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ACTIVE VIBRATION CONTROL OF A SMART BEAM UNDER ROTATION

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Abstract. *A rotating beam is fitted with piezoelectric actuators in conjunction with a PD control and a LQR control technique in order to minimize the deflection of the tip of the flexible beam due to the rotational motion. Both control techniques are used for comparison. Using the lagrangian approach, the discretized governing equation of motion is derived for the beam. The behaviour of the rigid body variable (angular displacement of the slewing axis) is given by a prescribed profile. The bending moment from the piezo ceramic is subsequently added to the lagrangian of the whole system and inserted into the governing equation of motion for the flexible beam. The position of the piezoactuator will be varied from the root to the end of the beam. The length of the piezo is a third of the beam length. These positions of the piezo are tested for three modes of vibration. For each position both control techniques are used to generate the voltage control on the piezo and the results for the different techniques are compared to each other.*

Keywords: *piezoelectric ceramic actuators, active vibration control, assume modes method, PD control, LQR control*

1 INTRODUCTION

The lightweight and flexible structures are aesthetically appealing. Less weight normally translates to lower energy consumption and increased performance, and they are easily deployed and assembled. These structures are defined as structures where the ratio of their mass to their length is relatively low. However these benefits, flexible structures present the challenge of stability maintenance because of their susceptibility to vibration. Many parts of these structures can be modeled as Euler-Bernoulli beams. Long and slender objects are traditionally modeled as Euler-Bernoulli beams. In this work one investigates an Euler-Bernoulli beam like structure undergoing rotation and subject to vibration. The purpose of this research is to investigate the use of piezoelectric material to minimize the beam vibration.

Investigations about the use of piezoelectric material to control beam vibrations is vast. Finite element models have been used to analyze the behaviour of a vibrating beam due to the introduction of a disturbing force with a piezo actuator and sensor acting to restore the beam to the desired position (Narayanan and Balamurugan, 2002). In this same reference, the effects of different control techniques, namely constant-gain negative velocity feedback, Lyapunov feedback and LQR (Linear Quadratic Regulator) is explored as well as the effects of temperature on the properties of the piezo material. The piezo material can be placed at a single location or at multiple locations (Molter et al., 2009). The Heaviside function can be used for the location of the piezo and investigation about the effects of its placement.

The first mode of vibration is the most important for the beam analysis and it is most effectively controlled by the placement of the piezo near the root of the beam (Molter at al., 2009). Similar results can be obtained by using a cost function in order to determine the optimal placement of the piezo actuator and sensor (Abreu et al., 2003). An optimal position will not be investigated here. In fact, the piezo here is located in three different positions: the root of the beam, the middle of the beam, and the free end of the beam. The length of the piezo is considered here a third of the length of the beam. Similar idea was employed by (Gani et al., 2003). In this last reference, the first and second modes of vibration were considered, and from the results, the most effective position of the piezo was shown to be at the root of the beam, concurrent with most other research.

The piezoelectric ceramic is a type of intelligent material due to its properties: direct and inverse piezoelectric effects and the ability to be used as the sensor or actuator in active vibration control (Zhang and Wang, 2010). Care must be taken in order that the voltage supply to the piezo does not exceed the break down voltage at which the piezo material loses its ability to function as a smart martial. Two control techniques are used here: the Proportional Differential (PD) technique and the Linear Quadratic Regulator (LQR) technique. These techniques are applied in several other research papers.

Traditionally, the methods of controlling mechanical vibration fall into two classifications: active and passive control. Passive methods involve the addition of some constant external force to act as a damper, or construction using more rigid materials. Many lightweight and flexible structures are employed in high performance and precision applications; hence, for optimal performance, it is required that these structures possess the ability to sensing and responding to changes in their environment.

The use of the piezoelectric ceramic adhered to the surface of the structure to counteract the effects of vibration will not add significant weight to the original structure and will not affect the initial flexibility characteristic.

2 GOVERNING EQUATION OF MOTION FOR THE ROTATING FLEXIBLE BEAM

The geometric model of the dynamic system investigated in this work is presented in Figure 1. This system comprises a flexible beam-like structure in slewing motion.

In this figure, the inertial axis is represented by XY, the moving axis (attached to the slewing axis and moving with it) is represented by xy, the beam deflection (as a space-time variable) is represented by $v(x, t)$ and the slewing angle is represented by $\theta(t)$.

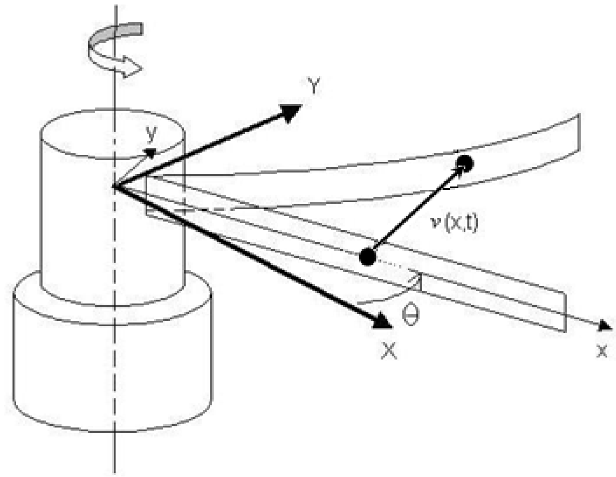

Figure 1. The slewing flexible beam system.

The governing equations of motion are obtained through the energy method [8-12]. In order to apply this method one needs to know the kinetic and potential (strain) energies stored in the slewing beam-like flexible structure during the time evolution.

The kinetic energy, T, of the rotating beam is given by:

$$T = \frac{1}{2}\int_0^{\ell} \rho A\,[\,(\dot{v} + x\dot{\theta})^2 + (v\dot{\theta})^2\,]dx\,. \tag{1}$$

In Eq. (1), $\theta(t)$ represents the angular displacement of the beam, $v(x, t)$ represents the transversal displacement along the beam, ρ represents the linear mass density of the beam, A represents the cross-section area of the beam and ℓ represents the length of the undeflected beam.

In the mathematical model considered here, linear curvature is assumed in the modeling of the flexible structure (Fenili, 2000; Popov, 1968; Sah and Mayne, 1990).

The potential energy, V, of the rotating beam is given by:

$$V = \frac{1}{2}\int_0^{\ell} EIv''^2 dx\,. \tag{2}$$

In Eq. (2), E represents the Young's modulus of the material of the beam, and I represents the moment of inertia of the cross-section area of the beam.

The lagrangian, L, therefore, is given by:

$$L = T - V\,. \tag{3}$$

Substituting Eq. (1) and Eq. (2) in Eq. (3) results:

$$L = \frac{1}{2}\int_0^{\ell} \left\{ \rho A \left[(\dot{v} + x\dot{\theta})^2 + (v\dot{\theta})^2 \right] - EI v''^2 \right\} dx \,. \tag{4}$$

The lagrangian is discretized using the assumed modes method [8, 11, 12]. According to this method, the variable $v(x, t)$ is written by means of the expansion:

$$v(x,t) = \sum_{i=1}^{n} \Phi_i(x)\, q_i(t) \,. \tag{5}$$

In Eq. (5), n represents the number of normal modes, $\Phi_i(x)$, considered in the discretization. In this expansion, the time variables $q_i(t)$ are unknown variables to be numerically integrated and controlled. The space functions $\Phi_i(x)$ are known.

In this work, the normal modes associated with the clamped-free beam are considered. Only the first flexural mode is considered in the expansion given by (5).

Substituting (5) in (4) results:

$$\begin{aligned} L = {} & \frac{1}{2}\rho A \sum_{i=1}^{n}\sum_{j=1}^{n} \dot{q}_j(t) \int_0^{\ell} \Phi_i(x)\Phi_j(x)\,dx + \rho A\dot{\theta}\sum_{i=1}^{n}\dot{q}_i(t)\int_0^{\ell} x\Phi_i(x)dx + \frac{1}{2}\rho A\dot{\theta}^2\int_0^{\ell} x^2 dx + \\ & \frac{1}{2}\rho A\dot{\theta}^2 \sum_{i=1}^{n}\sum_{j=1}^{n} q_i(t)\, q_j(t)\int_0^{\ell}\Phi_i(x)\Phi_j(x)\,dx - \frac{1}{2}EI\sum_{i=1}^{n}\sum_{j=1}^{n} q_i(t)\, q_j(t)\int_0^{\ell}\Phi_i''(x)\Phi_j''(x)\,dx \end{aligned} \,. \tag{6}$$

In Eq. (6):

$$\int_0^{\ell} \Phi_i(x)\Phi_j(x)\,dx = \gamma_i \qquad \text{if} \quad i = j \tag{7}$$

$$\int_0^{\ell} \Phi_i(x)\Phi_j(x)\,dx = 0 \qquad \text{if} \quad i \neq j \tag{8}$$

Considering, as stated before, only one mode of vibration, i = j = 1. Then:

$$L = \frac{1}{2}\rho A\,\gamma_1 \dot{q}_1^2 + \rho A \alpha_1 \dot{\theta}\dot{q}_1 + \frac{\ell^3}{6}\rho A\dot{\theta}^2 + \frac{1}{2}\rho A\,\gamma_1\dot{\theta}^2 q_1^2 - \frac{1}{2}EI\lambda_1 q_1^2 \,. \tag{9}$$

In Eq. (9):

$$\alpha_1 = \int_0^{\ell} x\Phi_1 dx \tag{10}$$

$$\lambda_1 = \int_0^{\ell} \Phi_1''^2\,dx \tag{11}$$

Equation (11) can be rewritten as:

$$\lambda_1 = \int_0^{\ell} \Phi_1''^2 dx = \int_0^{\ell} \Phi_1^{iv}\Phi_1 dx = \int_0^{\ell} \left(\frac{\rho A\omega_1^2}{EI}\Phi_1 \right)\Phi_1 dx = \frac{\rho A\omega_1^2}{EI}\int_0^{\ell} \Phi_1^2 dx = \frac{\rho A\omega_1^2\gamma_1}{EI} \tag{12}$$

Lagrange's equations are given by:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\frac{\partial L}{\partial q_i}=0 \tag{13}$$

Using the lagrangian given in Eq. (9), the governing equation of motion for the variable $q_1(t)$ is given by:

$$\ddot{q}_1+\frac{\alpha_1}{\gamma_1}\ddot{\theta}-\dot{\theta}^2 q_1+\omega_1^2 q_1=0 \tag{14}$$

The boundary conditions for the beam (considering one mode) are given by:

$$\Phi_1(0)=0 \tag{15}$$

$$\Phi_1'(0)=0 \tag{16}$$

$$\Phi_1''(\ell)=0 \tag{17}$$

$$\Phi_1'''(\ell)=0 \tag{18}$$

In Eq. (14), the variable θ (t) and its time derivatives are prescribed.

3 STRAIN ENERGY OF THE PIEZOELECTRIC ELEMENT

The strain energy induced by the piezoactuator can be used to incorporate the modal force/moment and stiffness caused by the piezoactuator into the mathematic mode1 of the flexible structure as given by Eq. (14). The total induced strain energy of the piezoactuator can then be written for n modes of vibration as (Yousefi-Koma, 1997):

$$U_a=\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}K_{ij}\,q_i q_j-\sum_{i=1}^{n}Q_i\,q_i \tag{19}$$

where:

$$K_{ij}=\frac{1}{2}\left(\frac{1}{2}E_a W_a t_a t_b^2\right)\left(\frac{4}{3}\bar{t}^2+2\bar{t}+1\right)\int_{x_1}^{x_2}\frac{\partial^2\Phi_i}{\partial x^2}\frac{\partial^2\Phi_j}{\partial x^2}dx \tag{20}$$

$$Q_i=-V(t)\left(E_a W_a d_{31a} t_b\right)\left(\bar{t}+1\right)\int_{x_1}^{x_2}\frac{\partial^2\Phi_i}{\partial x^2}\,dx \tag{21}$$

In Eqs (20) and (21), E_a represents the elastic modulus of the piezoactuator, W_a represents the width of the piezoactuator, t_a represents the piezoactuator thickness, t_b represents the beam thickness, $\bar{t}$ represents the thickness ratio given by $\bar{t}=t_a/t_b$, d_{31a} represents the piezoactuator strain constant, V(t) represents the voltage applied to the piezoactuator and x_1 and x_2 represents the coordinate value of the piezoactuator.

Considering one mode of vibration (n=1 in Eq. (5)), Eq. (19) is written as:

$$U_a=\frac{1}{2}K_{11}q_1^2-Q_1 q_1 \tag{22}$$

Equation (22) presents the strain energy induced by the piezoactuator considering only one mode of vibration.

4 THE COMPLETE MATHEMATICAL MODEL: ROTATING FLEXIBLE BEAM + PIEZOELECTRIC ELEMENT

Equation (22) must now be added to Eq. (9). The extended lagrangian must be inserted in Eq. (13). The resulting governing equation of motion for the rotating beam connected with the piezoactuator is then finally given by:

$$\ddot{q}_1 + \frac{\alpha_1}{\gamma_1}\ddot{\theta} - \dot{\theta}^2 q_1 + \left(\omega_1^2 + \frac{K_{11}}{\rho A \gamma_1}\right) q_1 = \frac{1}{\rho A \gamma_1} Q_1 \tag{23}$$

Let:

$$C_a = \frac{E_a W_a d_{31a} t_b \left(\bar{t} + 1\right)}{\rho A\, \gamma_1}$$

Using Eq. (21), the term on the right side of Eq. (23) can be better written as:

$$\ddot{q}_1 + \frac{\alpha_1}{\gamma_1}\ddot{\theta} - \dot{\theta}^2 q_1 + \left(\omega_1^2 + \frac{K_{11}}{\rho A \gamma_1}\right) q_1 = -V(t)\, C_a \int_{x_1}^{x_2} \frac{\partial^2 \Phi_1}{\partial x^2}\, dx$$

or:

$$\ddot{q}_1 + \frac{\alpha_1}{\gamma_1}\ddot{\theta} - \dot{\theta}^2 q_1 + \left(\omega_1^2 + \frac{K_{11}}{\rho A \gamma_1}\right) q_1 = -V(t)\, C_a \int_{x_1}^{x_2} \frac{\partial}{\partial x}\left(\frac{\partial \Phi_1}{\partial x}\right) dx$$

or:

$$\ddot{q}_1 + \frac{\alpha_1}{\gamma_1}\ddot{\theta} - \dot{\theta}^2 q_1 + \left(\omega_1^2 + \frac{K_{11}}{\rho A \gamma_1}\right) q_1 = -V(t)\, C_a \left.\frac{\partial \Phi_1}{\partial x}\right|_{x_1}^{x_2}$$

or, finally:

$$\ddot{q}_1 + \frac{\alpha_1}{\gamma_1}\ddot{\theta} - \dot{\theta}^2 q_1 + \left(\omega_1^2 + \frac{K_{11}}{\rho A \gamma_1}\right) q_1 = -V(t)\, C_a \left[\Phi_1'(x_2) - \Phi_1'(x_1)\right] \tag{24}$$

The voltage V(t) in Eq. (24) is the quantity to be substituted by the control law. In state space form Equation (24) gives:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{\alpha_1}{\gamma_1}\ddot{\theta} + \dot{\theta}^2 q_1 - \left(\omega_1^2 + \frac{K_{11}}{\rho A \gamma_1}\right) q_1 - V(t)\, C_a \left[\Phi_1'(x_2) - \Phi_1'(x_1)\right] \end{aligned} \tag{25}$$

Equation (25) in matrix form gives:

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -\left(\omega_1^2 + \dfrac{K_{11}}{\rho A \gamma_1}\right) & 0 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + \begin{Bmatrix} 0 \\ \dot{\theta}^2 x_1 - \dfrac{\alpha_1}{\gamma_1}\ddot{\theta} \end{Bmatrix} - \begin{bmatrix} 0 \\ C_a \left(\Phi_1'(x_2) - \Phi_1'(x_1)\right) \end{bmatrix} V(t) \tag{26}$$

5 THE LINEAR CONTROLLERS PD AND LQR

The linear controllers are used in this work are the PD and the LQR. For the PD controller (Ogata, 2009), the control law is given by:

$$V(t) = -K_P(q_1 - q_{1ref}) - K_D(\dot{q}_1 - \dot{q}_{1ref}) \tag{27}$$

In Eq. (27), K_P and K_D are the proportional and derivative gains respectively and q_{1ref} and $\dot{q}_{1ref}$ are the reference signals.

For the LQR controller (Elgerd, 1967), the control law is given by:

$$V(t) = -R^{-1}\, B^T\, P\, \vec{x} \tag{28}$$

In Eq. (28), $\vec{x}$ is the state vector given by $\vec{x} = \{z_1 - z_{1ref} \quad z_2 - z_{2ref}\}^T$ where q_1 is the state z_1, $\dot{q}_1$ is the state z_2, z_{1ref} and z_{2ref} are reference values for the system states and P is the solution of the Riccati equation:

$$PA + A^T P + Q - PBR^{-1}B^T P = 0 \tag{29}$$

In Eqs. (28) and (29), Q and R are weighting matrices that satisfy the positive definiteness condition Q > 0 and R > 0. In these same equations, A and B are matrices obtained when the system is written in the state space form:

$$\dot{x} = Ax + Bu \tag{30}$$

where u = V(t).

6 NUMERICAL SIMULATIONS

For the numerical simulations six cases are considered for each mode of vibration. In CASE 1A the piezoactuator is located at the first third of the beam (near the root) and the PD control law is used. In CASE 1B the piezoactuator is located at the first third of the beam (near the root) and the LQR control law is used. In CASE 2A the piezoactuator is located at the second third of the beam (middle of the beam) and the PD control law is used. In CASE 2B the piezoactuator is located at the second third of the beam (middle of the beam) and the LQR control law is used. In CASE 3A the piezoactuator is located at the last third of the beam (near the free end) and the PD control law is used. In CASE 3B the piezoactuator is located at the last third of the beam (near the free end) and the LQR control law is used.

The parameters used in the numerical simulations are presented in Table 1.

The controller gains used in the numerical simulations are:

$$K_P = 800 \tag{31}$$

$$K_D = 650 \tag{32}$$

$$R = [\,0.006\,] \tag{33}$$

$$Q = \begin{bmatrix} 50000 & 0 \\ 0 & 0.01 \end{bmatrix} \tag{34}$$

Table 1. Parameters used in the numerical simulations.

Constitutive relation	Nomenclature	Value
Linear mass density (beam)	ρ	2700 kg/m^3
Elastic modulus (beam)	E	$0.700*10^{11}$ Pa
Beam length	L	1.200 m
Beam thickness	b	$1.500*10^{-3}$ m
Beam width	w	$2.000*10^{-2}$ m
Elastic modulus (piezoactuator)	E_a	$0.714*10^{11}$ Pa
Piezoactuator strain constant	d_{31a}	$200*10^{-12}$ m/V
Piezoactuator thickness	t_a	$0.305*10^{-3}$ m
Piezoactuator width	W_a	$1.270*10^{-3}$ m
Piezoactuator length	$x_2 - x_1$	0.400 m

The profile for the angular displacement, $\theta(t)$, is prescribed according to the curve presented in Figure 2. The first and second derivatives of $\theta(t)$ are numerical derivatives of this curve.

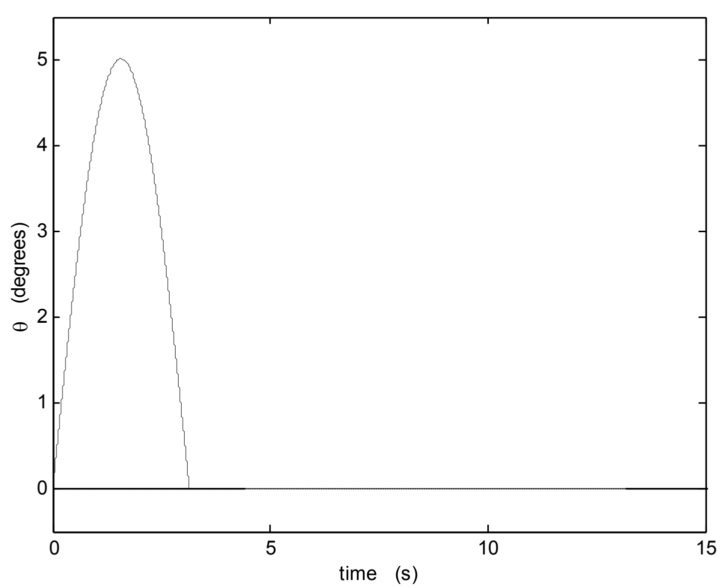

Figure 2. Profile for angular displacement $\theta(t)$ **.**

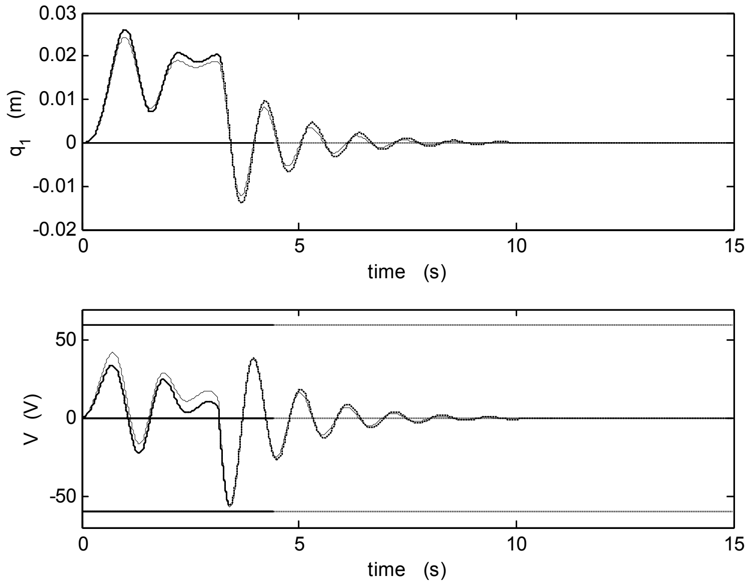

Figure 3. Piezoactuator near the beam root - First mode: beam deflection (q_1(m)) and control voltage (V (V)). PD: solid black and LQR: dashed black.

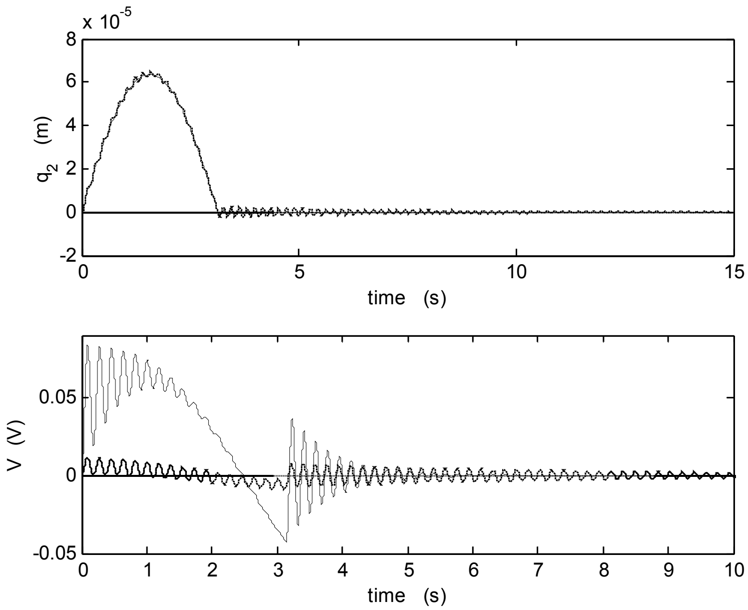

Figure 4. Piezoactuator near the beam root - Second mode: beam deflection (q_2(m)) and control voltage (V (V)). PD: solid black and LQR: dashed black.

According to Figures (3) to (11) the linear control law named LQR stabilizes the system for all the situations presented. The values of the PD gains and LQR weighting matrices are kept constant along all the numerical simulations discussed here. These values are presented in Eqs. (31) to (34). According to Figures (5), (7) and (10), the linear control law named PD is not stable for all the cases investigated here. These unstable results are not ploted in the figures

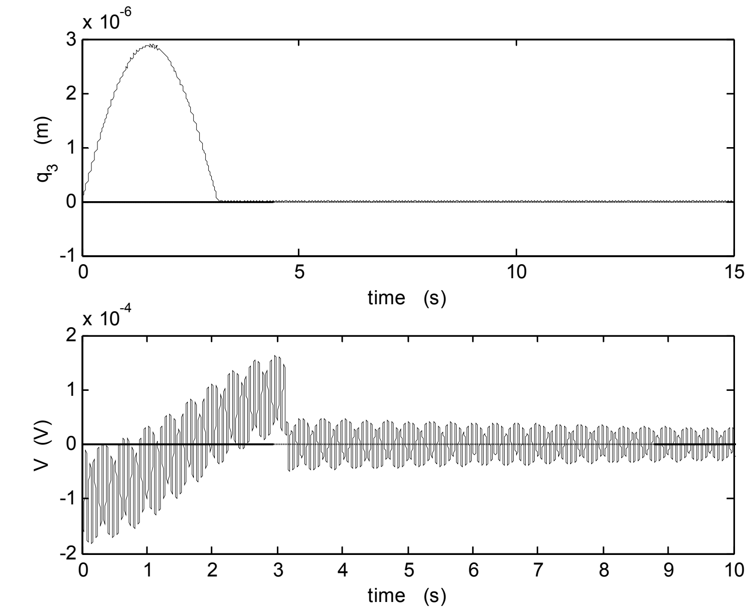

Figure 5. Piezoactuator near the beam root - Third mode: beam deflection (q_3(m)) and control voltage (V (V)). LQR. The result for PD is unstable and is not ploted here.

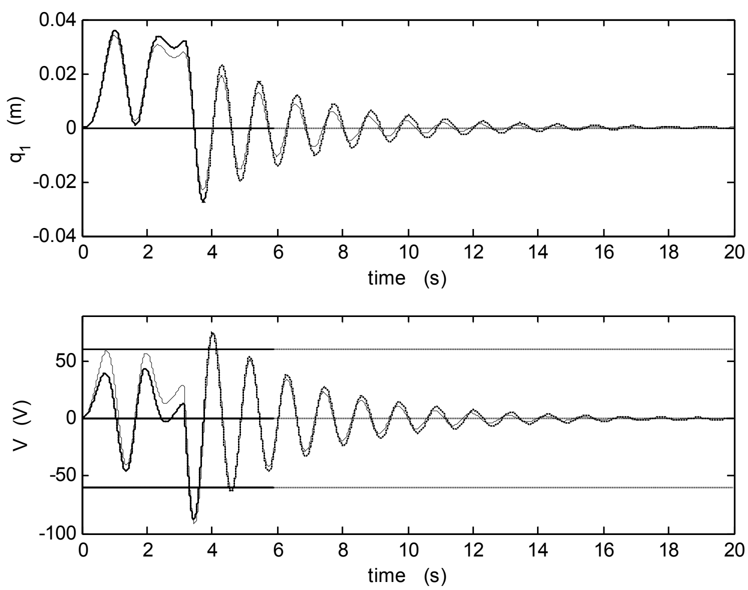

Figure 6. Piezoactuator in the middle of the beam - First mode: beam deflection (q_1(m)) and control voltage (V (V)). PD: solid black and LQR: dashed black.

The advantage of the LQR controller is that matrix A is a function of the natural frequencies of the system (ω_1, ω_2 and ω_3) system and this matrix is used for the obtaining of the LQR gains. In this sense, if the vibration mode to be controlled changes, new gains are automatically calculated in order to stabilize the new system (for the same weighting matrices Q and R). Once the PD gains are chosen for a specific mode, a new mode of vibration may require the choosing of a new set of values for these gains

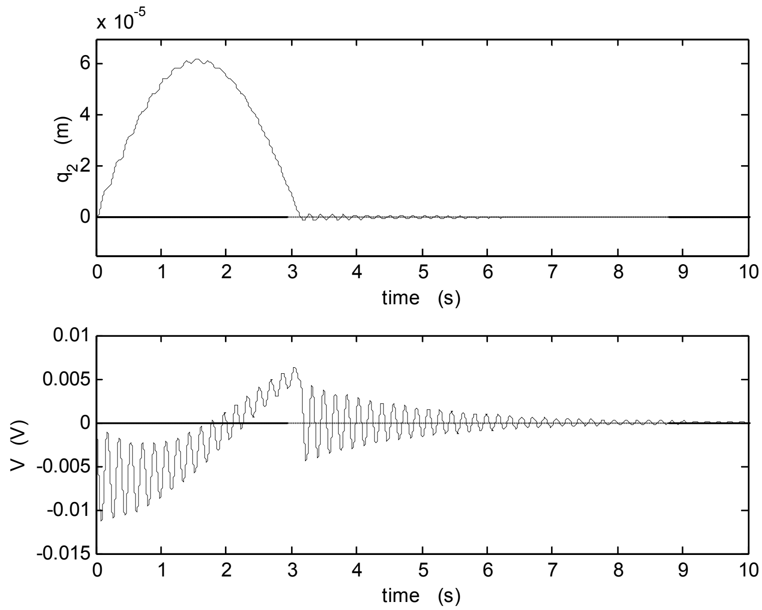

Figure 7. Piezoactuator in the middle of the beam - Second mode: beam deflection (q_2(m)) and control voltage (V (V)). LQR. The result for PD is unstable and is not ploted here.

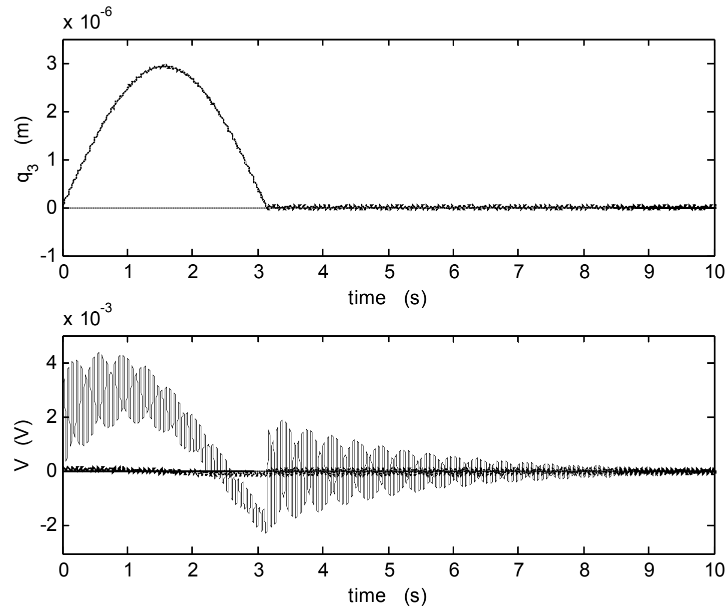

Figure 8. Piezoactuator in the middle of the beam - Third mode: beam deflection (q_3(m)) and control voltage (V (V)). PD: solid black and LQR: dashed black.

For different locations of the piezoactuator different maximum values are obtained for the control voltage. The values of the gains K_P, K_D and the matrices Q and R used in these numerical simulations are chosen in order that the voltage in Figure (3) may be between -60V and 60V and maintained along all this paper. Besides the fact that the PD response and the LQR response for q_1 are quite similar, the gains calculated for the LQR are considerably smaller when compared to the PD gains considered here

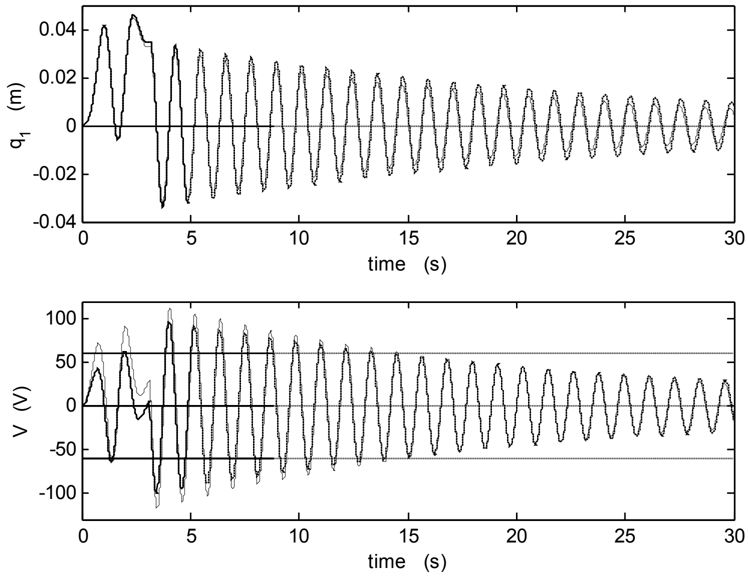

Figure 9. Piezoactuator near the free end of the beam - First mode: beam deflection (q_1(m)) and control voltage (V (V)). PD: solid black and LQR: dashed black.

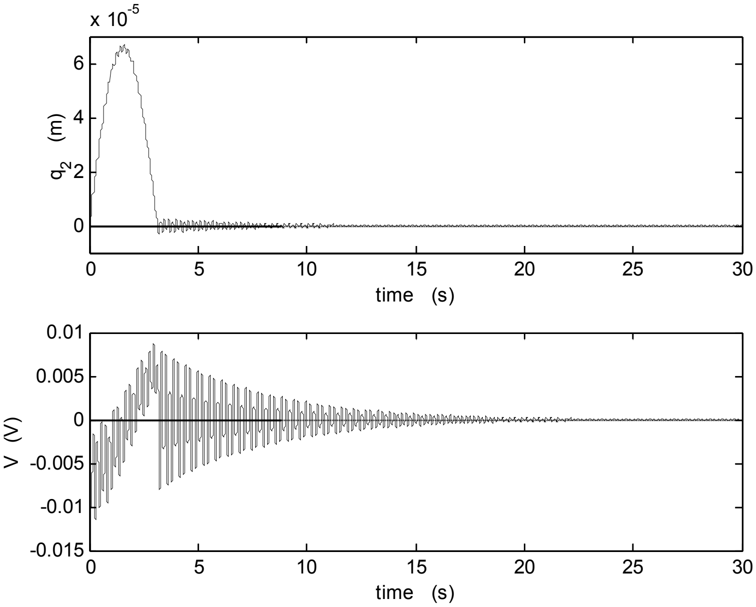

Figure 10. Piezoactuator near the free end of the beam - Second mode: beam deflection (q_2(m)) and control voltage (V (V)). LQR. The result for PD is unstable and is not ploted here.

According to the results presents, the best location for the control of the first mode is near the beam root. For the second and third modes the piezoactuator locations do not have significant influence in the system responses. In all the cases the control voltage necessary to control the system (in all the modes) has considerably smaller amplitudes for the LQR control law. In all the numerical simulations in this work no structural damping is considered in the mathematical model of the flexible beam. This assumption is made in order that all the attenuation in the system responses shown here should be provided by the piezoelectric actuator

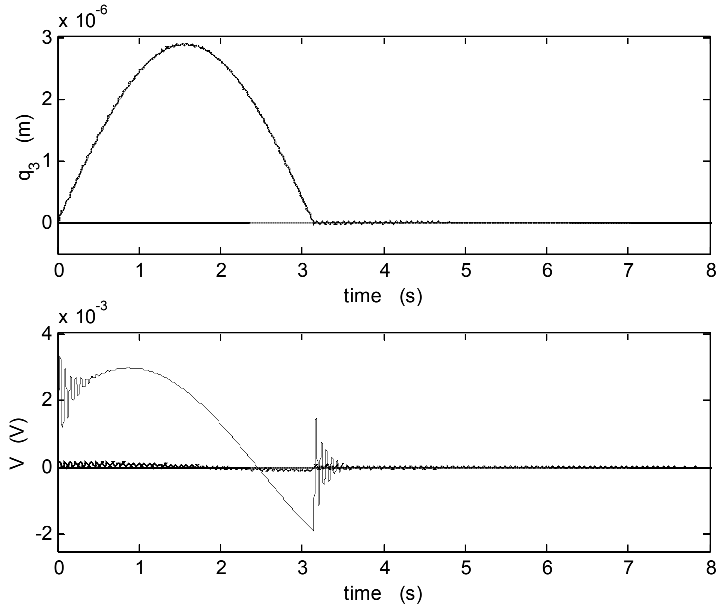

Figure 11. Piezoactuator near the free end of the beam - Third mode: beam deflection (q_3(m)) and control voltage (V (V)). PD: solid black and LQR: dashed black.

7 CONCLUSIONS

Comparing the numerical solutions obtained in this work, the control law named LQR is the most effective and robust when compared to the control law named PD for the attenuation of vibration in a flexible beam under piezoelectric actuation. The LQR is stable in all the cases considered. The PD is unstable in some cases. Considering the control of the first mode of vibration – which has the largest amplitude of vibration when compared to the second and third modes – the location of the piezoactuator that provides the best attenuation of vibration is near the beam root (first third of the beam). For the second and third modes of vibration apparently there is not a best location for the piezoactuator.

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