



Instrumentation Of a Nonlinear Pendulum Using Arduino Microcontroller

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Abstract. *In this paper the Arduino microcontroller together with the gyroscopic sensor for measuring vibrations in mechanical systems are used. The objective is to present the Arduino platform as an alternative for low cost signal acquisition, easy use and good precision for educational purpose. In this application the system presents a nonlinear behavior for certain initial conditions. The characterization of the system's nonlinearities are done by the comparison between the signal measured and a numerical solution of the classic equation of motion of the pendulum. It is also presented a scenario where the pendulum is excited by an magnetic interaction. The results showed in conformity with the literature.*

Keywords: *Arduino microcontroller, sensor, vibrations, nonlinear, education*

1 INTRODUCTION

In undergraduate courses of engineering and physics, the students are often taught how to model some physical phenomenon. The modeling generally makes a use of an abstract mathematics which is far no intuitive, giving the students only a superficial idea of the models. In order to make the students understand more the physical phenomenon as well as have an intuitive idea of it, the use of experimentation is often a good way to go.

The problems that one faces when decides to go this way is the cost and, sometimes, the complexity of the experimental procedure needed. With the purpose of making these procedures more easy and inexpensive, in this paper the use of the Arduino microcontroller is proposed. The use of the Arduino as an acquisition system present satisfactory precision for educational proposes (Sarik et al., 2010; Balogh, 2010; Cavalcante et al., 2011; Varanis et al., 2016).

In this paper the Arduino microcontroller is used to measure the kinematic quantities of a nonlinear pendulum with a magnetic interaction. The signals measured will be compared with a mathematical model. Our focus is to present the use of the microcontroller as an alternative to measuring vibrations in mechanical systems with an educational purpose. Moreover, magnets are added in order to let the pendulum's movement with a nonlinear character.

First, measurements of the free response of the pendulum with five different initial conditions will be done, and then the signals obtained will be compared to an analytical model. This comparison will show the outcome of the use of the Arduino as a acquisition system and its use for educational purpose is stressed. Also, the phase space portrait will be used to compare the numerical solutions and the experimental signals.

After that, the movement of the pendulum subjected to magnetic interaction will be characterized. As it will be discussed, the movement under magnetic interaction showed a behavior near to chaotic, but for a real characterization of the movement as one of this kind, more advanced mathematical tools are needed, which are out of the scope of this paper. This application present the use of the Arduino as an alternative acquisition system for studies of nonlinear behavior for students.

2 Arduino Microcontroller

The Arduino is an open-source microcontroller board, which has become very popular among the general public by being easy to use, simple programming and low prices, making it a great tool to quickly assemble and test electronic prototypes (Bhmer, 2012).

In this paper, the Arduino Mega 2560 Rev. 3 board was used, based on the Atmel ATmega2560 chip. This board contains 54 digital pins I/O (input/output), of which 15 pins can be used as PWM outputs, 16 analog inputs and 4 serial ports. It can be powered either by USB port of a computer or by an external power source.

The purpose of this paper is to use the Arduino microcontroller to obtain angular velocity signals of the physical pendulum proposed. In order to allow access to undergraduate students in physics and engineering courses for the experimental study of linear and nonlinear oscillations, through sensors like accelerometer and gyroscope. Since such sensors have low cost, because of the popularization of smartphones, tablets and other devices that use them widely (Shoaib et al., 2013; König et al., 2011).

The gyroscope sensor used is a sensor capable of measuring angular speed. The sensor used in this paper is contained in the MPU-6050 chip, which also have an accelerometer sensor inside it, capable of measuring accelerations. However, only the angular velocity signals given by the gyroscope sensor was used. The sensor used has a measuring range of 2000 degree/s in the three axes x, y, z .

3 Pendulum's Equation of Motion

The pendulum is a mechanical system which perhaps has been already fully described, and yet it still being studied due to its simplicity and the generalizations that can be made with it. A commonly use of this system is for educational purposes. An example of a use for that purpose is the introduction of some ideas of nonlinear motion, since the equation that describe the pendulum is nonlinear.

The equation of motion of a physical pendulum with damping is given by,

$$\ddot{\theta}(t) + \frac{\alpha}{I}\dot{\theta}(t) + \frac{mgL}{I}\sin\theta(t) = 0 \quad (1)$$

where $\theta(t)$ is the angular displacement in function of time, m is the mass, g the gravity, α the damping coefficient, I the moment of inertia from the rotating axis and L is the pendulum's length. Equation 1 is a second-order nonlinear ordinary differential equation. For small displacements the approximation $\sin\theta(t) \simeq \theta(t)$ is generally done. Thus Eq. 1 is commonly written as,

$$\ddot{\theta}(t) + \frac{\alpha}{I}\dot{\theta}(t) + \frac{mgL}{I}\theta(t) = 0 \quad (2)$$

which is now a second-order linear ordinary differential equation.

The solution of Eqs. 1 and 2, will be compared with the experimental signals, so that one can see which equation best represents the real physical pendulum proposed. The equations will be solved numerically. The analytical solution of Eq. 1 can be seen in (Beléndez A. et al., 2007).

4 Acquisition and Methods

The pendulum which its kinematic quantities will be measured is shown in Fig. 1, and consist in a physical pendulum made with aluminum which has a nonlinear mass distribution. It has a cylindrical geometry with a diameter of 16 mm, and a sliding joint at its end which can alter its length, which has a maximum value of 560 mm an minimum of 465 mm. The total mass of the system is 0.275 kg and the moment of inertia is 0.91 kg-m². The distance of the center of mass from the axis of rotation is 180 mm.

The sensor was fixed in the middle of the pendulum, and its position is shown in Fig. 2. For the data acquisition the MPU-6050 chip was used, and its link was made with the Arduino in the same way as in (Varanis et al., 2016). The data that were given by the sensor were passed to the computer through the USB port and processed by the software Matlab. As the signal is not given by the sensor in SI units, a conversion was made to rad/s in the same way as in (Varanis et al., 2016). The signals were obtained with a sample frequency of 40 Hz and 2048 points.

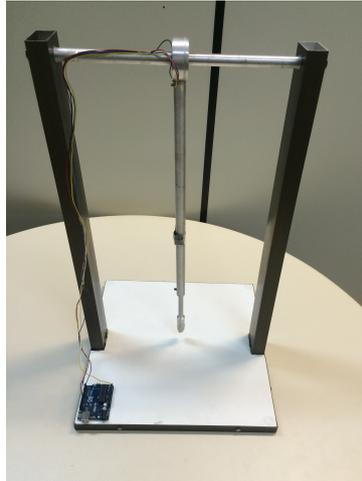


Figure 1: Physical Pendulum

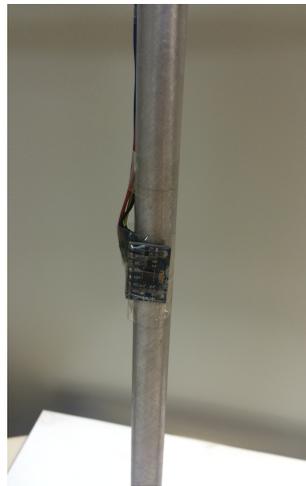


Figure 2: Localization of the sensor



Figure 3: Position of the Magnets

In the end of the pendulum a magnet was placed to interact with the others magnets placed in the base. The position of the magnets that will excite the pendulum can be seen in Fig. 3. The magnets have a cylindrical geometry with a 10 mm diameter and different lengths, they are made of neodymium and their grade is N35. It was used three magnets and different arrangements of their positions for measuring the pendulum's kinematic quantities. Since the goal of this paper is to present the use of the Arduino only, the positions of the magnets were

chosen randomly.

5 Results and Discussions

It was executed four measurements with four different initial conditions: 20, 45, 90 and 180 degrees. The pendulum was released from rest, thus the initial velocity was zero. these initial conditions were used to solve Eqs. 1 and 2, to make the comparison between the experimental signal. In addition, it will be compared the displacement given by the sensor and by the numerical solution of Eqs. 1 and 2. Since the gyroscopic sensor only gives the angular velocity, the displacement was obtained by numeric integration.

Figures 4, 5, 6 and 7, show the comparison between the signals measured and the solutions of the equation of motion. The red graphs represent the experimental signal, the blue graphs the solution of Eq. 1 and the green graphs the solution of Eq. 2. One can note by analyzing the figures that the pendulum presented a nonlinear damping. Because of that behavior, the movement of it is not well described by the solution of the equation of motion. The two responses were also out of phase. In order to make the numerical solution closer to the signal, a nonlinear damping must be considered in the equation of motion of the pendulum.

In Fig. 7 one can note that for large displacements the linear equation (Equation 2) does not describe appropriately the real motion of the pendulum. This is expected since this is an approximation for small displacements. On the other hand, Figs. 4, 5 and 6, showed that the approximation $\sin\theta \simeq \theta$ is plausible since the nonlinear and linear solutions do not differ very much.

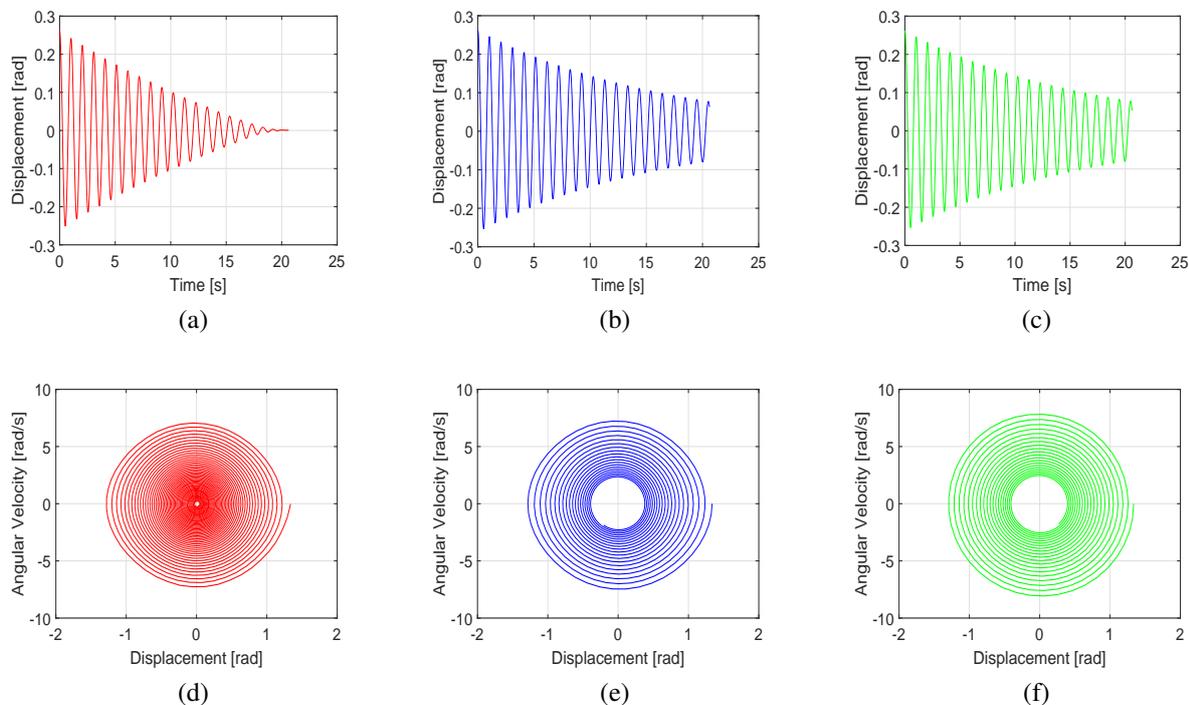


Figure 4: Experimental Signal (Fig. 4a), and Numerical Solutions (Figs. 4d and 4c), for 20 degrees

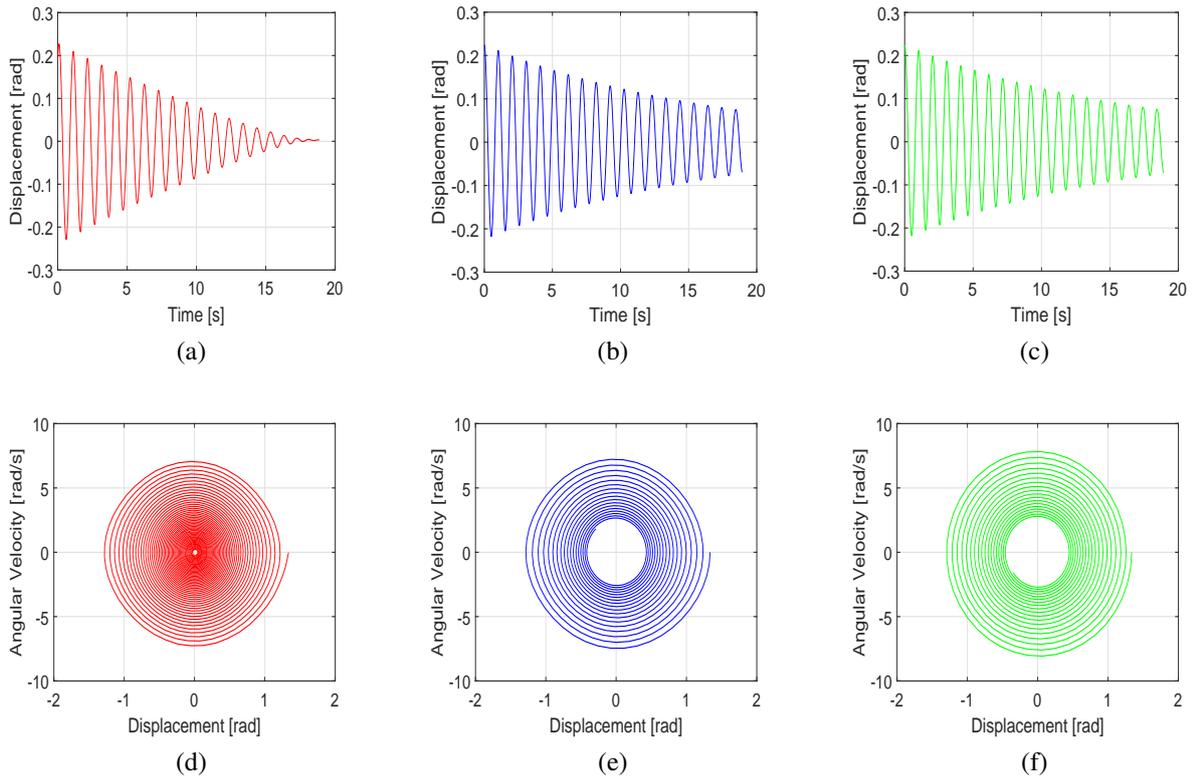


Figure 5: Experimental Signal (Fig. 5a), and Numerical Solutions (Figs. 5b and 5c), for 45 degrees

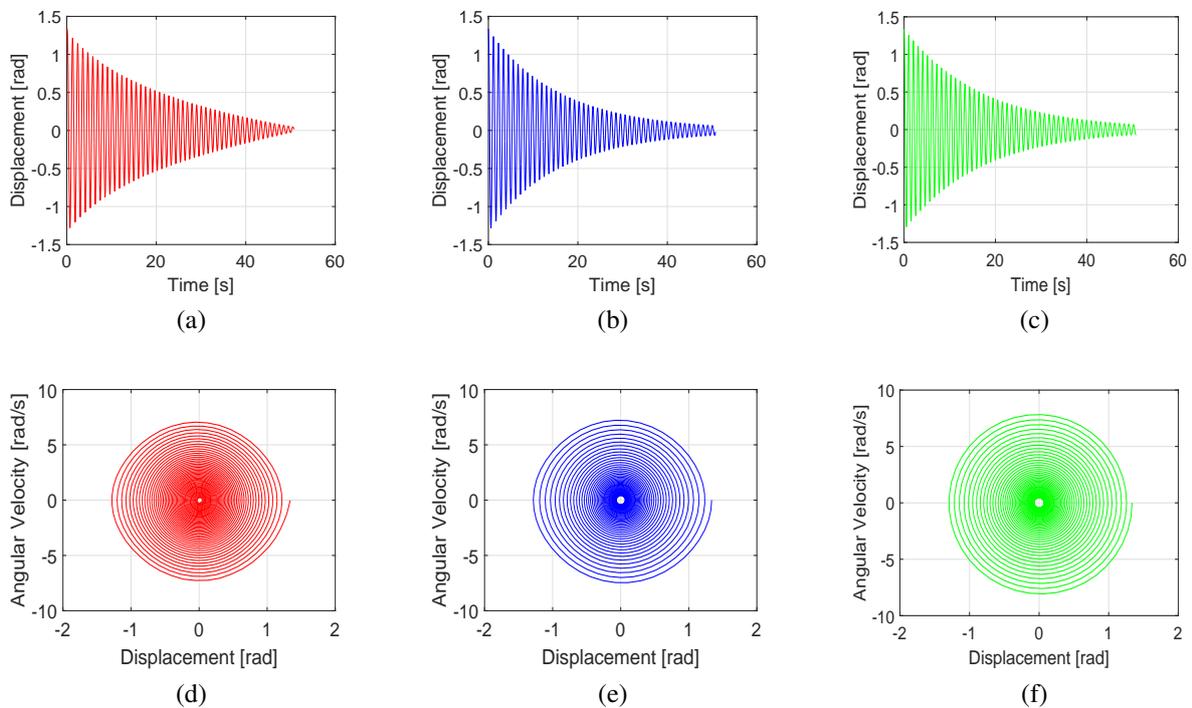


Figure 6: Experimental Signal (Fig. 6a), and Numerical Solutions (Figs. 6b and 6c), for 90 degrees

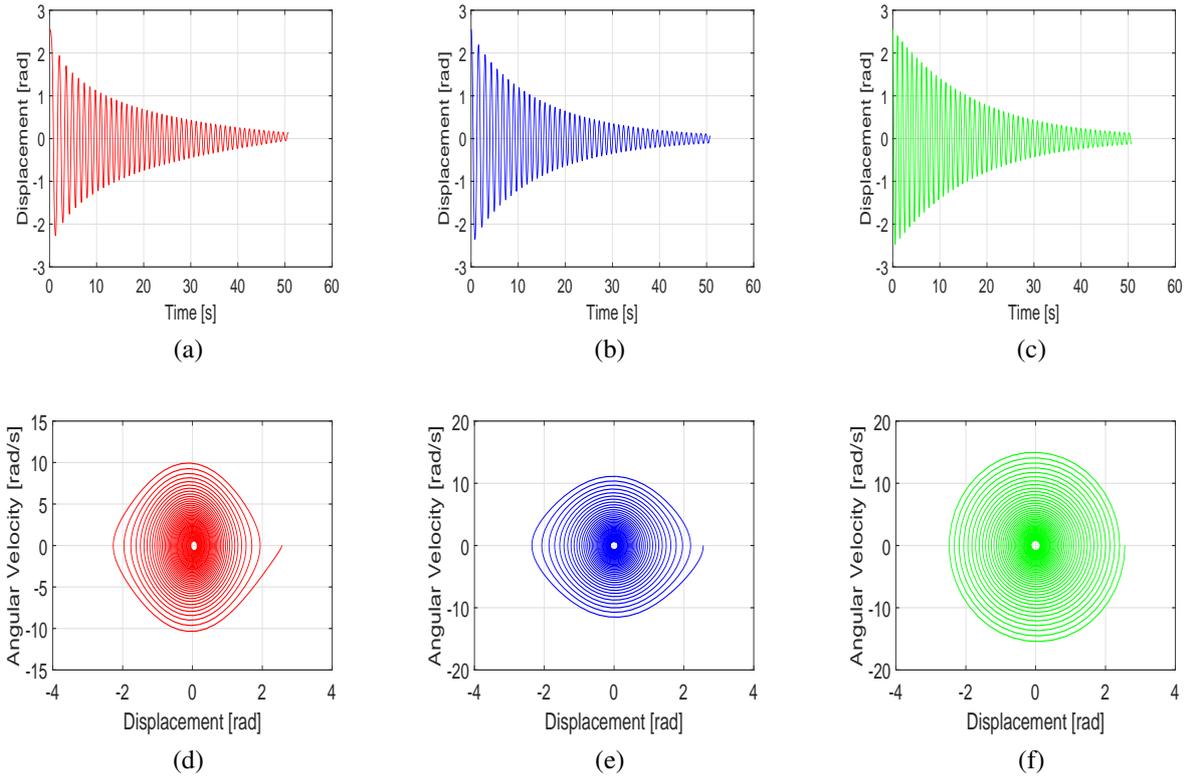


Figure 7: Experimental Signal (Fig. 7a), and Numerical Solutions (Figs. 7b and 7e), for 180 degrees

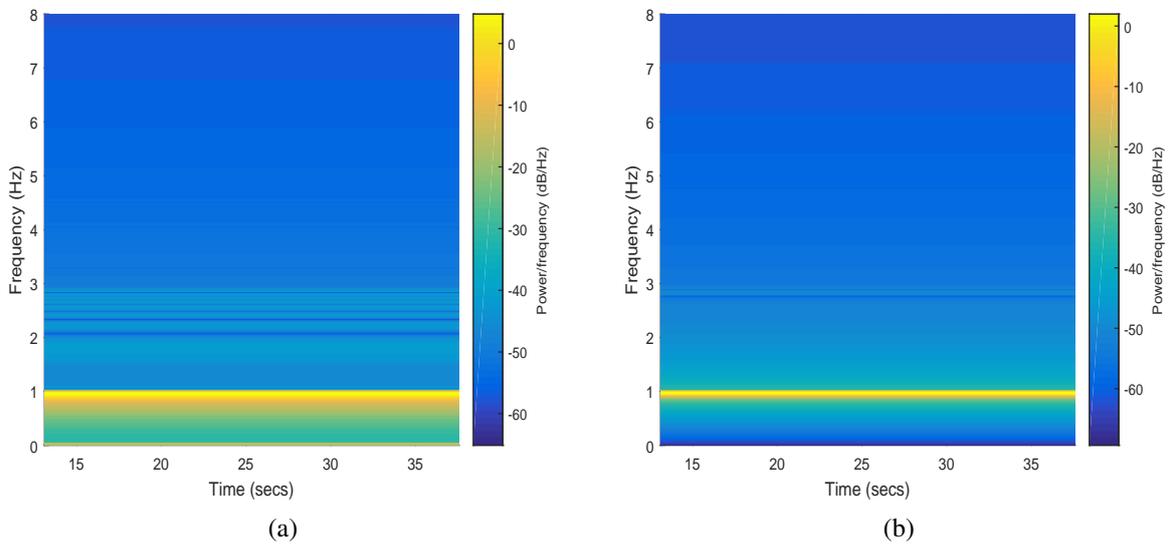


Figure 8: Short Time Fourier Transform of the Experimental Signal (Fig. 8a) and the nonlinear solution (Fig. 8b), for 180 degrees

Although the signal did not show similar to the numerical solution, the frequency measured by the sensor was reasonable close to the natural frequency of the pendulum, analytically obtained. To make that statement, the Short Time Fourier Transform (STFT) was passed in the

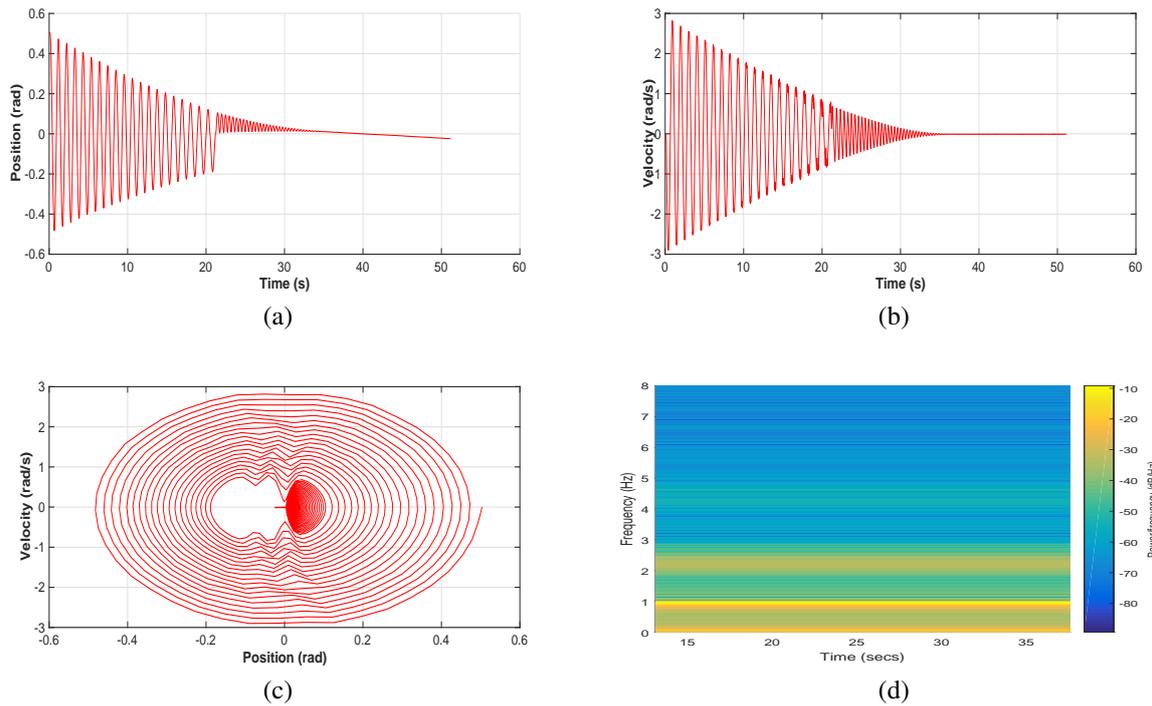


Figure 9: Position Signal, Velocity Signal, Phase Space and STFT (Figures 9a, 9b, 9c and 9d; respectively) case with magnetic interaction

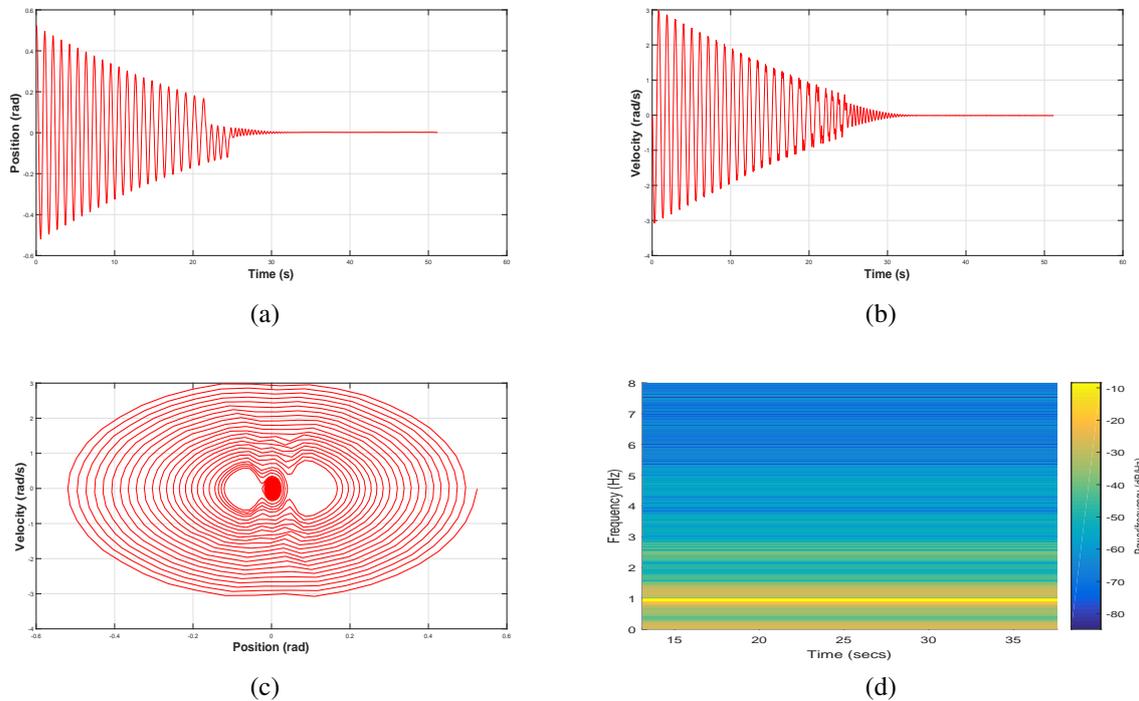


Figure 10: Position Signal, Velocity Signal, Phase Space and STFT (Figures 10a, 10b, 10c and 10d; respectively) case with magnetic interaction

experimental signal for the initial displacement of 180 degrees; the result is showed in Fig. 8. The frequency obtained by using the STFT was 0.9375 Hz, which was close to the analytical frequency, that is 0.957 Hz.

Figures 9 and 10 show the measurements of the pendulum with magnetic interaction. The nonlinear characteristic is clearly noted by analyzing the phase space portrait (Figs. 9c and 10c). The application of the STFT in the signals (Figs. 9d and 10d) gave an interesting results by showing some others low energy peaks besides the natural frequency peak.

In others measurements made, not showed here, it was observed that the pendulum had a tendency in resting next to the magnet with more influence, as one can see in the positions graphs which the magnetic interaction was on (Figures 9a and 10a). With appropriate tools, the real characterization of the pendulum's motion as a chaotic one can be done.

The results of the use of the Arduino microcontroller turned out to be reliable. The gyroscopic sensor did not catch too much noise in the measurements. For educational purposes, the use of the Arduino microcontroller is well justified.

6 Conclusions

In this paper, it was presented the use of the Arduino microcontroller as an acquisition system with the purpose of letting the experimental study of nonlinear phenomena in undergraduate courses as a possible accomplishment. Moreover, it was shown that the use of the microcontroller gives results that are suitable for education purposes.

As the results obtained are in agreement with the literature and they present a satisfactory precision for the application proposed, one can assert that the Arduino microcontroller is an excellent tool to be used as a support in classrooms, making them more dynamic and intuitive. It's worth noting that for more complex mechanical systems a better acquisition system is required

In our future work, a nonlinear damping will be added on the pendulum's equation of motion, to let the model more suitable to the real pendulum; and also make a mathematical model of the magnetic force that acted on the pendulum, in order to have a better understanding of the nonlinear motion.

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