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ATTITUDE CONTROL SYSTEM DESIGN FOR A RIGID-FLEXIBLE SATELLITE USING THE H-INFINITY METHOD WITH PARAMETRIC UNCERTAINTY.

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Abstract. *This paper presents the Attitude Control System (ACS) design for a rigid-flexible satellite with two vibrations mode, using the H infinity method considering the parametric uncertainty over the mass matrix. Usually the mathematic model obtained from the linearization and/or reduction of the rigid flexible model loses information about the flexible dynamical behavior and introduces some uncertainty. As a result, the ACS performance can be degraded when controlling large angle maneuvers. One way to recovery the dynamics information is to include in the ACS design the parametric and not parametric uncertainties of the system. The rigid flexible satellite dynamics can be represented by an ordinary differential equation (EDO), which coefficients are the matrices mass, damping and rigidly. In this paper one investigates the influence of the uncertainty in the mass matrix in the control law performance designed by the H infinity control methodology. The results of the simulations have shown that the control law designed using the H-infinity control method considering the parametric uncertainty is robust since the ACS performance has been improved in controlling the rigid flexible satellite attitude and suppressing vibrations.*

Keywords: *ACS, Satellite rigid-flexible, Uncertain, H infinity*

1 INTRODUCTION

The majority of the complex space mission the spacecraft control system can be divided basically in orbit control which can be related to the interplanetary movement (Mainenti, et al, 2016) and attitude control related to the angular rotation around its center of mass. In the ACS design the most well-known techniques of optimal control is the theory of Linear Quadratic Regulator (LQR) (Souza, 2006), which recently, has been generalized for non-linear systems, known as the State Dependent Riccati Equation (SDRE) technique (Souza and Bigot, 2016).

For rigid-flexible satellite, one of the most important requirements of the mission is the accuracy pointing of the satellite. Mainly at the end of the satellite big angle maneuver, since the satellite needs turn around its center of mass (attitude maneuvers) directing one of its faces to a target in space (Sidi, 1997). In order to perform this kind of maneuvers is necessary to design the ACS so that the rotational movement meets the attitude maneuvers while dampen possible vibrations of the panel. As a result, the design of the ACS must take into account the dynamics behavior of the structure such as vibrations of liquids, antennas and panels of the satellite. The flexible appendages vibrations can interact with the satellite rigid motion, during the translational and/or rotational maneuver, damaging the ACS pointing accuracy (Souza and Souza, 2014).

Another problem is to recovery the loss of the dynamics information associated with the inability to obtain the real model, since the model reduction (Pinheiro and Souza, 2014) can introduced some kind of perturbation which can be represented by the parametric and non parametric uncertainties where the first is associated with the parameters variation and the second with the unmodelled dynamics (unstructured uncertainty). In this paper one designs an ACS to perform the attitude maneuver and suppress the panels' vibrations taking into account the uncertainties. To do that one applies the H infinity control method which permits to obtain a robust control law, since this method has the property to incorporate the uncertainties in its formulation (Gasbarri et al., 2014). In particular, the parametric uncertainty considered is in the mass matrix since the flexible motion is dominated by this parameter. The model used is similar to the China Brazil Earth Resources Satellite – CBERS, as shown in Fig.(1a).

2 RIGID-FLEXIBLE DYNAMIC MODEL

To obtain the dynamics equations of the rigid-flexible satellite one admits that the model can be considered by a mechanic analogous like a rotatory beam, which preserves all physics characteristic of the movement (Junkins, 1993).

In Fig.(1b) the rigid central hub has radius R, the actuator rotor has viscous friction b_m , and inertia J_{ROTOR} and develops a torque τ(t); the flexible beam has uniform linear mass density ρ, uniform bending stiffness EI and length L; the flexible beam deformation is $w(x,t)$; and the tip mass is m_{tip} and inertia J_{tip}; $\alpha(t)$ represent the tip angle and $\theta(t)$ is the rigid angle displacement; and $s(x)$ represent the distance between the referential axis to an element of mass in the link.

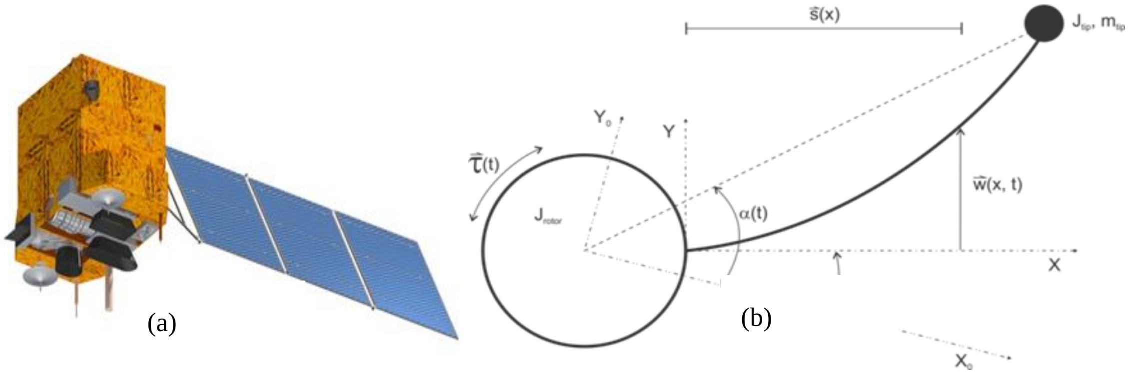

Figure 1. (a) CBERS (b) Satellite Analogous representation.

The flexible link is considered an Euler-Bernoulli beam. Like show in (Souza and Souza, 2015) the equations of the motion was derived using a Lagrangian approach combined with the assumed modes methods, admitting two vibrations modes and a dissipation of energy in a Rayleigh form (Bigot and Souza, 2013). Then the equations linearized of the dynamics are represented in a matrix form:

$$\begin{bmatrix} \mathbf{I}_t & \mathbf{M}_{rf}^T \\ \mathbf{M}_{rf} & \mathbf{M}_{ff} \end{bmatrix} \begin{bmatrix} \theta \\ \ddot{\mathbf{q}} \end{bmatrix} + \begin{bmatrix} b_m & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_{\mathrm{ff}} \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{\mathbf{q}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{\mathrm{ff}} \end{bmatrix} \begin{bmatrix} \theta \\ \mathbf{q} \end{bmatrix} = \begin{bmatrix} \tau \\ \mathbf{0} \end{bmatrix} \tag{1}$$

with,

$$\mathbf{B}_{ff} = k_e EI \int_0^L \boldsymbol{\phi}'' \boldsymbol{\phi}''^T dx\,;\ \mathbf{K}_{ff} = EI \int_0^L \boldsymbol{\phi}'' \boldsymbol{\phi}''^T dx\,;\ \mathbf{K}_{ff} = EI \int_0^L \boldsymbol{\phi}'' \boldsymbol{\phi}''^T dx$$

$$\mathbf{M}_{ff} = \rho \int_0^L \boldsymbol{\phi}\boldsymbol{\phi}^T dx + m_{tip} \boldsymbol{\phi}_L \boldsymbol{\phi}_L^T + \frac{1}{2} J_{tip} \boldsymbol{\phi}'_L \boldsymbol{\phi}'^T_L\,; \tag{2}$$

where $\theta$is the rigid angular displacement, $\mathbf{q}$ the flexible displacement vector and $\boldsymbol{\phi}$ is the function of the modal form. By analogies, the Eq.(1) represent the standard form $\mathbf{M}\ddot{x} + \mathbf{D}\dot{x} + \mathbf{K}x = \mathbf{Q}u$, where the matrix are called $\mathbf{M}$ is the mass matrix, $\mathbf{D}$ is the damping matrix and $\mathbf{K}$ is the rigid matrix. Writing the space state form, one has

$$\begin{bmatrix} \dot{\mathbf{X}}_1 \\ \dot{\mathbf{X}}_2 \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{M}^{-1}\mathbf{Q} \end{bmatrix} \mathbf{u} \tag{3}$$

Considering $\mathbf{X}_1 = \theta$ and $\mathbf{X}_2 = \mathbf{q} = [q_1 \ \ q_2]$ the state's vectors and the control law $\mathbf{u} = -\mathbf{K}_{\mathrm{H}\infty}\,\mathbf{X}$, where $\mathbf{K}_{H_\infty}$ is the gain calculate by the H infinity method.

3 $\mathrm{H_{OO}}$ CONTROL METHOD WITH PARAMETRIC UNCERTAINTY

The H infinity control method have the advantage to inserts structured uncertainty and/or unstructured uncertainty in the design of the controller (Zhou, 1998). The philosophy behind the control method is to found a gain $\mathbf{K}_{\mathrm{H}\infty}$ that minimizing the H infinity norm of the closed loop function (Eq.(6)) in a system subject a generalized plant model $\mathbf{P}$,

$$\left|\left|F_l(\mathbf{P}, \mathbf{K}_{\mathrm{H}\infty})\right|\right|_\infty = \max_\omega \bar{\sigma}\left(F_l(\mathbf{P}, \mathbf{K}_{\mathrm{H}\infty})(j\omega)\right);\ \left|\left|F_l(\mathbf{P}, \mathbf{K}_{\mathrm{H}\infty})\right|\right|_\infty < \gamma \tag{6}$$

$$F_l(\mathbf{P},\mathbf{K}_{\mathrm{H}\infty})\omega = \mathbf{P}_{11} + \mathbf{P}_{12}\mathbf{K}_{\mathrm{H}\infty}(\mathbf{I} - \mathbf{P}_{22}\mathbf{K}_{\mathrm{H}\infty})^{-1}\mathbf{P}_{21} \tag{7}$$

where $F_l(\mathbf{P},\mathbf{K}_{\mathrm{H}\infty})$ is the linear fractional transformation (LFT) of $\mathbf{P}$ and $\mathbf{K}_{\mathrm{H}\infty}$**,** and the γ is a design parameter which the smallest value is associated with the space state realization, such that the Hamiltonian $\mathbf{H}$ (Eq.(8)) has no eigenvalues on the imaginary axis (Skogestad, 2001),

$$\mathbf{H} = \begin{bmatrix} \mathbf{A} + \mathbf{B}\mathbf{R}^{-1}\mathbf{D}^T\mathbf{C} & \mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T \\ -\mathbf{C}^T(\mathbf{I} + \mathbf{D}\mathbf{R}^{-1}\mathbf{D}^T)\mathbf{C} & -(\mathbf{A} + \mathbf{B}\mathbf{R}^{-1}\mathbf{D}^T\mathbf{C})^T \end{bmatrix} \tag{8}$$

where $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ are the space state matrices and $\mathbf{R} = \gamma^2\mathbf{I} - \mathbf{D}^T\mathbf{D}$.

In order to design a robust control law one inserts uncertain ($\mathbf{\Delta}$) in the design procedure so as the conditions of robust stabilization are obtained and the gain $\mathbf{K}_{\mathrm{H}\infty}$ obeys the follow relations (Gu et al., 2005):

$$\min_{\mathbf{K}\,stabilising} \left|\left|\mathbf{\Delta}\mathbf{K}_{\mathrm{H}\infty}(\mathbf{I} + \mathbf{G}\mathbf{K}_{\mathrm{H}\infty})^{-1}\right|\right|_\infty ;\ \left|\left|\mathbf{\Delta}\mathbf{K}_{\mathrm{H}\infty}(\mathbf{I} + \mathbf{G}\mathbf{K}_{\mathrm{H}\infty})^{-1}\right|\right|_\infty < 1 \tag{9}$$

3.1 Robust Controller Design

The uncertainty of the mass matrix ($\mathbf{M}$) can be introduced re-writing Eq.(4) in the follow form:

$$\bar{\mathbf{M}}(\mathbf{I} + p_m\boldsymbol{\delta}_m)\ddot{x} + \mathbf{D}\dot{x} + \mathbf{K}x = \tau \tag{10}$$

where $\bar{\mathbf{M}}$ is the mass nominal value, p_m is the uncertain values and $\boldsymbol{\delta}_m$ is the amplitude of the uncertain $(-1 \le \boldsymbol{\delta}_{\mathrm{m}} \le 1)$. The matrix $\bar{\mathbf{M}}^{-1}$ can be represented by a linear fraction transformation (LFT) in δ_m , like:

$$\bar{\mathbf{M}}^{-1} = \bar{\mathbf{M}}^{-1}(\mathbf{I} + p_m\boldsymbol{\delta}_m)^{-1} = F_U(M_{mi}, \boldsymbol{\delta}_m) \tag{11}$$

The block diagram of the Fig. 2 shows how the uncertainty acts as a perturbation in the system interconnections.

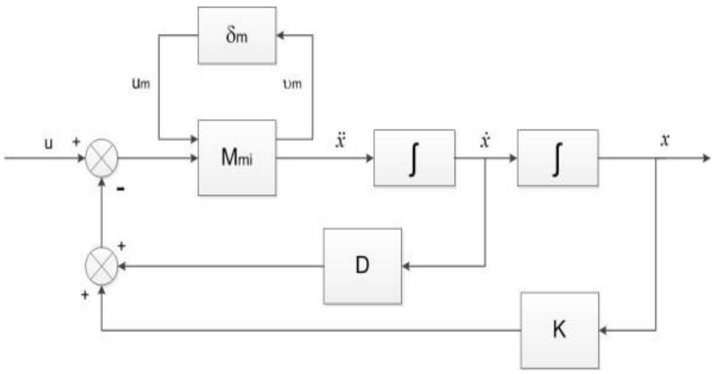

Figure 2. Block diagram of the system with uncertainty

The uncertain in this problem actuate in the matrices $\mathbf{M}$. Than the uncertain amplitude is given by $\boldsymbol{\delta}_m$. One have 27 possibilities of $\boldsymbol{\delta}_m$, or else, we have 27 cases of uncertainties for each matrix $\mathbf{M}$.

4 SIMULATIONS RESULTS

Figure 3 shows the angular displacement and its variation in time when designing a control law by the H infinity control method; for an initial condition of the link is $\theta = 10^0$, taking into account the 27 possible cases of uncertainties. An important observation is the controller gain chance for every case of uncertain, until the H_∞ control method find the most robust gain as possible. All simulations have been done for a time interval of 180s. The constant parameter used in the simulation are: R=0.05m, b_m=0.15 m^2/s, J_{rotor}= 0.3kgm^2, L=1m, ρ=2700 kg/m, ke=0.03, EI=18.4Nm^2,m_{tip}=0.25kg and J_{tip}=0.04kgm^2. One obtains:

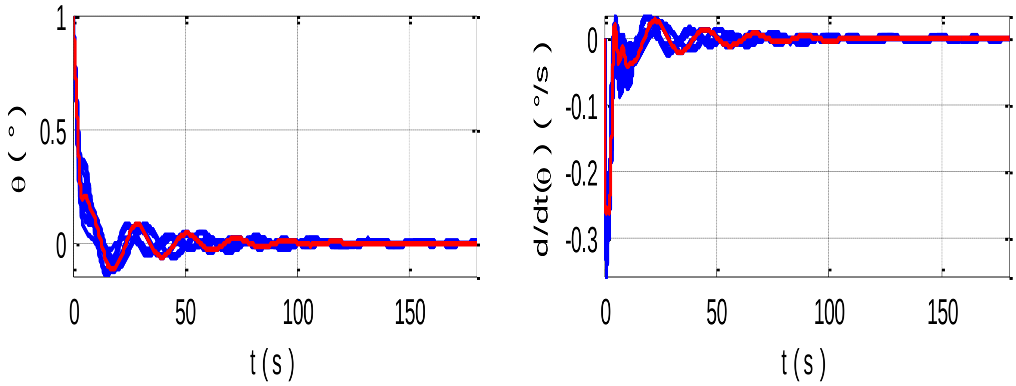

Figure 2. The angle θ and velocity $\dot{\theta}$, where red and blue lines are the nominal and the uncertainty cases.

Figure 4 shows the flexible displacement and its variation in time for the same control law and for same initial condition of the link is $\theta = 10^0$ for all the 27 uncertainties possible combinations. One observes that the attitude angle and the angular velocity return to the equilibrium position in approximately 100s.

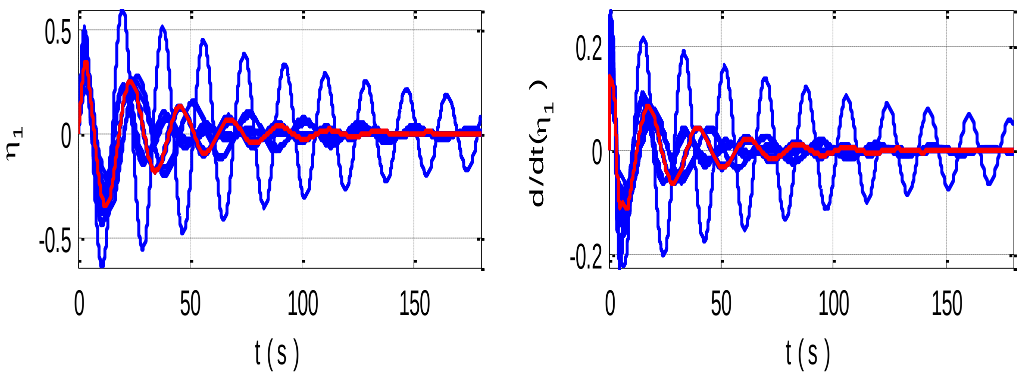

Figure 3. The flexible mode q_1 and rate $\dot{q_1}$ - red and blue lines are the nominal and the uncertainty cases.

For the first mode it’s clear that for some the controller designed has not able to suppress the vibration.

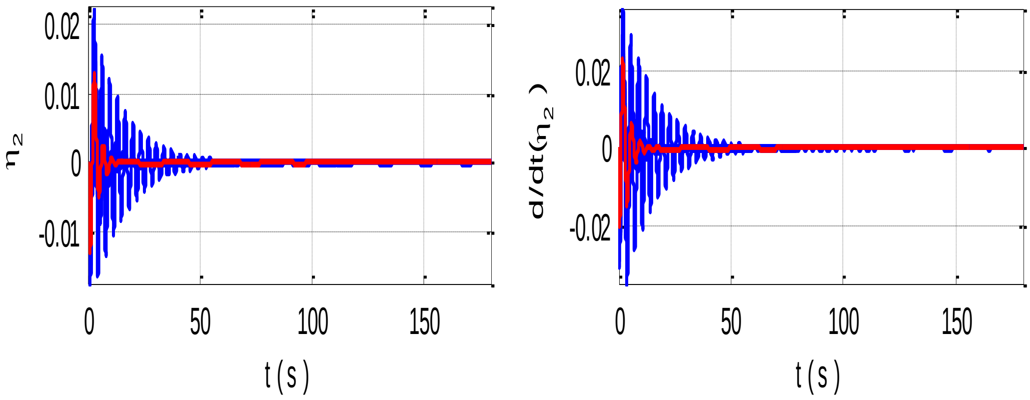

Figure 4. Variation of the second flexible mode q_2 and its rate $\dot{q}_2$, the red line is the nominal case and the blue lines are the uncertainty cases.

For the second mode the controller designed has able to control and suppress the vibration in approximately 70s for all cases.

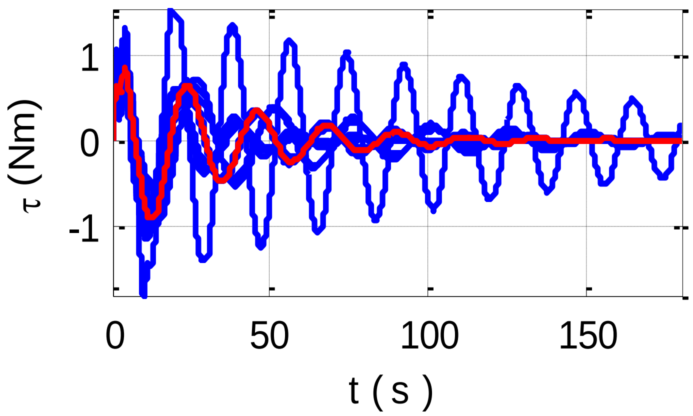

Figure 5. Response of the control effort τ, the red line is the nominal case and the blue lines are the uncertainty cases.

The response of the action of the torque shows that for some 11 cases the system archive the equilibrium in 180s.

5 CONCLUSIONS

The results of the simulations shows that the control law design, for the rigid flexible satellite, was able to deal with the parametric uncertainty, under the mass matrix, controlling the system and returning it to an equilibrium position in approximately 180s for 56% cases and for the other cases the rise time was biggest. In face of that, one can say that the ACS designed by H-infinity method proposed was robust enough in the presence of the uncertainty and manage to suppress the vibrations. In other words, the control law design was able to calculate new gains that provided a good response avoiding the destabilization of the system.

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