SATELLITE ATTITUDE CONTROL SYSTEM DESIGN WITH NONLINEAR DYNAMICS AND KINEMTICS OF QUATERNION USING REACTION WHEELS

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Abstract. *The success of a space mission where the satellite must perform rapid attitude maneuvers with great angles is extremely dependent of a careful investigation of the nonlinear dynamics of the satellite. Since these big maneuvers imply in the dynamic coupling between the satellites angular motion and the actuators such as reaction wheels and/or gas jets. As a result, this coupling must be taking into account in the Attitude Control System (ACS) design. This paper presents the nonlinear model derivation of a rigid satellite and the p*erformance comparison of two controllers designed by Lyapunov and LQR methods. *The dynamics of the satellite is initially derived in the non-linear Euler equations form and the kinematics is based on the quaternion parametrization which represent the rotation and attitude motion, respectively. In the sequel, the linear model is obtained where linearization is about an operating point of the arbitrary angular velocity and the reaction wheel angular momentum. From this model, several simulations are performed in order to investigate the influence of the nonlinear dynamics in the in the SCA performance which is designed by trial and error and by the Linear Quadratic Regulator approaches. The ACS performance is evaluated considering the capacity of the reaction wheels to maintain the stability and to control the angular velocity and the attitude of the satellite. The stability is investigated comparing the location of the poles and zeros of the open and closed loops. The ACS performance is evaluated comparing the amount of energy spend by each control law.*

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Keywords: *Satellite Control, nonlinear dynamics, LQR and Lyapunov theory*

1 INTRODUCTION

Currently the space missions are increasingly complexity due to its different tasks and due to the satellites structures with large number of solar panels, antennas, cameras and mechanical manipulators (Mainenti, et al, 2016). Such complexity results in mathematical models with non-linear dynamics. Therefore, the nonlinear terms play an important role in understanding the dynamics and performance of the attitude control system (ACS) of the satellite. Other important aspects in the study of the dynamics and control of structures in space are: the degree of interaction between the rigid and flexible movement, maintenance of SCA performance in the presence of model uncertainties, evaluation of control strategies to reduce residual vibrations in order to maintain the precision pointing and identification of system parameters such as vibration frequency, damping coefficient (Sidi, 1997) and liguid motion (Souza and Souza, 2014). Generally the ACS design involves sensors, estimators, actuators and controllers which the interface is the onboard computer. The key point to have an ACS with good performance for a satellite with nonlinear dynamics is to design a simplest as possible control law. Actuators used in the ACS are generally the reaction wheels that generate torques (linear) continuous and the gas jets that generate non-continuous torques (non-linear) (Pinheiro and Souza, 2014). In this paper one presents the derivation of satellite mathematical model using as actuators three reaction wheels so as the equations of motion are nonlinear (Curtis, 2010). The control law of the ACS is first designed by the Lyapunov approach (Junkins and Turner, 1986) which adopts a Lyapunov function proportional the kinetic energy. The second controller is designed using the theory of Linear Quadratic Regulator (LQR) (Souza, 2006). After that one compares the performance of the Lyapunov and LQR controller in order to evaluate the influence of the nonlinear dynamic in the controller performance. The linear model is associated with the normal operation mode the satellite when the angles and angular velocities are small. Some aspects related with the weight matrix Q and R of LQR and the performance of the control law are investigated having as performance criteria the overshoot and the settling time.

2 SATELLITE EQUATIONS OF MOTIONS

Three reference systems are relevant in the attitude control of an artificial satellite. The first, called "reference (almost) inertial" is a system with origin in the center of mass of the Earth and whose axis: X points to the Vernal Equinox (at the intersection of the Earth's equator plane to the plane of the ecliptic); Z points in the direction and sense of vector earth angular velocity; Y form a direct trihedral XYZ, (see Figure 1.a). The second, called "orbital frame referential" is a system with origin in the center of mass of the Earth, coinciding with one of the focus of the ellipse whose axes X_0 and Y_0 are contained in the satellite orbit plane being: X_0 in the direction and sense that focus to direct trihedral $X_0Y_0Z_0$ (Figure 1.b)., The third, called "satellite referential frame " is a system with origin in the center of mass of the satellite, with the axes x, y and z mutually perpendicular, fixed on the satellite body and conducted on its three main axes of inertia (Figure 1.c).

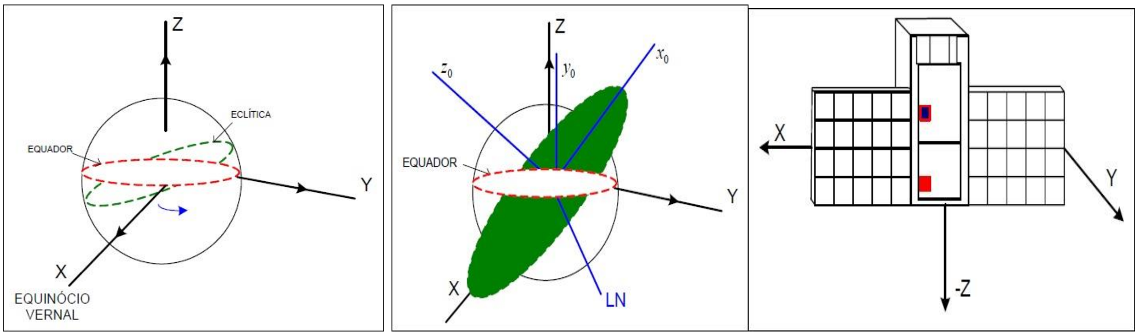

Figure 1- Inertial Frame (a) Orbital Frame (b) Satellite frame (c).

The equations that describe the dynamics of satellite motion relative to the inertial frame referential (X,Y,Z), are obtained considering that the total torque (N) acting on the satellite, which is the sum of external torques (due to the environment) (N_e) and the control torque (N_c) from the reaction wheel. The dynamic equation of the satellite (Blake and Larsen, 2010) can be obtained deriving the with respect time the total satellite angular momentum H given by

$$\dot{\mathbf{H}} = \mathrm{N_e} - \omega \times \mathbf{H} \tag{1}$$

where $\mathbf{H} = \mathbf{I}_S\omega + \mathbf{h}_w$, is the satellite plus the reaction wheel angular moment, respectively; $\mathbf{I}_S$ is the satellite matrix moment of inertia, $\boldsymbol{\omega}$ is the satellite angular velocity. As a result, after some derivation one has

$$\dot{\omega} = \mathbf{I}_S^{-1}(\mathrm{N_e} - \omega \times \mathbf{I}_S\omega - \omega \times \mathbf{h}_w - \dot{\mathbf{h}}_w) \tag{2}$$

Writing the vector product in the matrix form $\boldsymbol{S}(\omega)$ to simply notation one has

$$\boldsymbol{S}(\omega) = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{3}$$

Replacing the matrix $\boldsymbol{S}(\omega)$ into Eq.3 one has

$$\dot{\omega} = \mathbf{I}_S^{-1}(\mathrm{N_e} - \boldsymbol{S}(\omega)\mathbf{I}_S\omega - \boldsymbol{S}(\omega)\mathbf{h}_w - \dot{\mathbf{h}}_w) \tag{4}$$

The satellite dynamics equation is completed by identifying that the control torque is due to the reaction wheel

$$\dot{\mathbf{h}}_w = -\boldsymbol{N}_c \tag{5}$$

and replacing $\mathbf{I}_S, \omega$ e $\mathbf{h}_w$ into Eq.4 one has in the right hand side the angular acceleration of the satellite, given by

$$\begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{bmatrix} = \begin{bmatrix} \left(\omega_2\omega_3\left(I_{yy} - I_{zz}\right) + \left(\omega_3 h_2 - \omega_2 h_3\right) + N_C + N_e\right)I_{xx}^{-1} \\ \left(\omega_1\omega_3\left(I_{zz} - I_{xx}\right) + \left(\omega_1 h_3 - \omega_3 h_1\right) + N_C + N_e\right)I_{yy}^{-1} \\ \left(\omega_1\omega_2\left(I_{xx} - I_{yy}\right) + \left(\omega_2 h_1 - \omega_1 h_2\right) + N_C + N_e\right)I_{zz}^{-1} \end{bmatrix} \tag{6}$$

The satellite kinematics is represented by the quaternions q = (q_1 , q_2 , q_3 , q_4) which can be white in function of the angular velocity by

$$\dot{q} = \frac{1}{2}\boldsymbol{\Omega}(\omega)q = \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix} q \tag{7}$$

Another way of describing the satellite kinematics is separating the scalar part of the quaternion vector q_4 from the other three quaternions elements, which defines the Gibbs vector represented by $\boldsymbol{g} = (q_1 , q_2 , q_3)$. As a result, the kinematic equation in terms of the Gibbs vector is

$$\dot{\boldsymbol{g}} = -\frac{1}{2}\mathbf{S}(\omega)g + \frac{1}{2}q_4\omega \quad ; \quad \dot{q}_4 = -\frac{1}{2}\omega^T g \tag{8}$$

which in the compact form is given by

$$\begin{bmatrix} \dot{g} \\ \dot{q}_4 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -S(\omega) \\ -\omega^T \end{bmatrix} g + \frac{1}{2}q_4\begin{bmatrix} I_{3x3} \\ 0 \end{bmatrix}\omega \tag{9}$$

Finally, the satellite kinematic in matrix form in terms of the quatenions is given by

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \\ -\omega_1 & -\omega_2 & -\omega_3 \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} q_4 & 0 & 0 \\ 0 & q_4 & 0 \\ 0 & 0 & q_4 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \tag{10}$$

In order to simulate the satellite dynamics by its angular velocity and attitude (Gibbs vector) one must uses Eq.10 plus Eq. (11) considering the external torque $\mathbf{N}_g$ equal to zero.

$$\begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \end{bmatrix} = \begin{bmatrix} (\omega_2\omega_3(I_{yy} - I_{zz}) + N_C)I_{xx}^{-1} \\ (\omega_1\omega_3(I_{zz} - I_{xx}) + N_C)I_{yy}^{-1} \\ (\omega_1\omega_2(I_{xx} - I_{yy}) + N_C)I_{zz}^{-1} \end{bmatrix} \tag{11}$$

Considering the Lyapunov theory (Junkins and Kim, 1993) the control law U can be given by

$$U_i = N_c = -[k_i w_i + k_0 q_i (1 + q_i q_i + q_i q_i + q_i q_i)] \quad \text{with} \quad i=1,2,3 \tag{12}$$

The linear model of the satellite can be obtained considering the angular deviation of the satellite in terms of Euler angles of Yaw (β) , Pitch (ψ) and Roll (Φ) (Curtis,2010) are given by Eqs.13a,b,c (see Figure2).

$$\dot{\beta} = W_1 cos\psi + W_3 sin\psi \tag{13.a}$$

$$\dot{\psi} = W_2 - (W_3 \, cos\psi + W_1 \, sin\psi) \; tan\beta \tag{13.b}$$

$$\dot{\phi} = (W_3 \, cos\psi + W_1 \, sin\psi) \; cot\beta \tag{13.c}$$

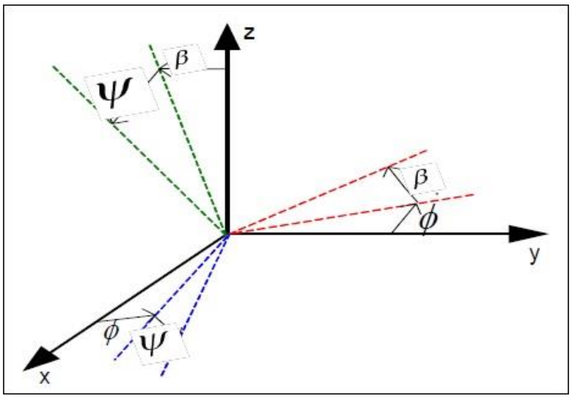

Figure 2 - Euler angles of Yaw (β) , Pitch (ψ) and Roll (Φ).

Assuming that the angular deviations are small, one can has

$$\dot{\beta} = W_1 \; ; \dot{\psi} = W_2 \; ; \dot{\phi} = W_3 \tag{14}$$

In order to simulates the linear model of the satellite, one defines the following state variables:

$$X_1 = W_1 \quad X_4 = \theta_1 \tag{15.a}$$

$$X_2 = W_2 \quad X_5 = \theta_2 \tag{15.b}$$

$$X_3 = W_3 \text{ e } X_6 = \theta_3 \tag{15.c}$$

Rewriting the equations 15a.b.c in the matrix form one has

$$\dot{X} = AX + BU \tag{16}$$

where

$$A = \begin{vmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{vmatrix} \quad B = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} \tag{17}$$

In this notation the state variables x = (w_1 , w_2 , w_3 , β ,ψ Φ) and U is the control variable, which when designed by the LQR technique, one must minimize the performance index given by

$$J(X_0, U(.)) = \int_0^{\infty} (X^T Q X + U^T R U) dt \tag{18}$$

$$Q = \begin{vmatrix} \lambda_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda_6 \end{vmatrix} \quad R = \begin{vmatrix} \lambda_7 & 0 & 0 \\ 0 & \lambda_8 & 0 \\ 0 & 0 & \lambda_9 \end{vmatrix} \tag{19}$$

That represents the weights related with the state and the control, respectively.

3 – SELECTION OF WEIGHT MATRICES

As for the simulation of the nonlinear satellite model and the design of its ACS using the Lyapunov theory by nonlinear feedback control law of Eq.12 which guarantees stability of the nonlinear closed-loop under the assumption of zero model errors. However, the determination of the particular gains values selection should be based on globally stabilization procedure (Junkins and Kim, 1993) which is a bit more complicate. Therefore, the determination of the gains used here will be based on simpler performance optimization criteria like system overshot and stabilization time.

As for the simulation of the linear satellite model and the design of its ACS using LQR theory one must understand the minimization of Eq. 18 which are function of the selection of the Q and R matrices. In order to do that one observes that an arbitrarily rapid reduction in the state can be achieved at the expense of an increase correspondingly large control, implying, in a practical impossibility to implement such a solution. Moreover, an arbitrarily large reduction in control may cause a significant elevation of the state, an undesirable situation in the attitude control process.

As a result, the selection of gain of the Lyapunov control law and the weight matrices of the LQR method becomes an extremely laborious process. However, in simulation of both case the choice will be made through trial and error, verifying, which values of these gains and matrices best meet criteria such as overshot, maximum control energy and settling time, reflecting a better performance system.

4 – SIMULATION RESULTS

In the simulation the initial values for angles (rad) and angular velocities (rad/s) are (w_1 = 0.1 , w_2 = -0.5, w_3 = 0.1, β = 0 ,ψ = -0.1 Φ = 0.5). The values are for the in final phase of pointing of the TD-1A satellite (Cubillos, 2005). The moment of inertia ($Kg.m^2$) are $I_x = 225$, $I_y = 207$, $I_z = 121$.

Figures 3 & 4; 5 & 6 ; 7 & 8 show the variation of angles, angular velocities and control law energy in the time function for the LQR controller with weights matrices $\lambda_{1 \dots 6} = 1000$; $\lambda_{8 \dots 9} = 0.1$ and $\lambda_{1 \dots 6} = 1000$; $\lambda_{8 \dots 9} = 0.01$, respectively.

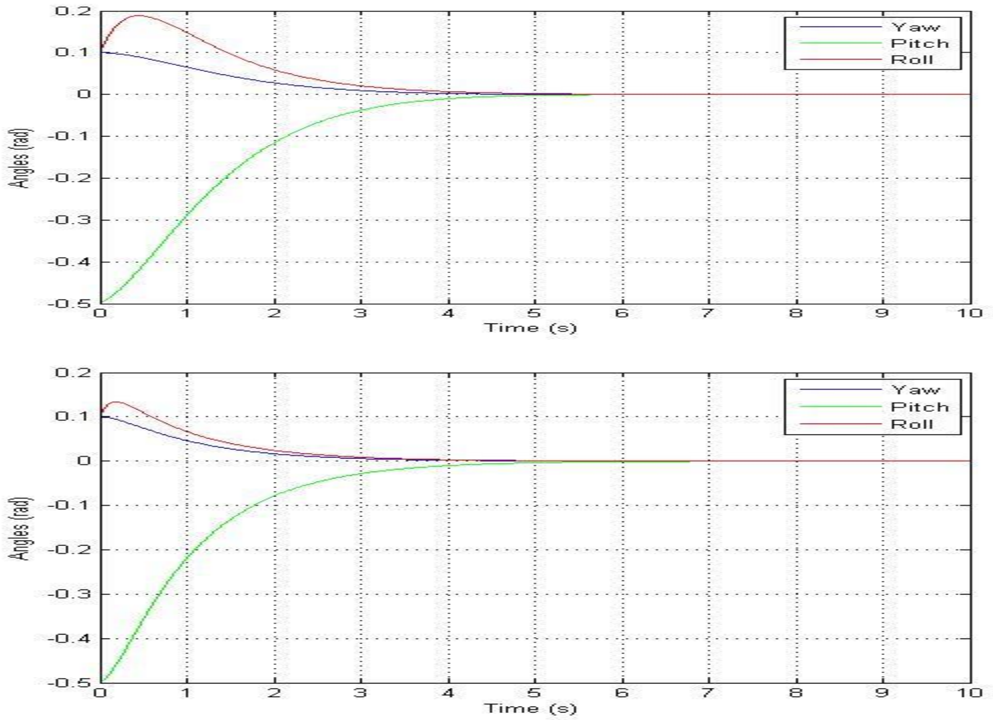

Figure 3 and 4 – Angles in function of time of the LQR Controller

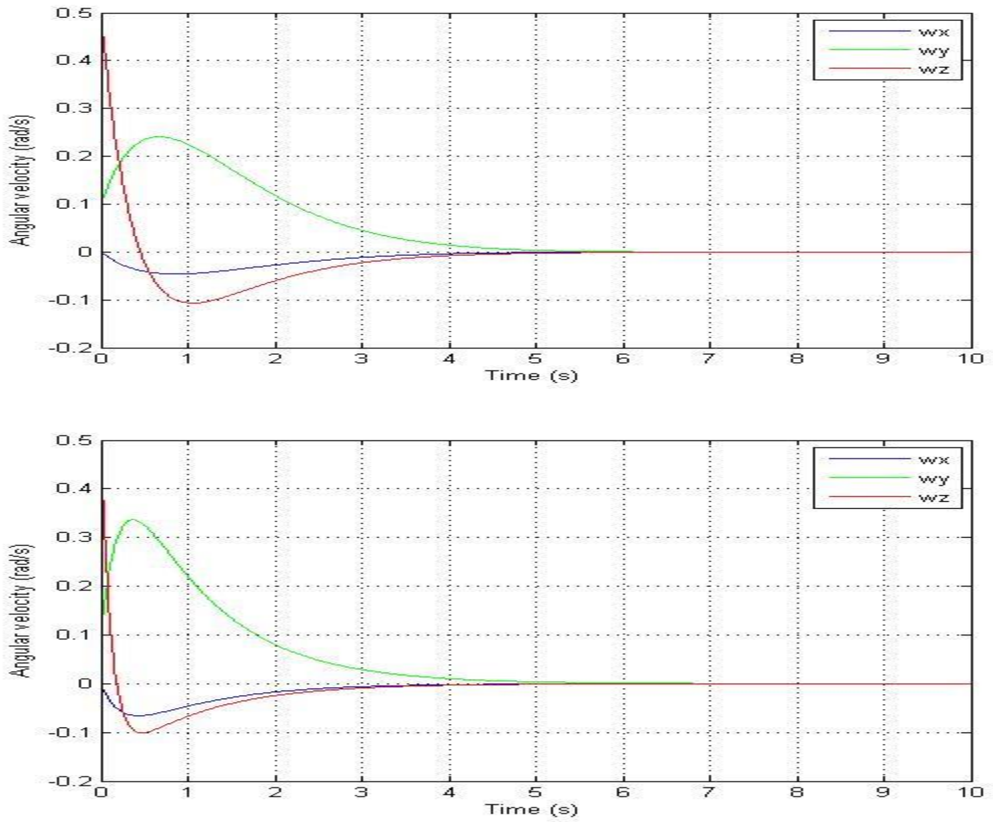

Figure 5 and 6 – Angular velocities in function of time of the LQR Controller

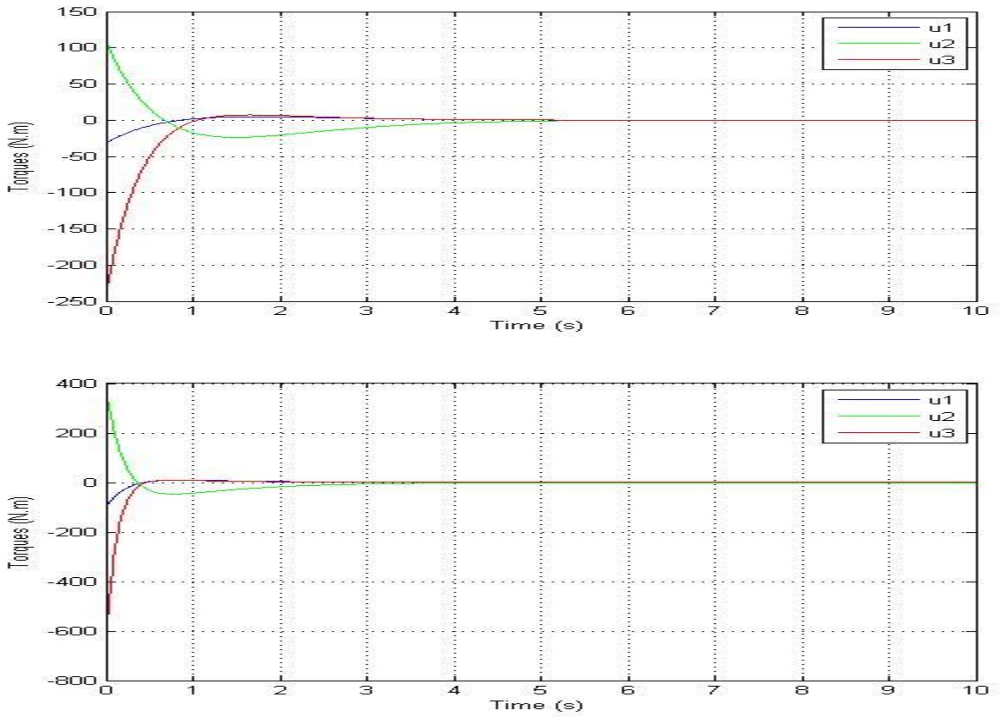

Figure 7 and 8 – LQR control law energy in function of time

Figures 9 & 10; 11 & 12 ; 13 & 14 show the variation of angles, angular velocities and control law energy in the time function for the Lyapunov controller with weights matrices $k_0 = 1000$; $k_{1 \dots 3} = 100$ and $k_0 = 1000$; $k_{1 \dots 3} = 1000$, respectively.

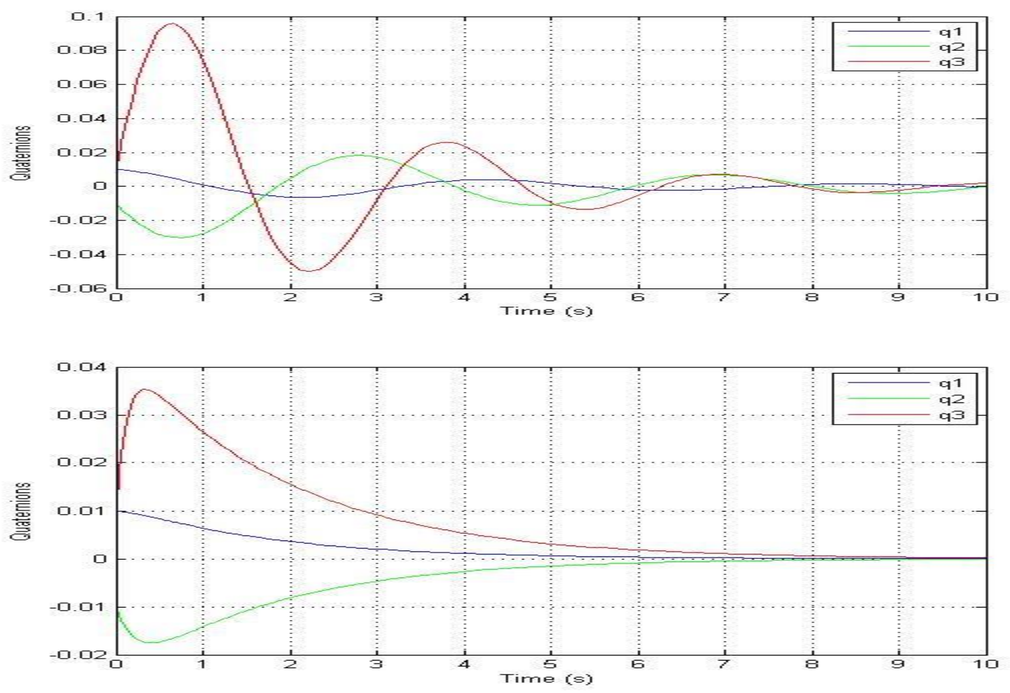

Figure 9 and 10 – Quaternions in function of time of the Lyapunov controller

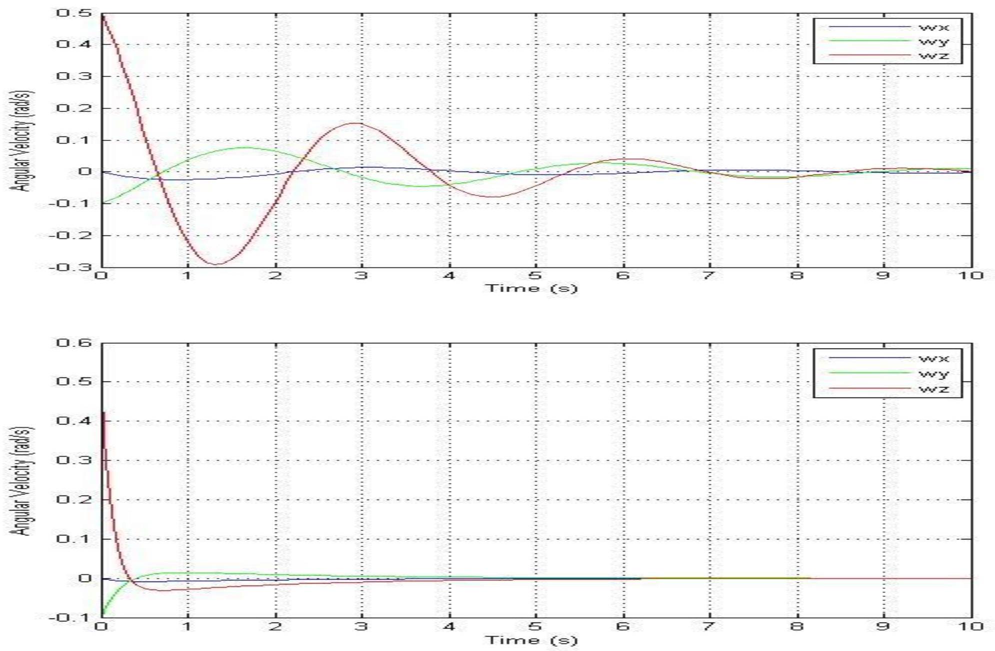

Figure 11 and 12 – Angular velocities in function of time of the Lyapunov controller

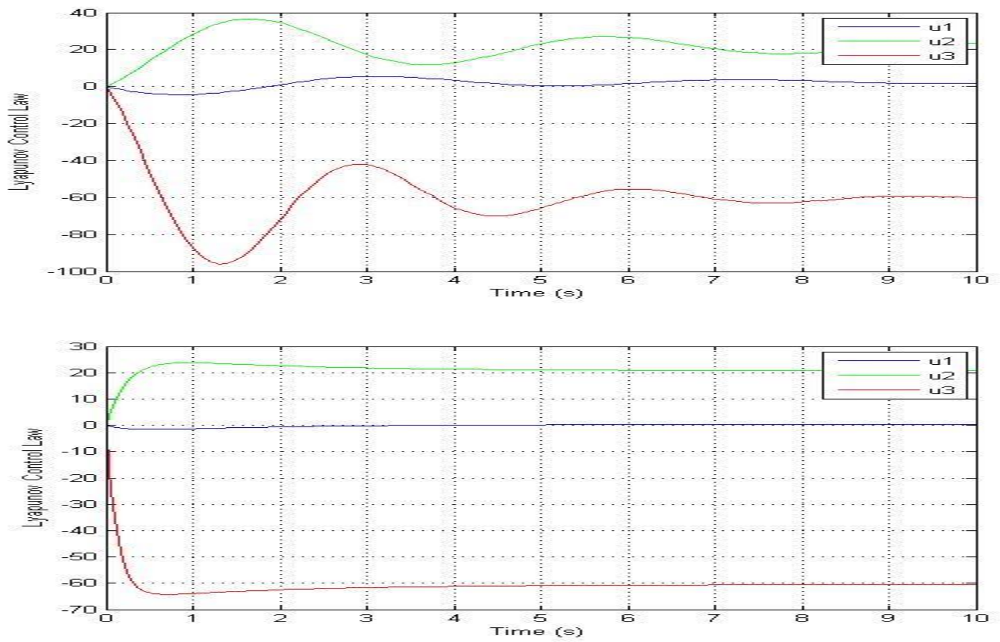

Figure 13 and 14 – Lyapunov control law energy in function of time

5- SUMMARY

From the results of the simulations it is possible to note that the performance of LQR controller is much better than the Lyapunov controller. Mainly, as for controlling the attitude and energy consumption. But it should be noted that the linear model used by LQR controller certainly does not represent the reality of the satellite since the basic dynamics of a satellite in rotation is 'given by the equations of Euler. Moreover, the fact that also calls attention in these simulations is the Lyapunov control law behavior, once it does not have its action taken to zero, that is, it continues to operate in order to maintain the angular velocities of the satellite reduced. This fact also indicates that the nonlinear terms of the equations of motion still alive. Finally, it is important to remember that the results for both, the LQR controller and the Lyapunov controller are function as the matrices weights, then better results can be achieved with other matrices weights.

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