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A PARAMETRIC ANALYSIS OF AN OCEAN WAVE ENERGY HARVESTING SYSTEM

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Abstract. In this paper, we present some parametric analysis of an ocean wave energy harvesting system. The mathematical model of a floating ocean platform attached to a DC motor is considered to use the ocean waves to generate electrical energy. A pendulum was coupled to the DC motor axis, transforming the vertical excitation of the floating platform into rotation movements of the generator, then varying the magnetic flux of the permanent

magnets of the motor, generating electric energy. In order to optimize the power generation, we present a study of mass settings of the pendulum and an ocean wave amplitudes analysis, based on Maranhão's coast characteristics. The dynamic behavior of the system is shown through numerical simulations, as well as the efficiency of conversion of the pendulum swings in electricity.

Keywords: Pendulum System, Mathematical Modeling, Energy Harvesting, Ocean Wave Energy.

1 INTRODUCTION

One of the great challenges of contemporary society is, undoubtedly, the discovery and economically viable exploitation of abundant and renewable energy sources. The global energy crisis is practically constant, given by the progressive increasing use of electrical and electronic devices by the modern man.

According to Burns et al. (2015), the first project of a device capable of transforming one kind of energy into another, it was a waterwheel. From there, both technology and engineering evolve drastically.

Researchers, through the time, that kept developing devices to explore the environmental energies and the ones that investigate new renewable energy sources divided their attentions searching the best way to explore and the higher potential in each kind of available energy sources (thermal, kinect, potential, magnetic, etc...).

Wave energy, a source of renewable energy, has the advantages of a high energy density and persistence and, therefore, is a competitive candidate for energy supply (Zhongxian et al., 2016).

The use of energy from vibration provided by pendulum systems can be a good application in ocean wave energy harvesting, by using the movement of the waves as an excitation source, for example, in a floating pendulum system.

Many different types of wave energy converters of different categories have been proposed.

Wiercigroch (2005) introduced a mathematical model for energy extraction from a parametric pendulum system whose base is excited, and investigate the dynamic of this system. Xu (2005) and Xu and Wiercigroch (2007) continue the work started in Wiercigroch (2005), investigating the possibilities of explore the rotations of the coupled pendulum system to maximize its velocities, aiming to a higher energy harvest.

In Ogai et al. (2010), the authors describe a new system of compressed air generation, through a pendulum oscillation power converter of the waves, installed in a coastal defense structure. One can further use the compressed air for power generation, for example using a pneumatic motor generator.

In Wiercigroch et al. (2011), the authors revisited the dynamic of a parametric oscillating system operating in rotation for the purpose of energy capture. The main idea of these authors, were based on the conversion of oscillatory motion into an oscillating rotation, as suggested Falnes (2007).

In Lenci and Rega (2011), an experimental apparatus was built to simulate the production of energy from the pendulum swings; it was possible to compare the results of experimental and simulations. The main contribution of this work is that the authors found that, in practice, there are rotations only in ranges where a measure of integrity of the dynamic system is large enough to support the experimental imperfections that lead to changes in initials conditions.

Nandakumar et al. (2012), optimize the dynamics worked by Wiercigroch et al. (2011), using a torque control for the pendulum system, so that it can work with great rotation periods for the harmonic excitation of the base of their pendulum system. The optimal solutions proposed by these authors proved to be about 25% better than the answers without torque control of that system.

The principle of a pendulum oscillation energy converter was introduced in Lin et al. (2013), and dual-medium pressurizer designs and dual-stroke hydraulic system are emphasized.

In this work, we approach a way to explore the movement of a parametrically excited pendulum to the vibrational energy extraction, as we did in Dos Santos et al. (2015), but we introduce here a parametric analysis in order to optimize the energy harvest. We use a pendulum to transfer movement of vertical oscillation of the base for rotational displacement, and the pendulum structure was coupled to a DC generator. Thus, the speeds developed by the pendulum are used to generate electricity and the energy harvested by the system is proportional to the rotational speed of the pendulum. In order to maximize the energy harvesting of the system, we made a parametric analysis of the pendulum mass, and the ocean wave amplitude based on Maranhão's coast characteristics.

The organization of this paper is as follows. Section 2 contains the mathematical modelling and the proposal of energy generating and harvesting. In Section 3 numerical simulations necessary to analyze the dynamics of the system are performed. Conclusions are given in Section 4.

2 MATHEMATICAL MODELLING

The representation of the pendulum coupled to an onboard generator model is shown in Fig. 1.



Figure 1. Floating energy harvesting system excited by a harmonic force

The model shown in Fig. 1, it is conceived as a pendulum coupled to a DC generator, fixed to a floating platform. The pendulum mass m, with a massless rod of length l, θ is the angle of the pendulum. The generator has mass M_2 , and the platform mass M_1 . We consider k as the fluid thrust reactions and the linear viscous damping of the fluid is represented by c. The angular viscous damping coefficient is represented by c_{θ} ; Y represents the vertical displacement of the floating platform and $F = A \sin(\omega t)$ the excitation source of the base. The dynamic equations of the system are obtained from de Lagrangian formulation. Generalized Coordinates:

For Floating Platform:

$$\begin{cases} X_1 = 0 \Longrightarrow X_1 = 0 \\ Y_1 = Y \Longrightarrow \dot{Y_1} = \dot{Y} \end{cases}$$
(1)

For Pendulum System:

$$\begin{cases} X_2 = l\sin(\theta) \Rightarrow \dot{X}_2 = l\dot{\theta}\cos(\theta) \\ Y_2 = Y + l\cos(\theta) \Rightarrow \dot{Y}_2 = \dot{Y} - l\dot{\theta}\sin(\theta) \end{cases}$$
(2)

Kinetic Energy of the System:

$$T = \frac{1}{2} [M_1 (\dot{Y}_1)^2 + M_2 (\dot{Y}_1)^2] + \frac{1}{2} [m (\dot{X}_2^2 + \dot{Y}_2^2)]$$
(3)

$$\therefore T = \frac{1}{2} (M_1 + M_2 + m) \dot{Y}^2 + \frac{1}{2} m l^2 \dot{\theta}^2 - m \dot{Y} l \dot{\theta} \sin(\theta)$$
(4)

Potential Energy of the System:

$$V = mgY_2 + \frac{1}{2}kY_1^2$$
 (5)

$$\therefore V = mgY + mgl\cos(\theta) + \frac{1}{2}kY^2$$
(6)

From Eq.(4) and Eq.(6), we obtain the Lagrangian function, \mathcal{L} :

$$\mathcal{L} = T - V = \frac{1}{2} (\mathbf{M}_1 + \mathbf{M}_2 + \mathbf{m}) \dot{\mathbf{Y}}^2 + \frac{1}{2} \mathbf{m} l^2 \dot{\theta}^2 - m \dot{Y} l \dot{\theta} \sin(\theta) - mgY + mgl\cos(\theta) - \frac{1}{2} kY^2$$
(7)

The Rayleigh's Function is defined by Eq.(8):

$$D = \frac{1}{2}c_{\theta}\dot{\theta}^{2} + \frac{1}{2}c\dot{Y}^{2}$$
(8)

From Eq.(7) and Eq.(8), we obtain the dynamic equations of the system.

$$\begin{cases} (\mathbf{M}_{1} + \mathbf{M}_{2} + \mathbf{m}) \ddot{\mathbf{Y}} - \ddot{\theta}ml\sin(\theta) - \dot{\theta}^{2}ml\cos(\theta) + mg + kY + c\dot{\mathbf{Y}} = A\sin(\omega t) \\ m l^{2}\ddot{\theta} - \ddot{\mathbf{Y}}ml\sin(\theta) + mgl\sin(\theta) + c_{\theta}\dot{\theta} = 0 \end{cases}$$
(9)

By simplifying the system shown in Eq. (9), we have:

$$\begin{cases} \ddot{\mathbf{Y}} - \frac{m}{(\mathbf{M}_{1} + \mathbf{M}_{2} + \mathbf{m})} l \sin(\theta) \ddot{\theta} - \frac{m}{(\mathbf{M}_{1} + \mathbf{M}_{2} + \mathbf{m})} l \cos(\theta) \dot{\theta}^{2} + \frac{m}{(\mathbf{M}_{1} + \mathbf{M}_{2} + \mathbf{m})} g + \frac{k}{(\mathbf{M}_{1} + \mathbf{M}_{2} + \mathbf{m})} Y + \frac{c}{(\mathbf{M}_{1} + \mathbf{M}_{2} + \mathbf{m})} \dot{Y} = \frac{A \sin(\omega t)}{(\mathbf{M}_{1} + \mathbf{M}_{2} + \mathbf{m})} \\ \frac{\ddot{\theta} - \frac{\sin(\theta)}{l} \ddot{Y} + \frac{g}{l} \sin(\theta) + \frac{c_{\theta}}{\mathbf{m} l^{2}} \dot{\theta} = 0 \end{cases}$$
(10)

In order to transform Eq.(10) into a dimensionless system, we consider:

$$\mu_{1} = \frac{c}{(M_{1} + M_{2} + m)\omega_{1}} \qquad \gamma_{1} = \frac{m}{M_{1} + M_{2} + m} \qquad G_{1} = \frac{mg}{(M_{1} + M_{2} + m)\omega_{1}^{2}l}$$

$$\mu_{\theta} = \frac{c_{\theta}}{ml^{2}\omega_{1}} \qquad \gamma_{2} = \frac{g}{\omega_{1}^{2}l} \qquad F_{0} = \frac{A}{(M_{1} + M_{2} + m)\omega_{1}^{2}l} \qquad \Omega = \frac{\omega}{\omega_{1}} \qquad (11)$$

$$\omega_{1} = \sqrt{\frac{k}{M_{1} + M_{2} + m}} \qquad x = \frac{Y}{l} \qquad \tau = \omega_{1}t$$

Rewriting the equations in Eq.(10), we have:

$$\begin{cases} x'' + \mu_1 x' + x - \gamma_1 \left(\theta'' \sin \theta + {\theta'}^2 \cos \theta \right) + G_1 = F_0 \sin \Omega \tau \\ \theta'' + \mu_\theta \theta' + (\gamma_2 - x'') \sin \theta = 0 \end{cases}$$
(12)

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Isolating the accelerations, we obtain:

$$\begin{aligned} x'' &= F_0 \sin \Omega \tau - \mu_1 x' - x - \frac{\gamma_1 \mu_0 \theta' \sin \theta}{(1 - \gamma_1 \sin^2 \theta)} - \frac{\gamma_1 \gamma_2 \sin^2 \theta}{(1 - \gamma_1 \sin^2 \theta)} + \frac{F_0 \gamma_1 \sin^2 \theta \sin \Omega \tau}{(1 - \gamma_1 \sin^2 \theta)} + \\ &- \frac{\gamma_1 \mu_1 x' \sin^2 \theta}{(1 - \gamma_1 \sin^2 \theta)} - \frac{\gamma_1 x \sin^2 \theta}{(1 - \gamma_1 \sin^2 \theta)} + \frac{\theta'^2 \gamma_1^2 \sin^2 \theta \cos \theta}{(1 - \gamma_1 \sin^2 \theta)} - \frac{G_1 \gamma_1 \sin^2 \theta}{(1 - \gamma_1 \sin^2 \theta)} + \theta'^2 \gamma_1 \cos \theta - G_1 \end{aligned}$$
(13)
$$\theta'' &= -\frac{\mu_0 \theta'}{(1 - \gamma_1 \sin^2 \theta)} - \frac{\gamma_2 \sin \theta}{(1 - \gamma_1 \sin^2 \theta)} + \frac{F_0 \sin \theta \sin \Omega \tau}{(1 - \gamma_1 \sin^2 \theta)} - \frac{\mu_1 x' \sin \theta}{(1 - \gamma_1 \sin^2 \theta)} - \frac{x \sin \theta}{(1 - \gamma_1 \sin^2 \theta)} + \\ &+ \frac{\theta'^2 \gamma_1 \sin \theta \cos \theta}{(1 - \gamma_1 \sin^2 \theta)} - \frac{G_1 \sin \theta}{(1 - \gamma_1 \sin^2 \theta)} \end{aligned}$$

$$\begin{bmatrix} x'' = \left[\frac{1}{1 - \gamma_1 \sin^2 \theta}\right] \left[-\mu_{\theta} \gamma_1 \theta' \sin \theta - \gamma_1 \gamma_2 \sin^2 \theta - \mu_1 x' - x - G_1 + \gamma_1 {\theta'}^2 \cos \theta + F_0 \sin \Omega \tau\right]$$
(14)
$$\theta'' = \left[\frac{1}{1 - \gamma_1 \sin^2 \theta}\right] \left[-\mu_{\theta} \theta' - \gamma_2 \sin \theta + \left(-\mu_1 x' - x + \gamma_1 {\theta'}^2 \cos \theta - G_1 + F_0 \sin \Omega \tau\right) \sin \theta\right]$$

From Eq.(13), we can rewrite the system in state space, Eq.(15), considering Eq.(14), below.

$$x_1 = x; \quad x_2 = x'; \quad x_3 = \theta; \quad x_4 = \theta';$$
 (15)

$$\begin{cases} x_{1}' = x_{2} \\ x_{2}' = \left[\frac{1}{1 - \gamma_{1} \sin^{2} x_{3}}\right] \left[-\mu_{\theta} \gamma_{1} x_{4} \sin x_{3} - \gamma_{1} \gamma_{2} \sin^{2} x_{3} - \mu_{1} x_{2} - x_{1} - G_{1} + \right] \\ + \gamma_{1} x_{4}^{2} \cos x_{3} + F_{0} \sin \Omega \tau \\ x_{3}' = x_{4} \end{cases}$$
(16)
$$x_{4}' = \left[\frac{1}{1 - \gamma_{1} \sin^{2} x_{3}}\right] \left[-\mu_{\theta} x_{4} - \gamma_{2} \sin x_{3} + \left(-\mu_{1} x_{2} - x_{1} + \right) + \gamma_{1} x_{4}^{2} \cos x_{3} - G_{1} + F_{0} \sin \Omega \tau \right) \sin x_{3} \right]$$

2.1 Proposal of Energy Generating and Harvesting

We can consider, for power generation effect, a system formed by the pendulum attached to the permanent magnet generator motor shaft, which is fixed to the floating platform. The pendulum will transfer the vertical oscillation of the waves for an angular displacement and velocity, in order to vary the magnetic flux and generate electric energy. We can consider that the generation of electric energy is proportional (75% - 100%) to the speed (rpm) of the generator, which is the same as the pendulum system.

Thus, we can define the following equation to calculate the energy for different speeds (rpm).

$$P_e = \frac{3}{4} \cdot \frac{P_n R_v}{R_n} \tag{17}$$

where: P_e is the estimated electric power, P_n is the nominal electrical power, supplied by the manufacturer in Watts; R_v is the variation of speed in rpm, and R_n is the nominal rotation, supplied by the manufacturer in rpm. We can obtain R_v by the RMS formulation, given by Eq. (17):

$$R_{v} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} R_{i}^{2}}$$
(18)

3 . NUMERICAL SIMULATIONS

The dynamic behavior of the system is analyzed through numerical simulations, using a Dormand-Prince based method of resolution (ode45), a 4th order Runge Kutta integrator, the algorithms were done in Matlab. The analysis occurred through phase-plans, responses in time, Poincaré Maps, Parameters Variation and Bifurcation Diagrams.

3.1 Dynamic Behavior of the System without the Generator

Considering the system formed by the platform and the pendulum, coupled through a shaft with bearing, disregarding mass and internal friction in this study: m = 50 kg and l = 0.7 m.

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Figure 2. Floating pendulum platform excited by a harmonic force

For numerical simulations, the parameters are: $M_1 = 200$ Kg, g = 9.81 m/s², k = 1800 Nm and c = 0.5 Ns/m.

The data below are confirmed by observations made in Golfão of Maranhão, Brazil, assigning values for the wave amplitude between 0.6 and 1.4 m, with overall average of 1 m (Feitosa, 1989). According to the author, the data on the height of the waves are grouped into classes with intervals of 0.6 to 0.8 m 0.9 to 1.1 m, and 1.2 to 1.4 m. The periods of waves oscillate between 11 and 19 sec., with relatively small amplitude. The average spread between 13.5 and 16.5 and the overall average is 15.3 crests (El-Robrini et al., 2006).

The external excitation representing the movements of the sea were adapted from Carvalho (2010) and El-Robrini *et al.* (2006), considering the amplitude A = 1 m and angular frequency $\omega = 1/19 \text{ s}^{-1}$.

It was considered $3 \cdot 10^5$ seconds of integration time and 25% of the final vectors were considered as the steady state of the system.



Figure 3. Vertical Displacement of the System: [A] Time Response; [B] Phase-Plan; [C] Phase-Plan in Steady State with Poincaré Map (red)

In Fig. (3), we can observe that the vertical displacement of the Floating Platform presents irregular behave, with low amplitudes of displacement, but in steady state, the vertical movement present periodic behavior, as can be seen in Fig.(3)-C.



Figure 4. Angular Displacement of the System: [A] Time Response; [B] Phase-Plan; [C] Phase-Plan in Steady State with Poincaré Map (red)

According to Fig (4), the pendulum system presents a very irregular behavior in the transient, with both high angular velocities and displacements, which is a good sign for the energy harvesting goal, but when in steady state, the system presents quasiperiodic behavior with very low amplitudes of displacement and velocities, negatively affecting the energy harvesting.

We can also observe in Figs (3) and (4) that the system has long transient periods and evolve to steady state very slowly, because of the low magnitude of both frequency and amplitude of the excitation source of the system.

3.2 Dynamic Behavior of the Energy Harvesting System

For energy harvesting, it was considered a permanent magnet generator, commercial model, of 1000 W generation capacity at 200 rpm, with mass $M_2 = 28$ kg and inertial torque $c_a = 0.15$ Nm.

It was considered $1.65 \cdot 10^5$ seconds of integration time and 25% of the final vectors were considered as the steady state of the system.



Figure 5. Vertical Displacement of the Energy Harvesting System: [A] Time Response; [B] Phase-Plan; [C] Poincaré Map

In Fig. (5), the Floating Platform shows maintain the periodic behavior with lower amplitudes of displacements, when compared with Fig. (3), evolving to even lower ones with the time.

We can also observe that, when considering the generator in the system, the trajectories evolve to steady state much faster.

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Figure 6. Angular Displacement of the Energy Harvesting System: [A-I] Time Response; [A-II] Time Response (ZOOM) [B] Phase-Plan; [C] Poincaré Map

As can be seen in Fig. (6), the pendulum system maintain higher displacements and velocities in the transient (Fig.(6)-[A-II]), which confirm the thesis of the higher harvest in the transient trajectories. According to Fig (6), the system initiate its orbits with great irregular trajectories evolving to lower amplitudes, presenting quasiperiodic behavior in steady state.



Figure 7. Time Evolution of the RPM of the Generator: [A] RPM full evolution in time [B]RPM evolution at first 1600s

Finally, in Fig. (7)-[A], which illustrates the instantaneous rotation of the generator, black, and the rms rotation, red, we can observe the evolution of the energy generation of the

system. Making use of a commercial generator with 1000 W of nominal power, a nominal speed of 200 rpm and inertial torque of 0.15 Nm, the system generated an estimated output of about 8.3 rms amount of energy, considering full vector. On the other hand, we had observed that the system evolve to steady state very slowly, which makes us to consider the transient trajectories for the energy harvesting, Fig.(7)-[B]. According to the transient analysis, the system indicates approximately 83.5 rms amount of power harvest in the initial transient, which indicates about 1000% more efficiency in harvesting.

In order to investigate the optimal parameters values for the power generation and what contribution which one could make into system dynamics, we made some analysis based in parameters variation.

According to El-Robrini *et al.* (2006), the coast of Maranhão, Brazil, presents waves with amplitudes in a range of 0.60 m to 1.40 m, in a period of 11s to 19 s. Based on these data, we made the following analysis.



Figure 8. Variation of the Amplitude of the Excitation Source and the response in : [A] Vertical displacement amplitudes; [B] Angular displacement amplitudes

According to Fig.(8), it can be seen that the system maintains its dynamic characteristics both in the vertical as the angular displacements, being periodic and quasiperiodic respectively, even with the variation of the amplitude of the excitation force.



Figure 9. Variation of the Frequency of the Excitation Source and the response in : [A] Vertical displacement amplitudes; [B] Angular displacement amplitudes

According to Figure 9, we realized analogously to the analysis described in Figure 8, that the system maintained its intact dynamic behavior, i.e., with the variation of the excitation frequency, the vertical component remained periodically while its angular component remained the quasiperiodic behavior.



Figure 10. Parameters Variation Analysis and the response in Power Harvested: [A] Amplitude of the Excitation Source; [B] Frequency of the Excitation Source; [C] Pendulum mass of the System

In Fig.(10) we can observe the parametric variation analysis of the system, considering the transient trajectories for energy harvesting. According to Fig.(10)-[A] and [B], the variation of the amplitude and the frequency of the excitation source, lightly interfere in the maximum energy harvesting.

Whereas the system begins collecting about 83.5 rms amount of energy, and the variation of these parameters, amplitude and frequency, have maximum power harvested in about 85 rms amount of energy, we can see a gain of approximately 2%. On the other hand, it can be seen that the mass variation of the pendulum system improves large gains in energy harvesting, with a jump of 83.5 rms amount of energy for a possibility to harvest approximately 143.3 rms amount of energy, which illustrates a gain of 171.6%.

4 CONCLUSIONS

In In this work was used numerical analysis to study the behavior of a floating pendulum platform attached to a DC generator. As can be seen in figures (3), (4), (5) and (6), the system with or without the generator, present periodic movement in the vertical displacement and

quasiperiodic behavior in the angular displacement. The major differences between both systems are the amplitudes of displacement. The system with the generator present lower amplitudes of displacement and evolve to steady state faster than the one without the generator. It may also be noted that the small amplitude of the frequency of the excitation source of the system, makes it presents quite long transients, providing a more efficient energy harvesting in the transient state. According to Fig.(7)-[A] and [B], the system can harvest between 8.3 rms and 83.5 rms amount of energy, considering a commercial generator with 1000 W of nominal power, a nominal speed of 200 rpm and inertial torque of 0.15 Nm.

In order to investigate the optimal parameters values for the power generation and what contribution which one could make into system dynamics, we made three analysis based in parameters variation.

According to Figs.(8) and (9), it can be seen that the system maintains its dynamic characteristics both in the vertical as the angular displacements, being periodic and quasiperiodic respectively, even with the variation of the amplitude and the frequency of the excitation force.

According to El-Robrini *et al.* (2006), the coast of Maranhão, Brazil, presents waves with amplitudes in a range of 0.60 m to 1.40 m, in a period of 11s to 19 s. Based on these data, we made variation analysis in these ranges, which resulted in lightly interference of these parameters in the maximum amount of energy harvested (approximately 2%), Fig.(10)-[A] and [B].

On the other hand, it can be seen, figure (10)-[C], that the mass variation of the pendulum system improves large gains in energy harvesting, with a jump of 83.5 rms amount of energy for a possibility to harvest approximately 143.3 rms amount of energy, which illustrates a gain of 171.6%.

The main idea to continue to explore this problem is consider the electrical part of the generator in the system and validate the mathematical model building and comparing the results to a physical model.

4.1 Responsibility Notice

"The authors are the only responsible for the content of this work".

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