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COMPARING TWO ANALYTICAL MODELS FOR ORBIT DETERMINATION USING GPS NAVIGATION SOLUTIONS

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Abstract. *This work presents the results of two analytical orbit models for orbit determination using the navigation solutions provided by on-board GPS receivers. Herein one proposes to analyze two analytical orbit models that can be used both on-board and on ground control centers for quick orbit determination. One model is the NORAD SGP8 elements (Hoots and Roehrich, 1980) where the two-line elements (TLE) are the orbit data needed to compute orbits. The second one is the analytical orbit model of the CBERS (China-Brazil Earth Resources Satellite) mission named COE (CBERS Orbit Elements). The orbit determination (OD) system to generate the TLE and COE datasets is being developed to be implemented in the INPE (Brazilian Institute for Space Research) ground control center. The TLE may be disseminated regularly to allow image receiving ground stations to track the CBERS-4 satellite. The COE can be used in the control center to monitor the orbit and compute maintenance maneuvers. The paper describes the basics of the two models, and implements such models to fit in the least squares sense the GPS long arc navigation solution measurements. Results will be shown based on actual GPS measurements from CBERS satellites, comparing aspects of the fittings and the models propagation.*

Keywords: *Orbit determination, GPS Navigation Solution, CBERS Orbit Elements, Two-Line Elements*

1 INTRODUCTION

The Satellite Control Centre (*SCC*) at *INPE* (National Institute for Space Research of Brazil) periodically conducts the orbit determination procedure of all satellites currently in operation in order to allow ground stations to track them and also to allow the control center to compute the orbit maintenance maneuvers.

The proposed orbit determination (*OD*) procedure is performed with the aid of a computer system that implements the least squares algorithm (Vallado, 1997; Montenbruck and Gill, 2000) to identify the orbital parameters which fit a orbit propagation model to the observed position and velocity data computed in an inertial reference system. As well, this computer system allows the *SCC* to perform the conventional OD procedure using radar tracking data called *ranging* and/or Doppler data type (Kuga, 1997; Orlando and Kuga, 2001).

Currently the *CBERS-4* (China-Brazil Earth Resources Satellite) is operated jointly by *SCC* and *XSCC* (Xian Satellite Control Center in China). There are two analytical orbit models used by SCC for implementing orbit related tasks such as prediction, ground tracking, maneuver, and upload of orbit parameters for on-board orbit computations. One is the *NORAD SGP8* (Hoots and Roehrich, 1980) whose orbital parameters are the two-line elements (*TLE*). The second is the specific model of the *CBERS* mission whose orbital parameters are called *COE* (*CBERS* Orbit Elements) (Kuga, 2002). The *TLE* can be disseminated regularly to allow users to track the *CBERS* satellites, while the *COE* can be used by the control center for orbit monitoring, update of on-board COE and maintenance maneuvers computation.

Since the *CBERS-1*, the first launched mission satellite, until the *CBERS-4*, current satellite placed in orbit in 2014 December, gradual improvements were inserted in terms of equipment. One improvement was the insertion of the recording and positioning system via on board *GPS* receivers. An analysis of the accuracy of orbital *GPS* ephemeris on board the *CBERS-2B* is shown in Kuga et al. (2009), where such ephemeris are compared to both the SCC determined orbit via *ranging* and Doppler, and to the *NASA* (National Aeronautics and Space Administration) *TLE*. In addition, it presents the result of orbit determination using *GPS* navigation solution ephemeris as observation data and *SGP8* model to generate the *TLE,* called *TLE GPS,* which is compared to the mentioned data. The result shows the appropriateness of using this data applied in rapid orbit determination procedures on ground and also on board, as an aid to on board attitude determination and control.

The *CBERS* mission objective is to perform Earth observation, especially over Brazil and China territory. Its orbit was designed to map the Earth every 26 days cycles. Its orbit is quasi polar (inclination of approximately 98.5 degrees), quasi circular, altitude of 778 km, and descending passin Equator at 10:30 local time (Kuga et al. 2009). These orbital characteristics allowed to develop the *CBERS* analytical orbit model with orbital parameters *COE* as frozen orbit elements (Kuga et al., 2004, Pardal et al., 2008).

It is currently being extended the orbit determination system in order to incorporate the use of navigation solutions provided by *GPS* receivers to obtain *TLE* as well as *COE.*

The present work aims to present an initial assessment of the results of the implementation of this computer system expansion using satellite *GPS* navigation solutions from *CBERS-2B* (Kuga et al., 2009) in the orbit determination with both models. Initially, the structure of the two models are presented and compared. Then, the results of orbit determination procedures are assessed.

The *GPS* data available from the *CBERS-2B* satellite are constrained to the field of view of one *INPE´*s ground station located in the brazilian territory (Cuiabá) and therefore they are formed by short arcs representing relatively small portions of the orbit. This implies some limitations in the TLE and COE OD procedures, which will be also discussed. It is intended to conduct a more thorough evaluation of the system with complete orbits covered by *GPS* navigation solutions of the *CBERS-4* satellite as a continuation of the work in the future.

2 MODELS DESCRIPTION

The propagation models implemented are formed by analytical solutions of the two-bodies disturbed problem equations. The difference between models formed by analytical solutions rely upon the forces that are considered. The orbital elements help to solve this problem because they allow a better understanding of the effects of perturbations over time through secular and periodic variations of short and long periods (Vallado, 1997).

One approach used is the series expansion to express the satellite movement, and different models are obtained depending on how this expansion is made and the order of the terms retained for its composition. The analytical solutions typically work by adding terms to a mean solution of the equations. This mean solution differs from the other solution approaches and, because of this, the models turn out to have different elements sets, specific from the employed rationale when developing the solution (Vallado, 1997).

2.1 Frozen Orbits

Frozen orbits are designed to minimize the global altitude variation through the cancellation of long period variations in the eccentricity e and perigee argument ω (Vallado, 1997). These orbits have practical implications of enabling, e.g., to obtain more uniform images in different satellite passages for the same latitude, due to the minimization of the altitude variation.

The frozen eccentricity and perigee argument depend only on the disturbing forces from the zonal harmonics for a given altitude or semi-major axis (Vallado, 1997). Usually it is studied the behavior of the orbital elements around the frozen orbit using a model that includes zonal terms and atmospheric drag. For a given semi-major axis, it is possible to verify that the geometry of time-histories is formed by stable closed trajectories in the phase-plane for different initial conditions of eccentricity and perigee argument around the fixed point represented by the frozen values of [e, ω].

The resulting equations for $\dot{\omega}$ and $\dot{e}$ from Brouwer and Hori theory (Brouwer and Hori, 1961) when including geopotential zonal disturbances up to J5 and the atmospheric drag are shown in (Kuga et al., 2004). For the *CBERS* mission, perigee argument value that defines the frozen orbit is around 90^{o} (Kuga et al., 2004).

2.2 *SGP8* Model

The *TLE* elements set was created for military use and is made available by *NASA* for the general public. The US Air Force tracks all detected objects in Earth orbit and provides the elements of "non-classified" objects. In addition to the mean elements, the available files have object identification information and the reference epoch where the elements are valid, and the format is properly described and standardized (Hoots and Roehrich, 1980).The *TLE* represent mean elements of the simplified general perturbation (*SGP*) models. Using one of these models, it is possible to perform orbit propagation and obtain the position and velocity ephemeris in the specified period. One of the models of this class is the *SGP8*.

The *SGP8* employs the Brouwer theory (Brouwer, 1959) developed in *SPACETRACK* project to describe the motion of satellites under the influence of the zonal harmonics J2, J3, J4 and J5 using Delaunay variables and incorporates a modification to include the atmospheric drag (Brouwer and Hori, 1961). The *SGP8* model also incorporates the Kozai theory (Kozai, 1959) and an improvement by using the analytical model of the atmosphere density (Lane and Crowford, 1969) to simplify the computational implementation, being different from *SGP4* model by the method employed in the equations integration (Hoots and Roehrich, 1980).

The *TLE* elements to be "solved for" in the identification procedure in *SGP8* model are $\bar{n}_0$, e_0, i_0, Ω_0, ω_0, M_0, B^*, which are, respectively, mean mean motion, eccentricity, inclination, right ascension of ascending node, perigee argument, mean anomaly and modified ballistic coefficient. They are mean values. The *TLE* provided by *NASA* further include $\dot{n}/2$ and $\ddot{n}/6$, where $\dot{n}$ and $\ddot{n}$ and are, respectively, the mean motion first and second time derivatives, which however are not used in SGP8 model.

The modified mean motion is related to the modified semi-major axis by the Kepler's Third Law - Eq. (1), where μ is the Earth's gravitational constant. The bars in n_0 e a_0 denotes the Kozai mean values. The value of a can be obtained by summing $\bar{a}_0$ to the short period term a_{SP} in *SGP8* model, as shown in Eq. (2) (Vallado, 1997).

$$\bar{n}_0 = \sqrt{\frac{\mu}{\bar{a}_0^{\,3}}} \tag{1}$$

$$a = \bar{a}_0 + a_{SP} \tag{2}$$

2.3 *COE* Model

The theory of frozen orbits was used in the quasi-circular orbit design of *CBERS* mission satellites (Kuga et al., 2004, Pardal et al., 2008). The *COE* are formed taking into consideration the frozen orbit characteristics, being represented by a type of non-singular specific elements in terms of eccentricity and the perigee argument, that resemble the long period terms of Brouwer theory, but also taking into account the atmospheric drag.

The *COE* are composed of the Keplerian elements, the variation of the orbital period and the variation of the eccentricity, in order to consider the effect of atmospheric drag (Kuga, 2002). The Keplerian elements are mean values where it is considered only the short period geopotential disturbances, corresponding to "prime" or long period elements in Brouwer theory (Kuga, 2002).

Thus, the *COE* elements to be "solved for" in the identification procedure are a', e', i', Ω', ω', M', $\dot{T}$ and $\dot{e}$, which are, respectively, mean semi-major axis, eccentricity, inclination, right ascension of the ascending node, perigee argument, mean anomaly, period time variation and eccentricity time variation.

Internally in the model, non-singular variables are used in terms of e and ω, according to Eqs. (3) and (4). This substitution allows quasi-circular orbits determination, because the equations have singularities when the eccentricity is zero.

$$\zeta = e\cos\omega \tag{3}$$

$$\eta = -e\sin\omega \tag{4}$$

The $\dot{e}$ and $\dot{n}$ values are obtained in *COE* model when considering the atmospheric drag. The $\dot{e}$ value comes from this formulation (Kuga, 2002). The $\dot{T}$ value is obtained when relating the Kepler's Third Law to the mean motion $n = 2\pi / T$, according to Eq. (5).

$$\dot{T} = -\frac{2\pi}{n}\dot{n} \tag{5}$$

Still comparing to the *SGP8* model, the "prime" *COE* elements are obtained by adding the long period *SGP8* terms (δ_{LP}) to mean values represented by the *TLE*, according to Eq. (6).

$$a', e', i', \Omega', \omega', M' = a_0, e_0, i_0, \Omega_0, \omega_0, M_0 + \delta_{LP}(a_0, e_0, i_0, \Omega_0, \omega_0, M_0) \tag{6}$$

3 ORBIT DETERMINATION WITH *CBERS-2B* DATA

It was performed an initial assessment of the computational implementations of the orbit determination procedures with *TLE* and *COE* parameters using *CBERS-2B* satellite and its on board *GPS* receiver data. The *GPS* navigation solutions *PVT* (position, velocity, time) are received along with image data by Cuiabá image receiving station.

The *GPS* data available for this evaluation are relatively poor, because they are relatively short with sparse portions of the orbit. The time-histories range from 10 to 15 minutes referring to orbit arcs comprised in approximately 0 ° to 30 ° south latitude. Furthermore, only daytime passages are available. The data available for orbit determination are shown in Table 1. The sampling time is 2 s. The epochs are *GMT* (Greenwich Mean Time).

Table 1. GPS PVT available for orbit determination.

Passage	Date	Start time	End time
1	20/07/2008	12:34:53.6	12:43:07.6
2	20/07/2008	14:09:49.6	14:25:03.6
3	21/07/2008	13:35:53.5	13:50:30.5
4	21/07/2008	15:16:45.5	15:26:54.5

These four data segments were concatenated and then used for orbit determination with both models. The *TLE* obtained are shown in Table 2, while the *COE* obtained are shown in Table 3.

Table 2. TLE computed with model SGP8

n (revs/day)	e	i (°)	Ω (°)	ω (°)	M (°)	B* (1/er)
14.35441931	0.0001294	98.5183	277.2635	113.0283	74.5565	0.1E-8

Table 3. COE computed with model COE

a (m)	e	i (°)	Ω (°)	ω (°)	M (°)	$\dot{T}$ (s/s)	$\dot{e}$ (1/s)
7148824	0.001307	98.5202	277.2638	93.2063	94.36	9.904E-7	-4.936E-10

The position least squares identification residues in *R*, *N* and *T* directions (radial, normal and transverse to the orbit) are shown in Figs.1, 2 and 3.

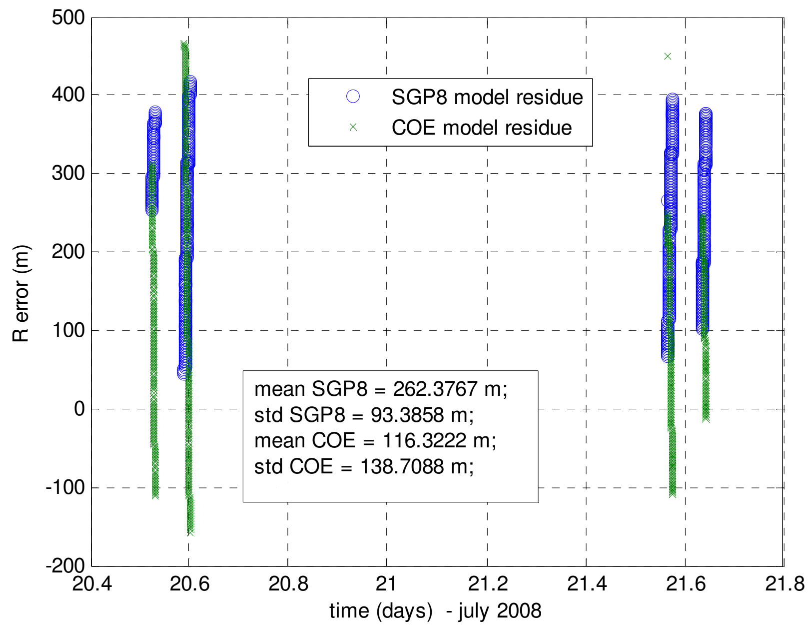

Figure 1. Position residue in radial direction. Residue statistics (mean and standard deviation) are also shown for the two models

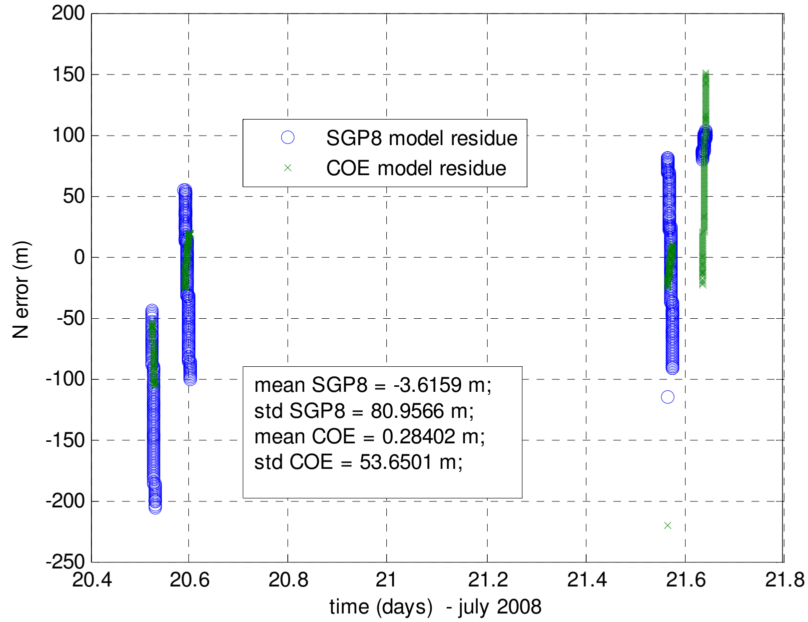

Figure 2. Position residue in normal direction. Residue statistics (mean and standard deviation) are also shown for the two models

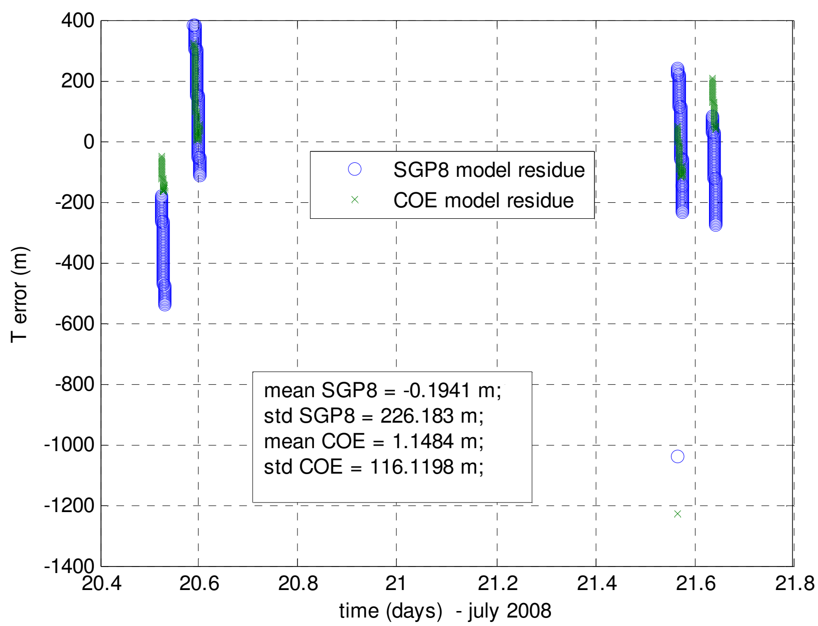

Figure 3. Position residue in transversal direction. Residue statistics (mean and standard deviation) are also shown for the two models

The computed statistics (mean and standard deviation) shown in Figs. 1, 2 and 3 refer to the complete set of data, with the four passes concatenated. It can be seen that residues have some bias, especially in radial direction, where the mean is relatively far from zero. This result is influenced by the sparse *GPS* data used to perform the orbit determination procedure, as mentioned above. The fitting to the identification data is relatively similar if considered all the three directions for the two models.

In addition to the *GPS PVT*, radar data were collected by the *SCC* on 22 and 23 July 2008 by the tracking stations of Cuiabá and Alcântara and an orbit determination procedure using this data was performed in this period for the *CBERS-2B*, called *SCC* solution. On 14: 47: 45.441 *GMT* of day 22 was performed an orbital correction maneuver, raising the orbit in about 40 m, introducing errors of diverse nature that contributed to deteriorate the accuracy of orbit determination computations (Kuga et al., 2009) .

With the 7 *TLE* elements and the 8 *COE* elements obtained from orbit determination, the *SGP8* and *COE* models were used for orbit propagation in the period range where the *SCC* solution via radar is available. The position differences between the two models propagations and *SCC* solution are shown in Figs. 4, 5 and 6 in radial, normal and transverse orbit directions.

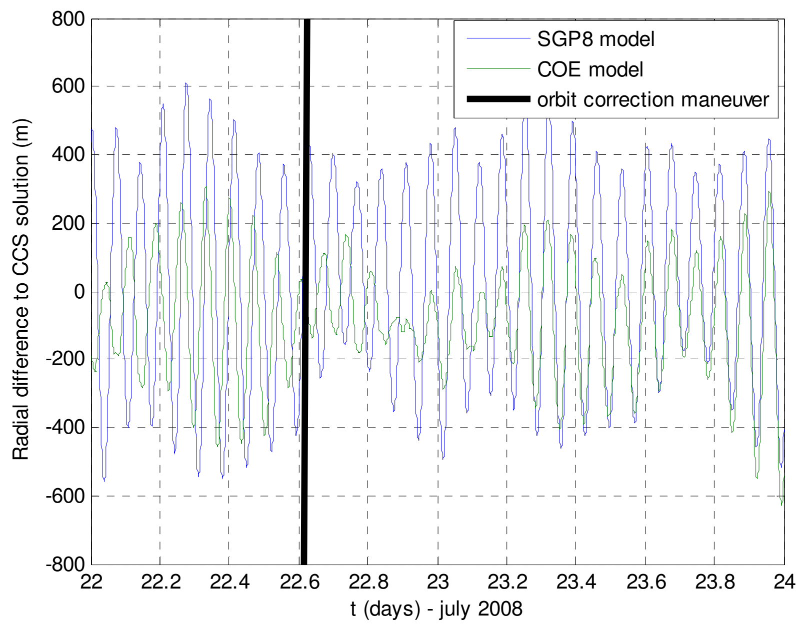

Figure 4. SGP8 and COE models propagation difference to SCC orbit determination solution based on radar data in radial direction

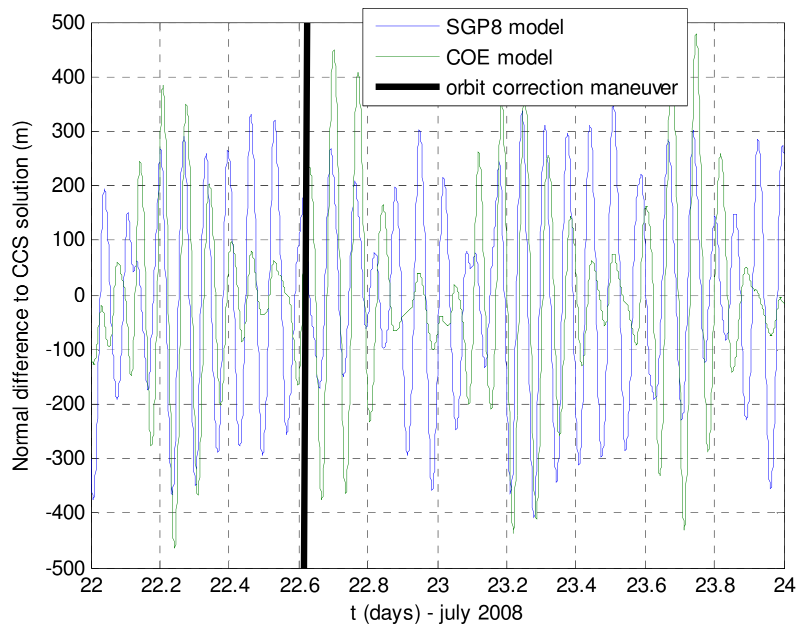

Figure 5. SGP8 and COE models propagation difference to SCC orbit determination solution based on normal data in normal direction

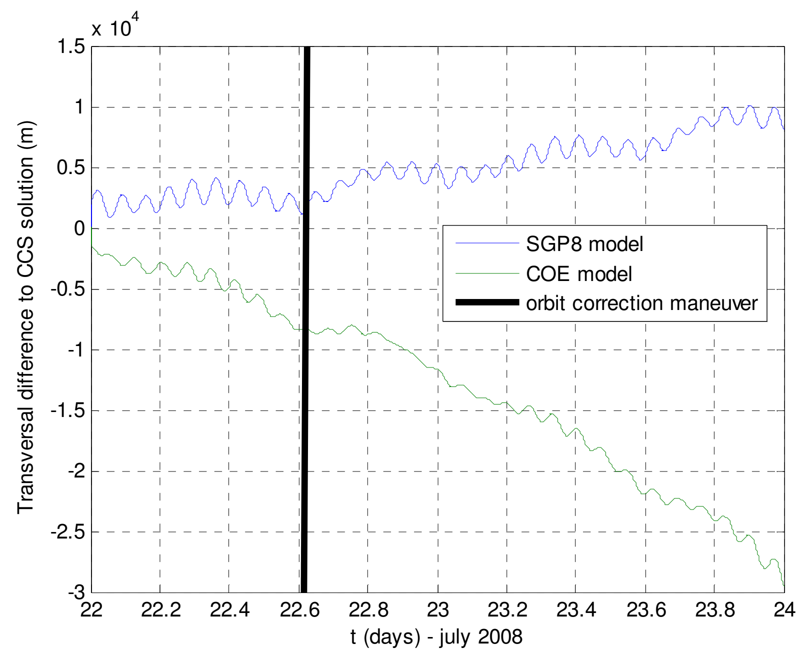

Figure 6. SGP8 and COE models propagation difference to SCC orbit determination solution based on transversal data in radial direction

The execution instant of the orbit correction maneuver carried out by *SCC* is marked in Figs. 4, 5 and 6. In general, the *COE* model has a better agreement with the *SCC* solution than the *SGP8* model in the radial direction, and the opposite occurs in the transverse direction, where the two propagations are divergent, but the *SGP8* diverges in a slower pace.

The influence of the maneuver carried out can be noticed mainly in the transverse direction, due to the nature of the orbit correction performed. From the moment of maneuver, it can be observed a divergent increase from the *SCC* solution. This is more noticeable in the *SGP8* model propagation.

Considering the relatively simplified structure of the *SGP8* and *COE* models, with geopotential zonal terms up to J5 and atmospheric drag model only, it is already expected a divergence in the propagation of these models over time, particularly in the transverse direction. Moreover, the limitation of the data used in the orbit determination and the orbit correction maneuver performed in the propagation time range contribute to introduce errors in the propagation models. Nevertheless, the *SGP8* model propagation would meet the level of accuracy required for tracking operation up to the maneuver time and the *COE* model propagation would meet this requirement until about 10 hours of day 22.

As an exercise, the data from the radar-based *SCC* solution was used as input data to the orbit determination procedures. They have been converted from inertial system to the *ECEF* system (where the *GPS* navigation solutions are referenced), placed in the *GPS PVT* format input and used for orbit determination with the two models. The data fitting are shown in Figs. 7, 8 and 9.

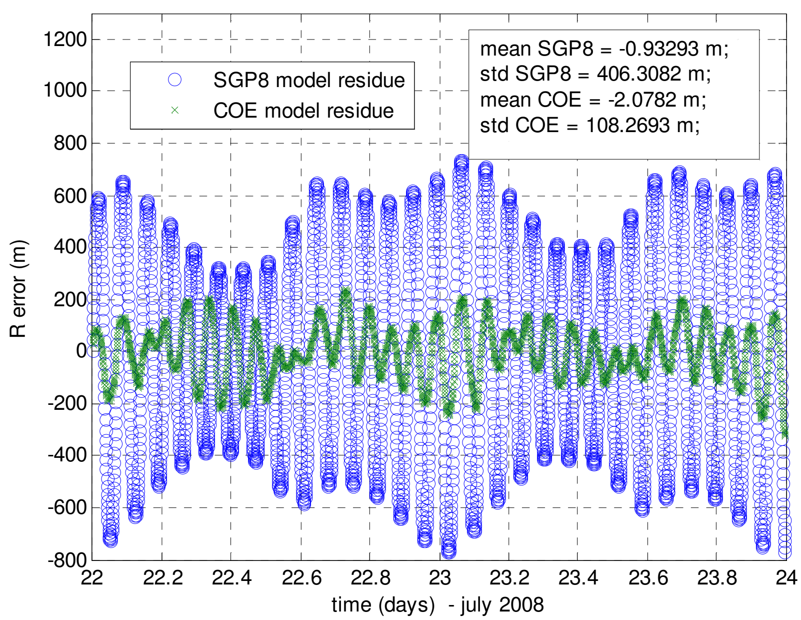

Figure 7. Position residue in radial direction - identification with radar data. Residue statistics (mean and standard deviation) are also shown for the two models

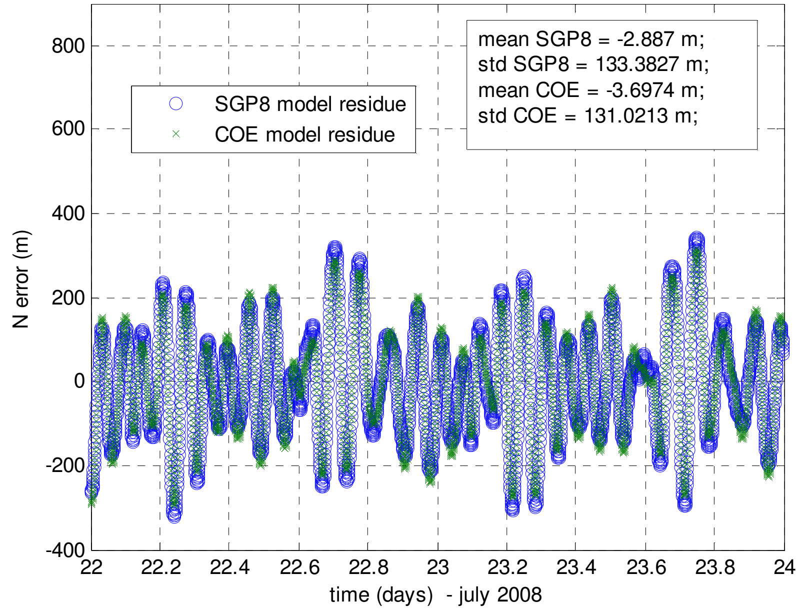

Figure 8. Position residue in normal direction - identification with radar data. Residue statistics (mean and standard deviation) are also shown for the two models

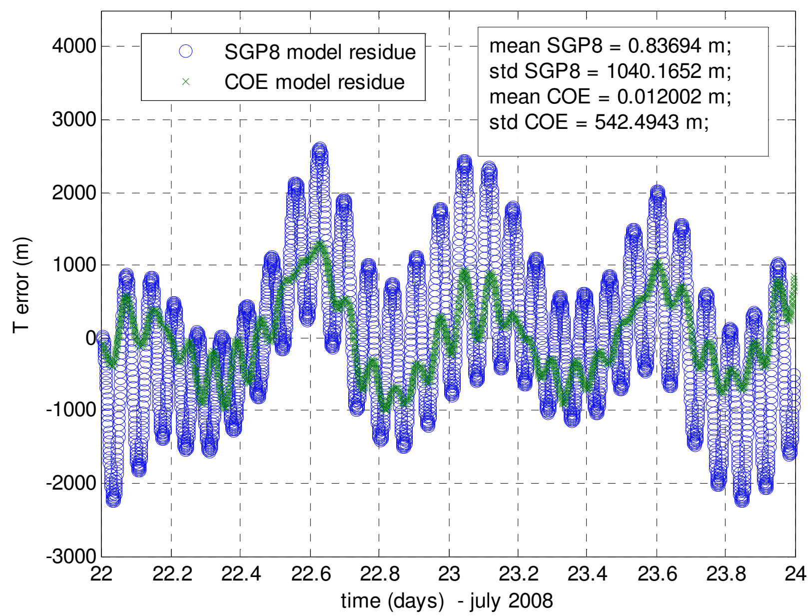

Figure 9. Position residue in transversal direction - identification with radar data. Residue statistics (mean and standard deviation) are also shown for the two models

Compared with the result of identification with the sparse dataset shown in Figs. 1, 2 and 3, it is possible to see more consistency in the identification results when working with the most complete dataset. It can be seen that the means are relatively close to zero and therefore not biased. Moreover, it can be observed that the residues dispersion of the identification with

COE model is less than the residues dispersion of the identification with *SGP8* model, indicating a better capability of the *COE* model for explaining the frozen orbits dynamics.

4 CONCLUSIONS

The use of *CBERS-2B GPS* receiver navigation solutions allowed for an initial validation of the *INPE*'s *SCC* orbit determination system extension to use *GPS PVT*, with the option to use the *SGP8* model for orbital parameters dissemination for image tracking ground stations and to use the *COE* model for internal tasks performed by the mission control center for orbit monitoring and frozen orbit maintenance maneuvers computation.

The two models are obtained from analytical solutions of the disturbed two-body problem, but have different structures due to the difference in their parameterization, which is reflected in the results obtained with *CBERS-2B* data. Despite the limitations of the data used, the level of agreement between the models and the radar-based *SCC* solution meet their application requirements.

Thus, now a computer system is available to work with *GPS PVT* data in the satellites currently in operation from *CBERS* mission and to work with *GPS PVT* data from future missions. It is intended to continue the system evaluation with *CBERS-4* satellite data in future work. The *CBERS-4* has storage capacity of *GPS* ephemeris representing complete orbits. The use of these data will allow the completion of the evaluation made here.

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