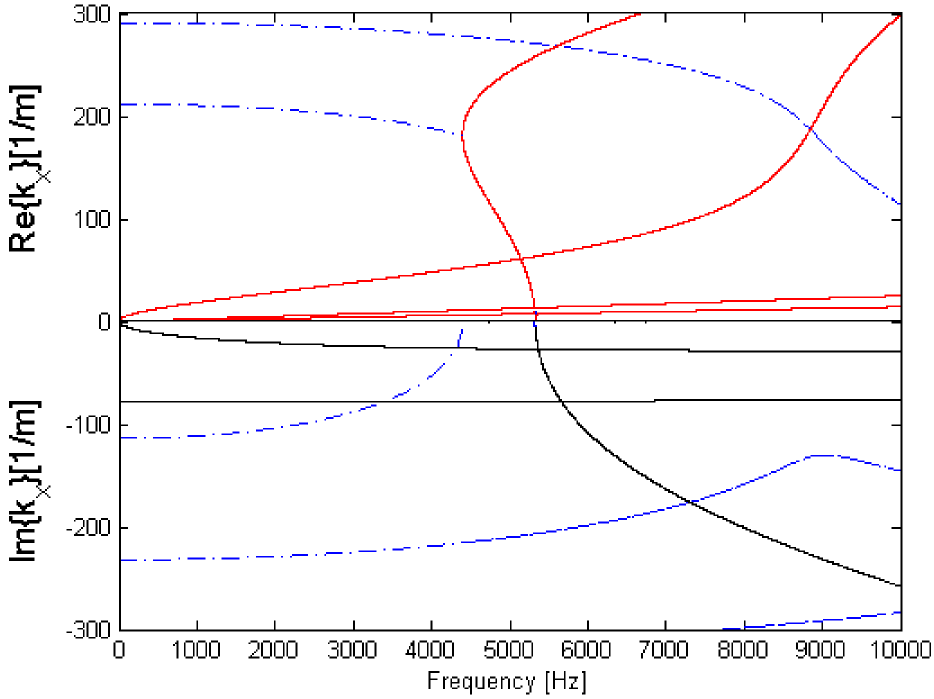


Figure 6. Dispersion relations in the x direction. ___for purely real or purely imaginary wavenumbers, _._ for complex wavenumbers

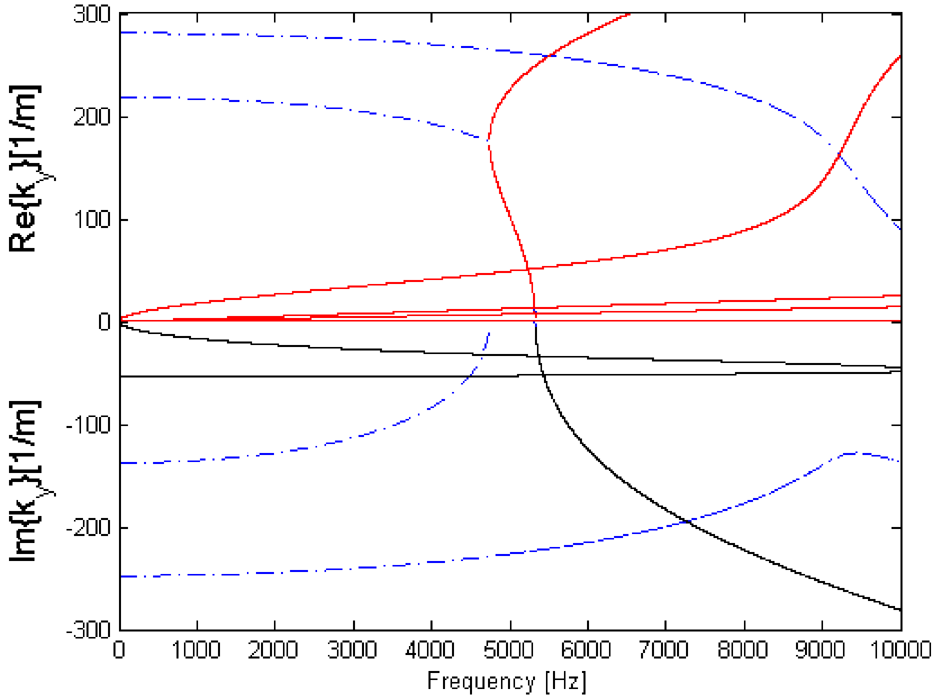


Figure 7. Dispersion relations in the y direction. ___for purely real or purely imaginary wavenumbers, _._ for complex wavenumbers

7 CONCLUDING REMARKS

In this work, the wave and finite element (WFE) approach was applied and the wave parameters of a homogenized honeycomb sandwich panel model for aerospace applications were presented and some numerical details discussed.

By using WFE it was possible to use a single section of the structure, meshed in 26 3D SOLID185 ANSYS elements, to obtain the wave parameters of the structure for reasonably high frequency. The fact that there are commercial software with extensive element libraries greatly facilitates the modeling and use of the WFE. Additionally, it allowed a simple configuration of the unique properties obtained experimentally.

For low frequencies, it is observed that the panel behaves as a simple plate and an equivalent structure could be made using a single material, up to 5kHz, however, for higher frequencies new and more complex wavemodes appear.

A preliminary analysis of the panel was performed and in the future a full analysis of the wavemodes and experimental validation of the wavenumbers will be performed.

ACKNOWLEDGEMENTS

The authors are thankful to the financial support of the Brazilian National Council of Research (CNPq), Process number 445773/2014, and the Federal District Research Support Foundation (FAPDF), Process number 0193001040/2015 and a scholarship for the first author.

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