PERIODICITY IN A HARMONICALLY EXCITED DAMPED PENDULUM

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Abstract.

We study, in this paper, the nonlinear dynamics of a damped and forced pendulum. This simple model can represent robotic arms, antennas and space solar panels, energy harvesting devices of vibrations present in waves etc.

The response of this system has a wealth of possible behaviors, depending on model parameters, initial conditions and the amplitude and frequency of loading. The answers may result periodic, of several different periods, almost periodic, chaotic etc.

This work intends to make a numerical parametric study. The problem is mathematically modeled by an ordinary differential equation obtained by Newton's laws. The evaluation of the response and the characterization of its stability is given by numerical integration of this mathematical model by Runge-Kutta 4th order algorithm, implemented in MATLAB environment.

In this paper, we show an interesting aspect of the dynamic behavior of this model, namely periodic damped free vibration responses depending on certain parameters and initial conditions. Some preliminary periodic forced responses are also shown.

Keywords: *Nonlinear dynamics, damped and forced pendulum, periodic behavior.*

1 INTRODUCTION

We study, in this paper, the nonlinear dynamics of a damped and forced pendulum represented in Fig. 1. This simple model can represent robotic arms, antennas and space solar panels, energy harvesting devices of vibrations present in sea waves etc.

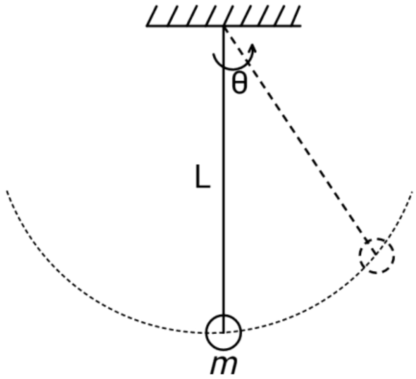

Figure 1. Damped and forced pendulum

The response of this system has a wealth of possible behaviors, depending on model parameters, initial conditions and the amplitude and frequency of loading. The answers may result periodic, of several different periods, almost periodic, chaotic etc.

This work intends to make a numerical parametric study. The problem is mathematically modeled by differential equations obtained by Newton's Laws. The evaluation of the response and the characterization of its stability is given by numerical integration of this mathematical model by Runge-Kutta 4th order algorithm, implemented in MATLAB environment.

A geometric study is carried out to detect possible periodic damped free vibration behavior depending on adequate choice of model parameters and initial conditions. Some preliminary periodic forced simulations are also shown.

This study is a Scientific Initiation exercise based on material by, among others, BRASIL, CLOUGH and PENZIEN, FETTER and WALECKA, GUCKENHEIMER and HOLMES, JACKSON, LAKSHMANAN and RAJASEKAR, MAZZILLI and BRASIL, MEIROVITCH, OTT, RASBAND, SAVI, STROGATZ, TABOS, THOMPSON and HUNT.

2 MATHEMATICAL M ODEL

Our physical model is represented in Fig. 1. It is a mass *m* bob connected to an *L* length pinned massless rigid rod. The generalized coordinate of the model is the angular displacement of the rod. Some friction is considered in form of linear viscous damping. The model also considers an exciting torque applied to the pinned joint.

Quantities used are:

L: Length (m)

m: Mass (kg)

c: Damping coefficient (*N.s/m*)

θ: Angle between initial and current position (rad)

g: gravity acceleration (m/s^2)

T: torque (Nm)

Newton's Second Law was used to get resultant torque. The excitation torque and non-zero initial conditions tends to take the pendulum out of its equilibrium position, while gravity and damping tend to make it return to it.

In this way,

$$T_{result} = T(t) - mgLsen\theta - c\dot{\theta} \tag{1}$$

Using Newton's Second Law,

$$mL^2\ddot{\theta} = T(t) - mgL\,\text{sen}\theta - c\dot{\theta} \tag{2}$$

and dividing both sides by mL^2,

$$\ddot{\theta} + \frac{c}{mL^2}\dot{\theta} + \frac{g}{L}\text{sen}\theta = \frac{T(t)}{mL^2} \tag{3}$$

The excitation torque is supposed to have the form of

$$T(t) = t_0 \sin\Omega t \tag{4}$$

3 NUMERICAL SIMULATION

To study the nonlinear dynamical effects of excited damped pendulum model for dynamic loading we use our own numerical time integration program of nonlinear ordinary differential equations of motion via the Runge-Kutta algorithm of 4th order, in the environment MATLAB.

We have carried out a parametric study of the problem through response time histories and phase plans, specifically for the case of damped free vibrations que may occur for certain initial conditions. Some preliminary forced simulations are also shown.

4 RESULTS

As previously stated, simulations were first done with ($T(t)$) equal zero and variable initial conditions. Latter, some preliminary periodic responses of forced vibrations are presented.

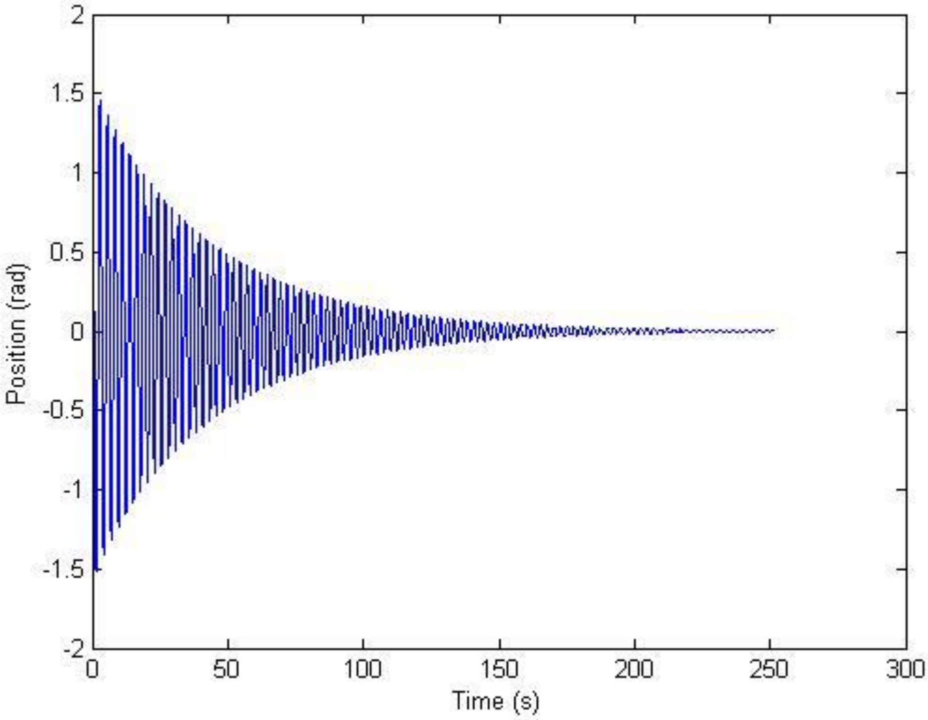

Figure 2. Time history with initial position=π/2, m = 2, c=0.2, g = 9.8 e L = 1.5.

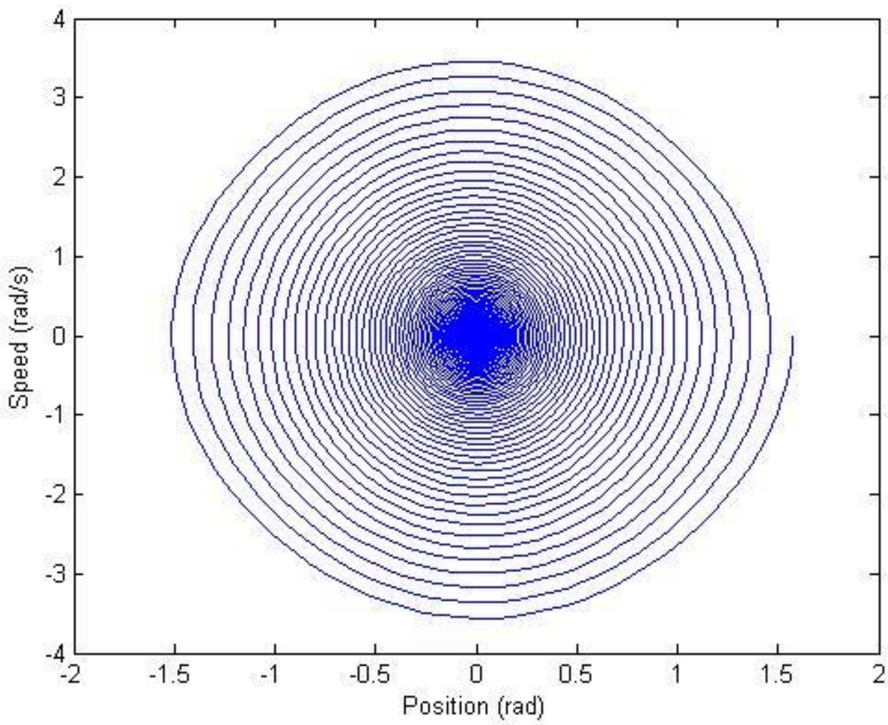

Figure 3. Phase Plane with initial position=π/2; m = 2; c=0.2; g = 9.8; L = 1.5.

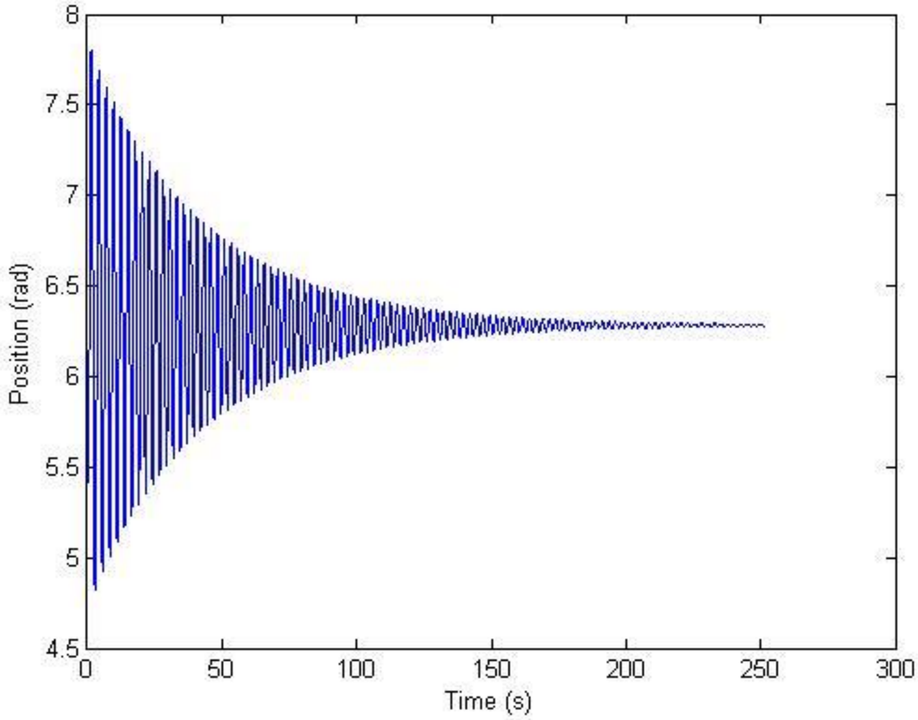

Figure 4. Time history with conditions: initial position=3π/2; m = 2; c=0.2; g = 9.8; L = 1.5.

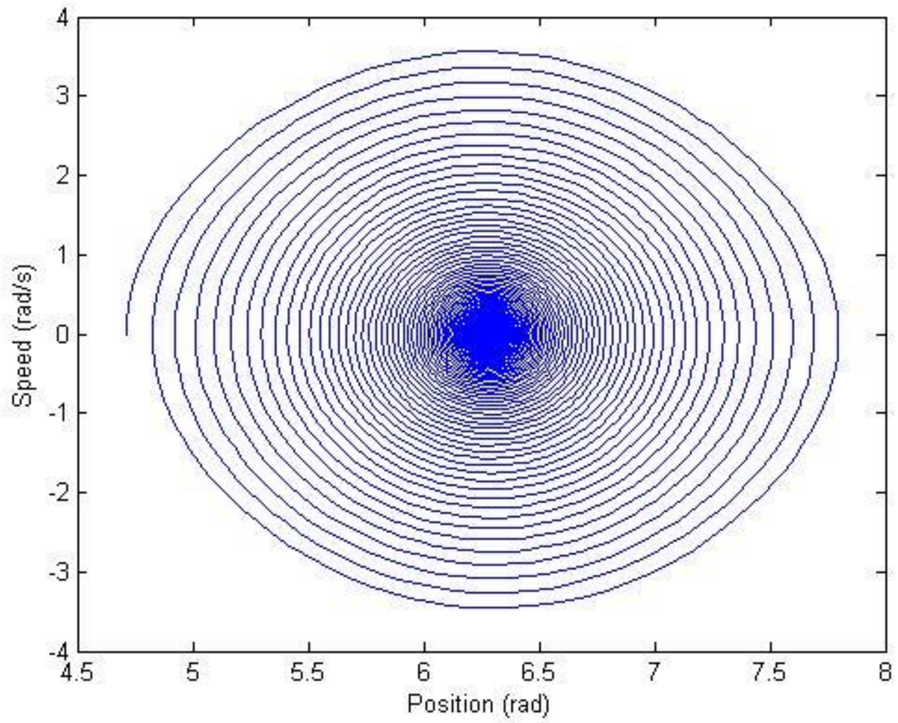

Figure 5. Phase Plane with conditions: initial position=3π/2; m = 2; c=0.2; g = 9.8; L = 1.5.

Initial speed and damped variable.

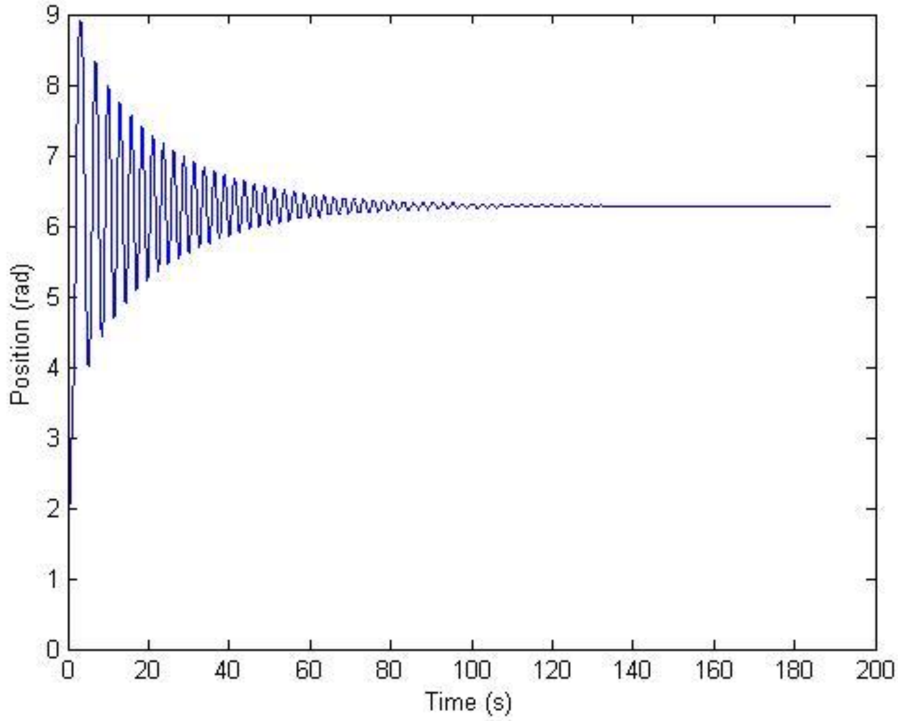

Figure 6. Time history with initial position=0; initial speed=5.5; m = 2; c=0.4; g = 9.8; L = 1.5.

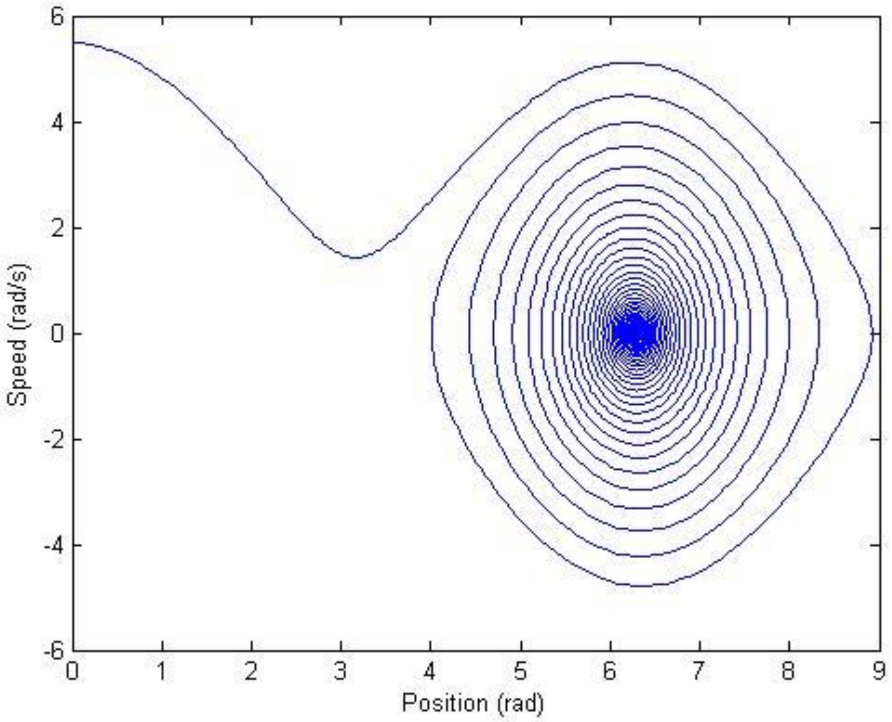

Figure 7. Phase Plane with initial position=0; initial speed=5.5; m = 2; c=0.4; g = 9.8; L = 1.5.

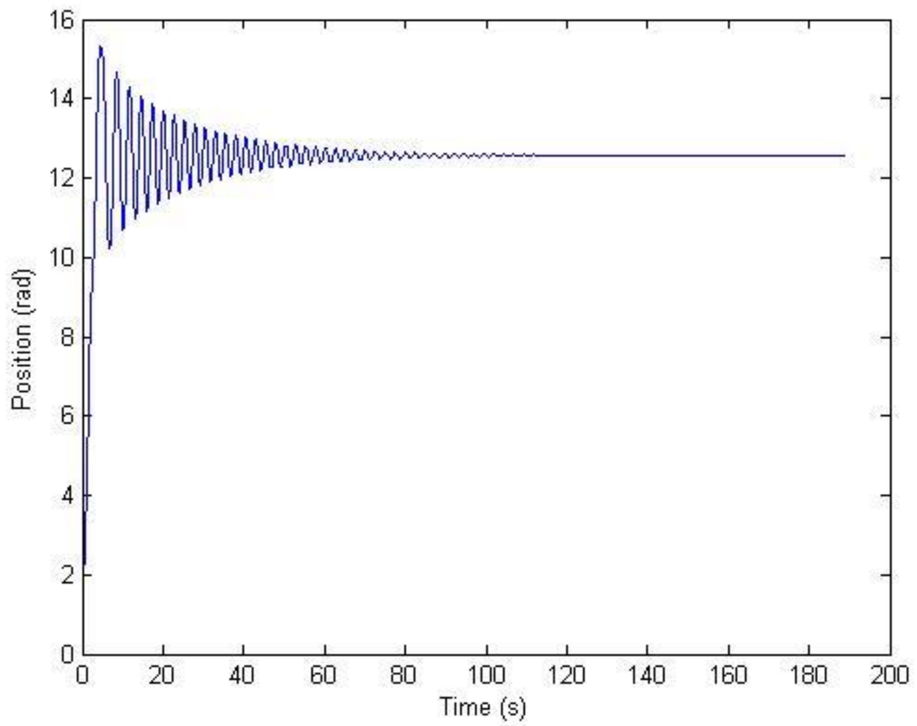

Figure 8. Time history with initial position=0; initial speed=6; m = 2; c=0.4; g = 9.8; L = 1.5.

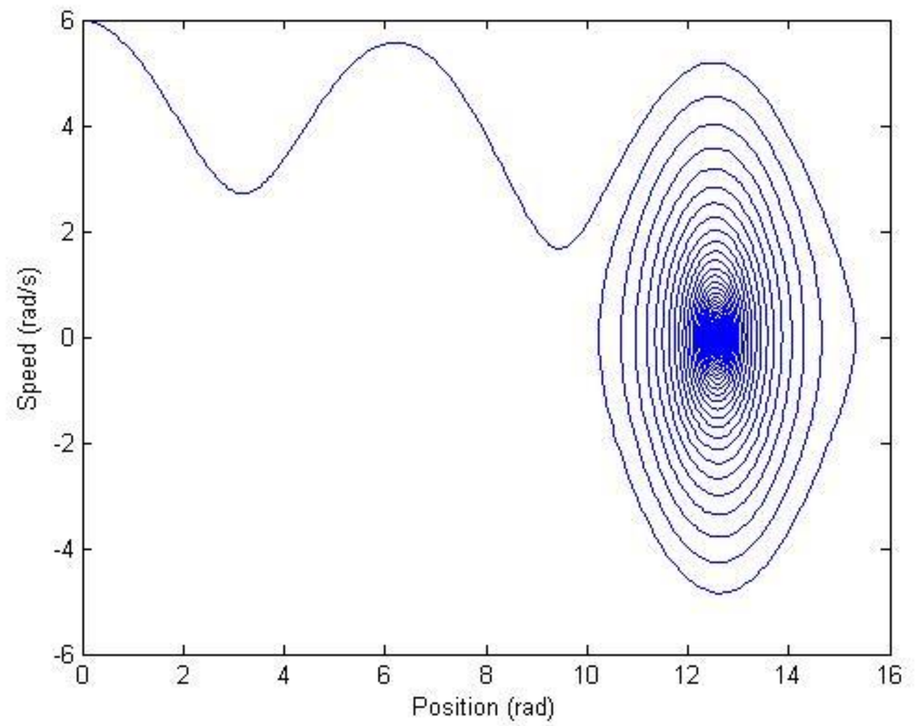

Figure 9. Phase Plane with initial position=0; initial speed=6; m = 2; c=0.4; g = 9.8; L = 1.5.

For the next simulations, a nonzero periodic forcing torque is applied.

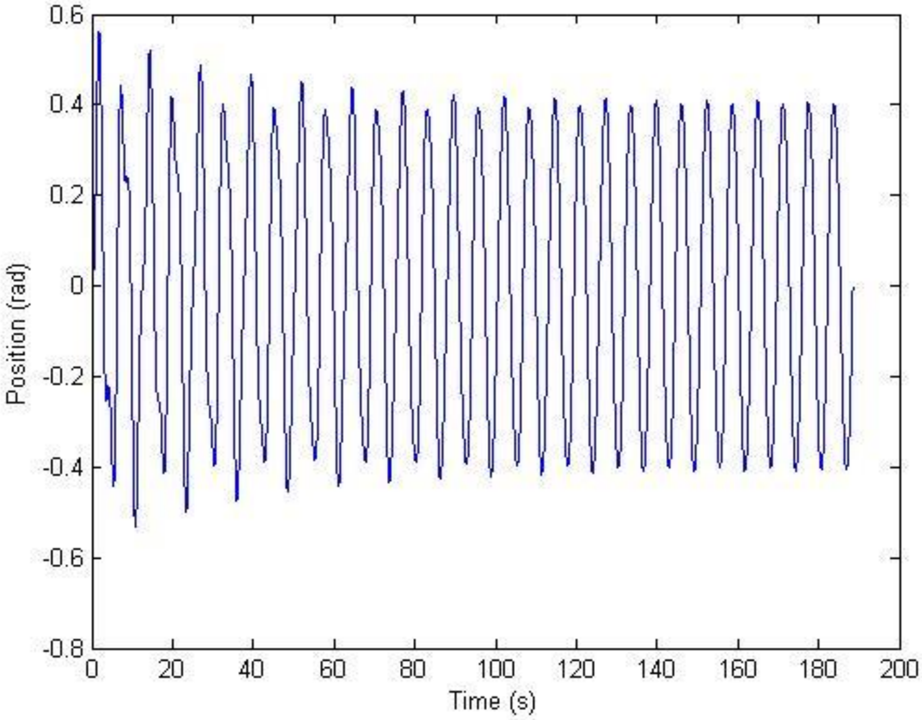

Figure 10. Time history with t0=10; Ω=1; m = 2; c=0.4; g = 9.8; L = 1.5.

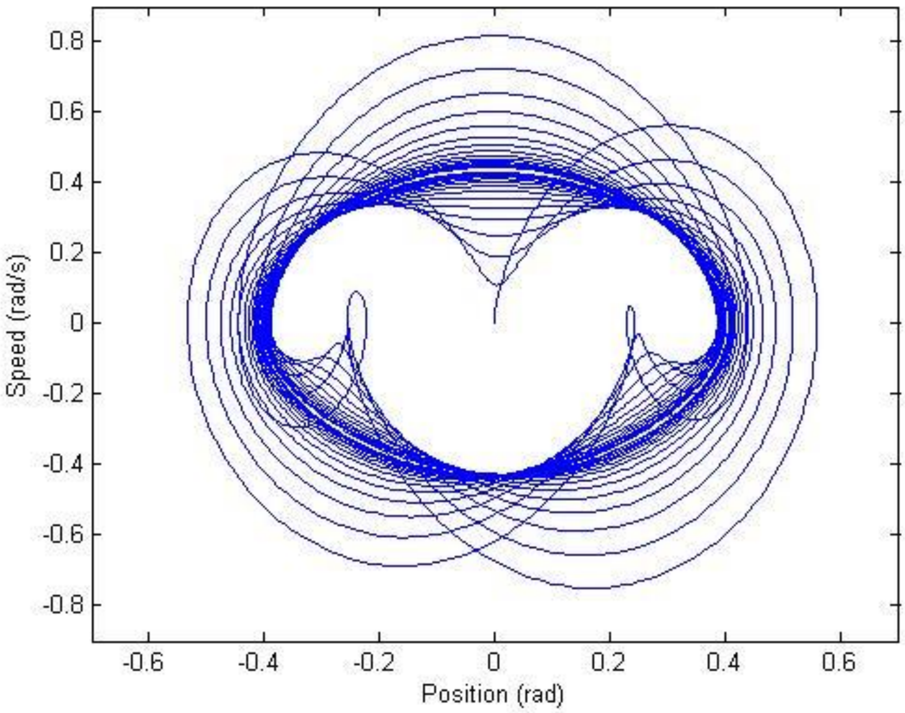

Figure 11. Phase Plane with t0=10; Ω=1; m = 2; c=0.4; g = 9.8; L = 1.5.

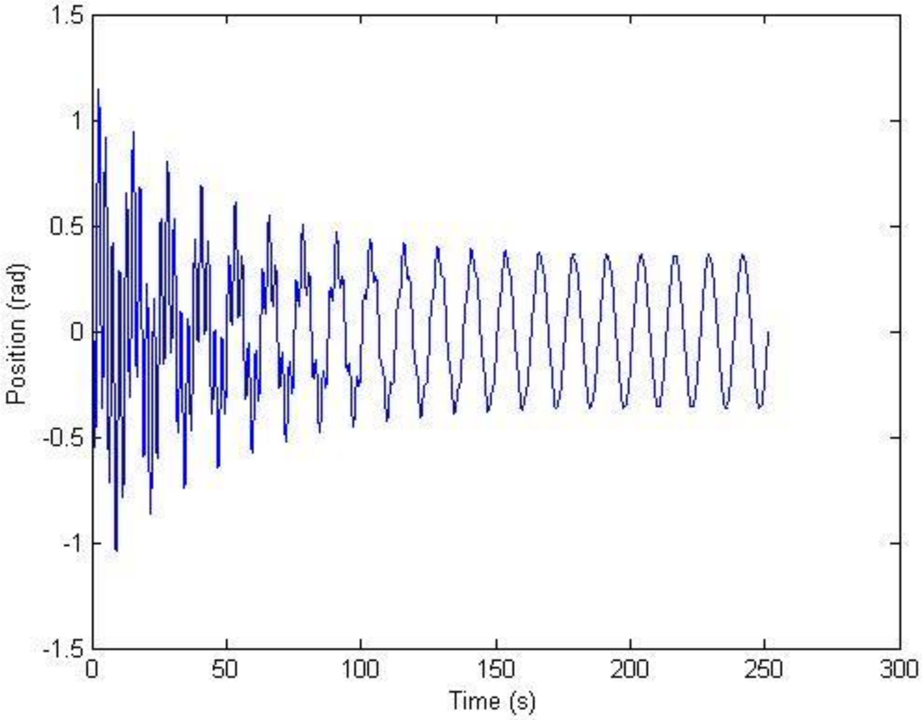

Figure 11. Time history with conditions: t0=10; Ω =1; initial position=π/2; m = 2; c=0.4; g = 9.8; L = 1.5.

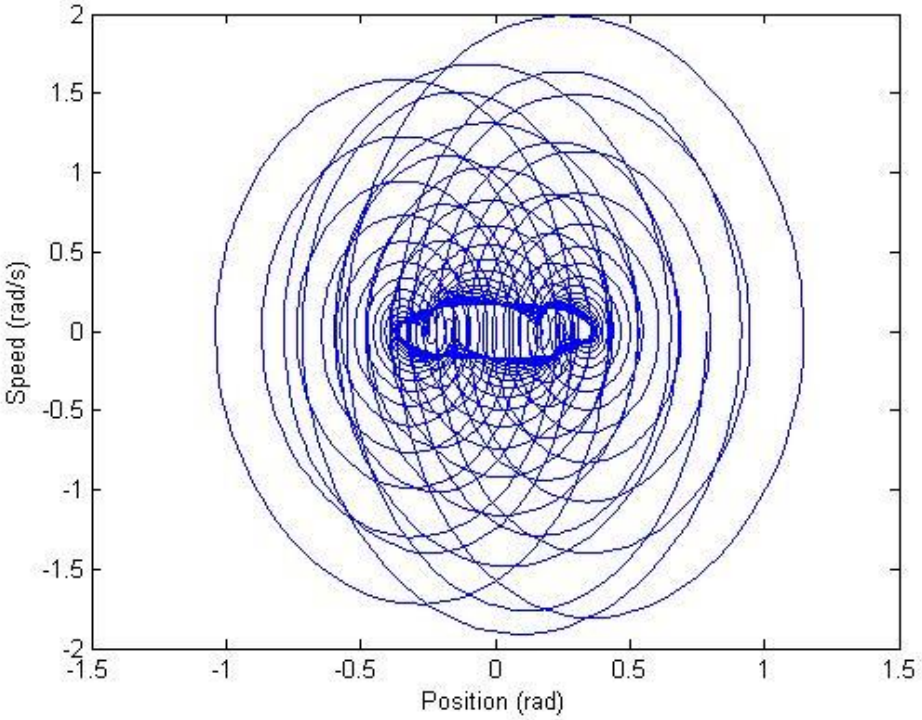

Figure 12. Phase Plane with condition: t0=10; Ω =1; initial position=π/2; m = 2; c=0.4; g = 9.8; L = 1.5.

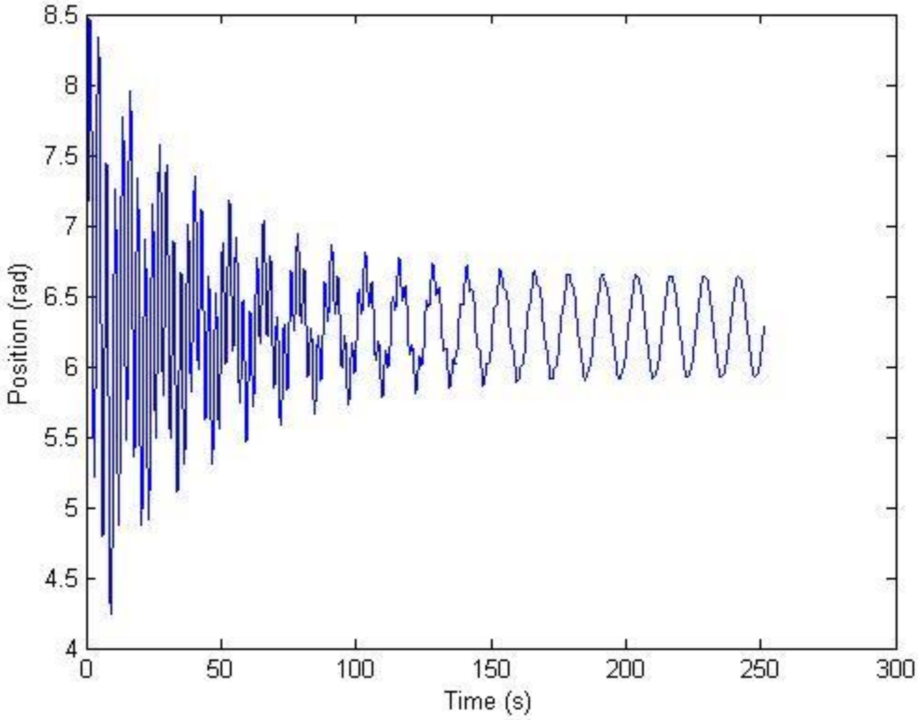

Figure 13. Time history with conditions: t0=10; Ω =1; initial position=3π/2; initial speed=2; m = 2; c=0.4; g = 9.8; L = 1.5.

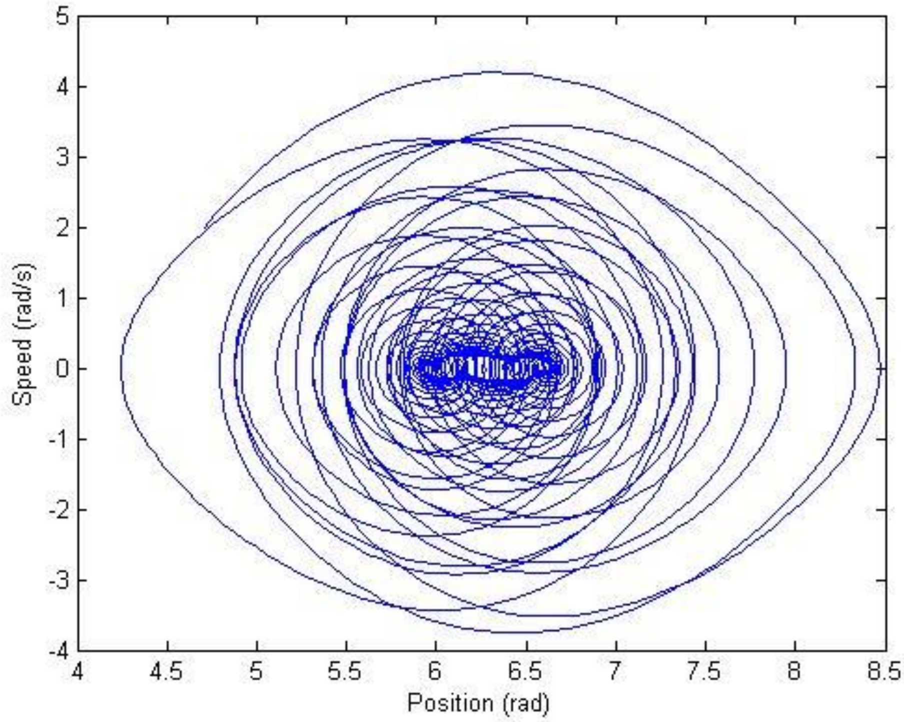

Figure 14. Phase Plane with conditions: t0=10; Ω =1; initial position=3π/2; initial speed=2; m = 2; c=0.4; g = 9.8; L = 1.5.

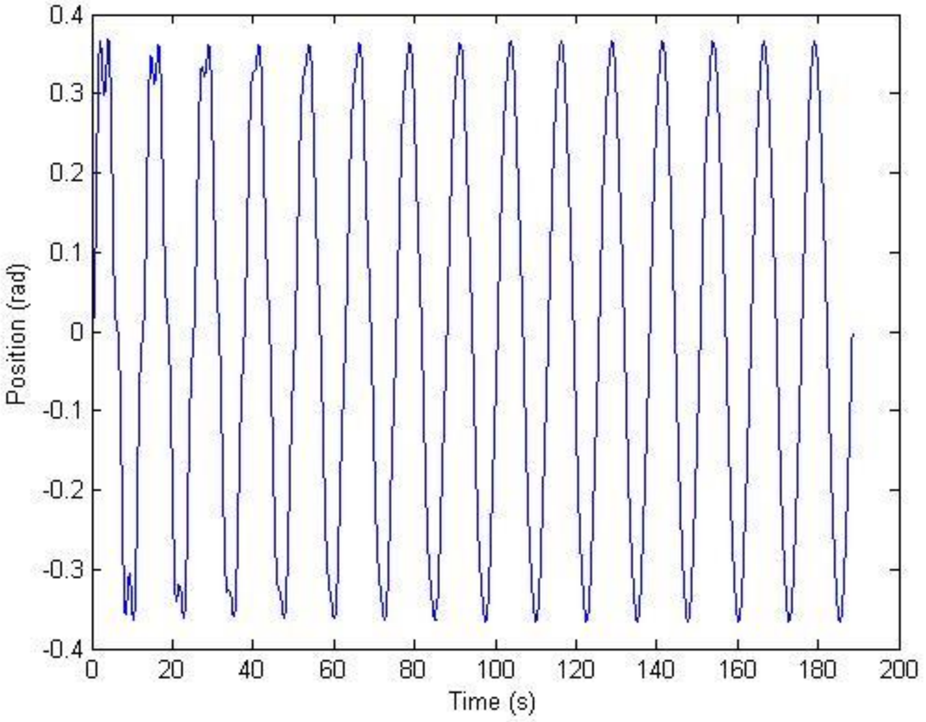

Figure 15. Time history with conditions: t0=10; Ω =0.5; m = 2; c=0.4; g = 9.8; L = 1.5.

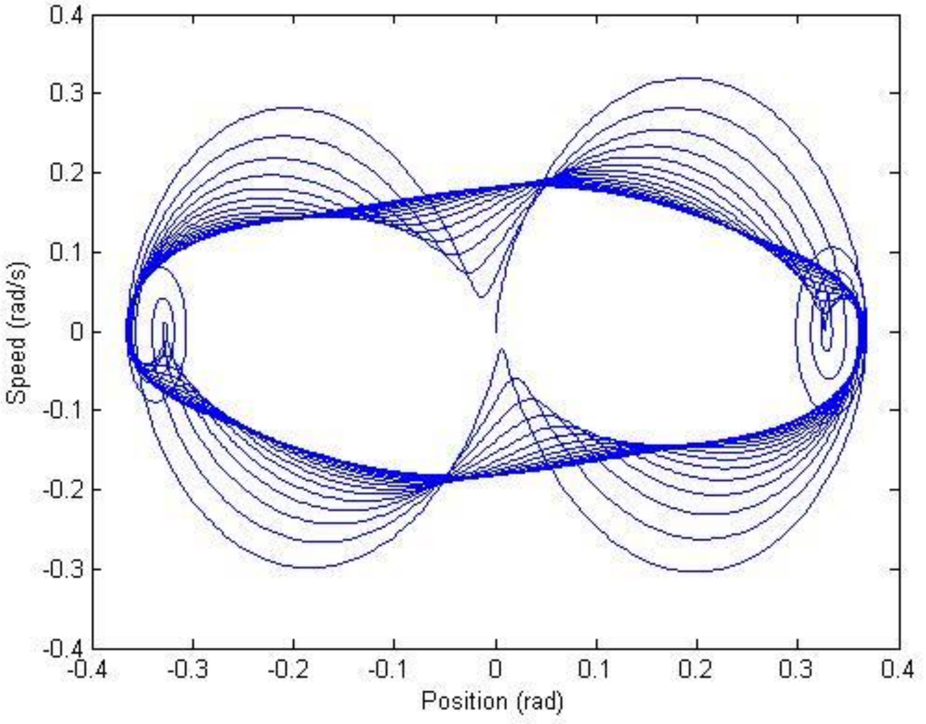

Figure 16. Phase Plane with conditions: t0=10; Ω =0.5; m = 2; c=0.4; g = 9.8; L = 1.5.

5 CONCLUSIONS

An initial numerical nonlinear dynamic study of periodic motions of excited and damped pendulum vibrations for several parameter settings and initial conditions was presented.

Future work will lead to an as complete as possible geometric study of this model, including Poincaré maps, bifurcation diagrams, Lyapunov exponents etc.

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