Simulations of the Formation of STIPs (Systems of Tightly Packed Inner Planets)

PROJECT OF: ROSSETTO, PEDRO H.B.*

UNDER SUPERVISION OF: PROF. M. DUNCAN

September 4, 2017

Abstract

This paper investigates the process of migration of planets of masses ranging from 1 to 20 $M_\oplus$ in the protoplanetary disc (Type I Migration) and how it influences the architecture of Planetary Systems.The integrator SyMBA (Duncan et al., 1998) was used to generate data for analysis.

Keywords:Planet formation, planet migration, Type I Migration, STIP, computer simulations.

1   Introduction

Planets of masses ranging from 1 to 20 $M_\oplus$ can generate a disturbance in the protoplanetary disk that modifies the dynamics of the planet itself but does not modify greatly the structure of the disc. This modification can be divided into two torques: one due to the spiral wave created by the planet in the disc and another caused by the material near to the planet's orbit enclosed in the horseshoe region around the planet (*Baruteau et al.* (2014), see Figure 1). Then, the planet start to migrate in a process called *Type I Migration*.

The spiral wave torque, also referred as the Lindblad torque, is typically negative causing the planet to migrate inwards. A formula that with a good agreement describes this kind of torque in a adiabatic gas disk was given by *Paardekooper et al.* (2010):

$$\gamma\Gamma/\Gamma_0 = -(2.5 + 1.7\beta - 0.1\alpha) \qquad (1)$$

where γ is the ratio of specific heats and we assume that:

$$\Sigma = \Sigma_0 \left(\frac{a}{a_0}\right)^{-\alpha}$$

with Σ being the surface density and:

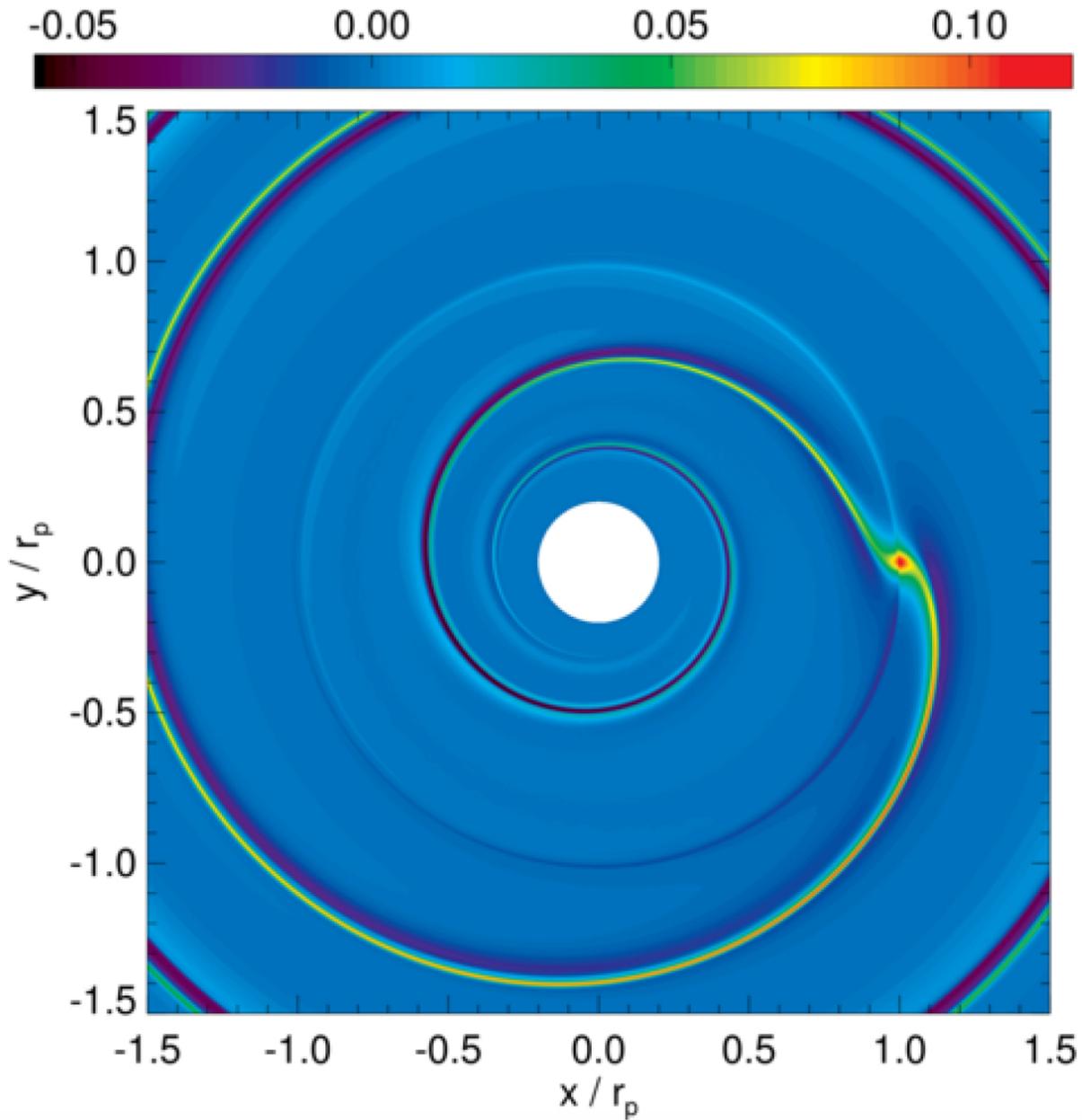
*pedrohenriquerossetto@gmail.com

Figure 1: *Disturbance created by a planet in the disk. Figure from Baruteau et al., 2014. The red dot shows the planet's location, at a distance r_p from the central star.*

$$T = T_0 \left(\frac{a}{a_0}\right)^{-\beta}$$

with T being the temperature. Γ_0 is the normalization factor given by:

$$\Gamma_0 = \frac{q^2}{h^2}\Sigma a^4 \Omega^2$$

where: q is the planet mass in units of the mass of its star, Σ the surface density of the gas disk at the planet location, a is the semimajor axis of the planet and Ω is its frequency; and, h the scale height of the disc in units of a.

It is already known that the Lindblad torque sets a really short time scale for planet migration and this is a challenge to the models since planets are observed to be common in solar systems. This problem has been approached by the addition of the corotation torque. This torque is exerted by the part of the disk inside the horseshoe region of the planet and it is normally directed outwards, balancing in some situations the Lindblad torque.

Paardekooper et al. (2010) presented a formula for the non-linear corotation torque $(\Gamma_{c,HS})$. It is:

$$\gamma\Gamma_{c,HS}/\Gamma_0 = 1.1\left(\frac{3}{2} - \alpha\right) + 7.9\frac{\xi}{\gamma} \tag{2}$$

where $\xi = \beta - (\gamma - 1)\alpha$. This formula is used for a non-saturated corotation torque, in an adiabatic disk.

The total torque that acts in a low-mass planet in a protoplanetary disk is therefore the sum of equations (1) and (2):

$$\Gamma = \Gamma_L + \Gamma_{c,HS}$$

that is:

$$\gamma\Gamma/\Gamma_0 = -0.85 - 1.7\beta - \alpha + 7.9\frac{\xi}{\gamma} \tag{3}$$

A similar formula was developed in the same paper by *Paardekooper* for isothermal disks, it is:

$$\Gamma/\Gamma_0 = -0.02 - 3.578\alpha \tag{4}$$

2 Methodology

The simulations here present were run in SyMBA (*Duncan et al., 1998*). The method of implementation of Type I Migration was through time scales. The adopted convention for the time scale is:

$$\tau = -\frac{a}{da/dt}$$

Considering an adiabatic disk, equation (3) yields the time scale:

$$\tau_{adi} = \gamma \frac{\tau_0}{c_a} \tag{5}$$

where $c_a = 0.85 + 1.7\beta + \alpha - 7.9\xi/\gamma$ and τ_0 being giving by:

$$\tau_0 = \frac{1}{2}\frac{h^2}{q}\frac{M}{\Sigma_P a^2}\Omega_P^{-1} \tag{6}$$

Considering an isothermal disk, equation (4) yields the time scale:

$$\tau_{iso} = \frac{\tau_0}{c_a} \tag{7}$$

where $c_a = 0.02 + 3.578\alpha$ and τ_0 is given by equation (6).

3   Tests and Simulations

Several simulations were run in order to test whether the code agrees with the behaviour proposed by the formulas (3) and (4). Some of these tests are shown in the subsections below.

After the tests were realised, it was observed that it was possible to generate a planetary trap for some specific values of α just by imposing the condition that the disc in a certain location changes its thermodynamic characteristics, i.e., goes from adiabatic in the inner region to isothermal in the outer region. Some of the simulations done exploiting this are also presented below.

All tests and simulations had gas surface density at 1 AU equal to $3.9 \cdot 10^4 g/cm^2$, which is the value used in what is called the Minimum Mass Solar Nebula.

3.1   Tests of the Code

The test simulations, from now on 'tests', were done with a wide range of planet masses and orbital radii. All tests were run until 10^5 years and had gas disk time scale equals to 10^7 years.

Equation (6) dictates how the time scale can possibly vary with a. In our models some assumptions were made on how certain quantities depends on a, such as:

$$H = H_0 a^{5/4}$$

Which led to the result:

$$\Sigma = \int_{-\infty}^{\infty} \rho dz = \sqrt{\pi}\rho H$$

Since $\Sigma = \Sigma_0 a^{-\alpha}$, we have that:

$$\rho = \rho_0 a^{-\delta}$$

where $\rho_0 = \Sigma_0/(H_0\sqrt{\pi})$ and $\delta = \alpha + 5/4$.

Now it is possible to check how τ_0 depends on a. According to the relationships stated above and with formula (6):

$$\tau_0 \propto \left(\frac{a^{5/4}}{a}\right)^2 \frac{1}{a^{-\alpha}a^2}a^{3/2} \propto a^\alpha$$

That means that if $\alpha = 0$, τ_0 yields the analytical answer that the migration is exponential. This is:

$$a(t) = a_0 e^{-t/\tau}$$

It is necessary to notice that this calculation was done ignoring the fact that the gas disk decays exponentially. This assumption is reasonable for situations like the ones presented in the tests because the whole simulation was run for 10^5 years and the gas disk decay time was 10^7 years.

Then in order to check that the simulations were done correctly it sufficient to check if at a certain time the relationship below holds:

$$\tau = t\left(\ln(\frac{a_0}{a(t)})\right)^{-1} \tag{8}$$

Figure 2 shows the graphic results for the test where the gas was completely isothermal and Figure 3 for when it was completely adiabatic. Tables 1 and 2 show the data produced by the simulations as well as the value of τ calculated by formula (8) (i.e., value of τ in the simulations) and by formulas (5) and (7) (i.e., analytical value of τ).

Table 1: *Data of the isothermal gas disk test and the calculated values of τ. The value of a was analyzed at the time 10^5 yrs in order to get a considerable migration of all planets from their original positions.*

Planet	Initial a	a after 10^5 yrs	Simulation τ	Analytical τ
1	1.00E+00	9.28E-01	1.33E+06	1.32E+06
2	3.00E+00	2.83E+00	1.76E+06	1.76E+06
3	5.00E+00	4.84E+00	3.17E+06	3.17E+06
4	7.00E+00	6.91E+00	7.92E+06	7.93E+06

Table 2: *Data of the adiabatic gas disk test and the calculated values of τ.*

Planet	Initial a	a after 10^3 yrs	Simulation tau	Analytical tau
1	1.00E+00	1.02E+00	-6.59E+04	-6.57E+04
2	3.00E+00	3.03E+00	-8.86E+04	-8.76E+04
3	5.00E+00	5.03E+00	-1.67E+05	-1.58E+05
4	7.00E+00	7.02E+00	-4.09E+05	-3.94E+05

The value of τ calculated analytically by formulas (5) and (7) was the same as, or close to, the value obtained by the simulations. Therefore, we can assume that the code is correct.
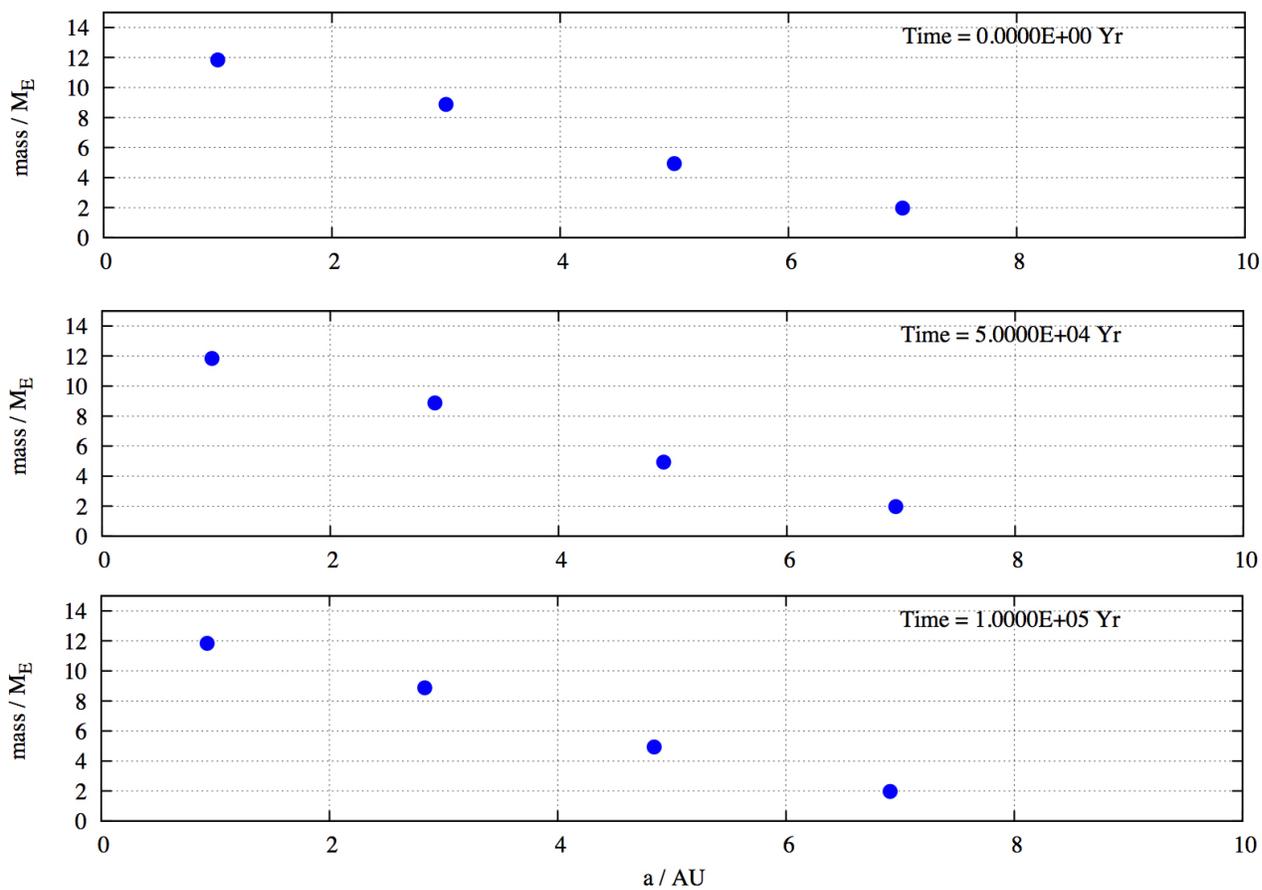
Figure 2: *Slight inwards migration of the planets in an isothermal gas disk with* α = 0. *On the x-axis is the semimajor axis a in AU of each planet and on the y-axis their masses in* $M_\oplus$.
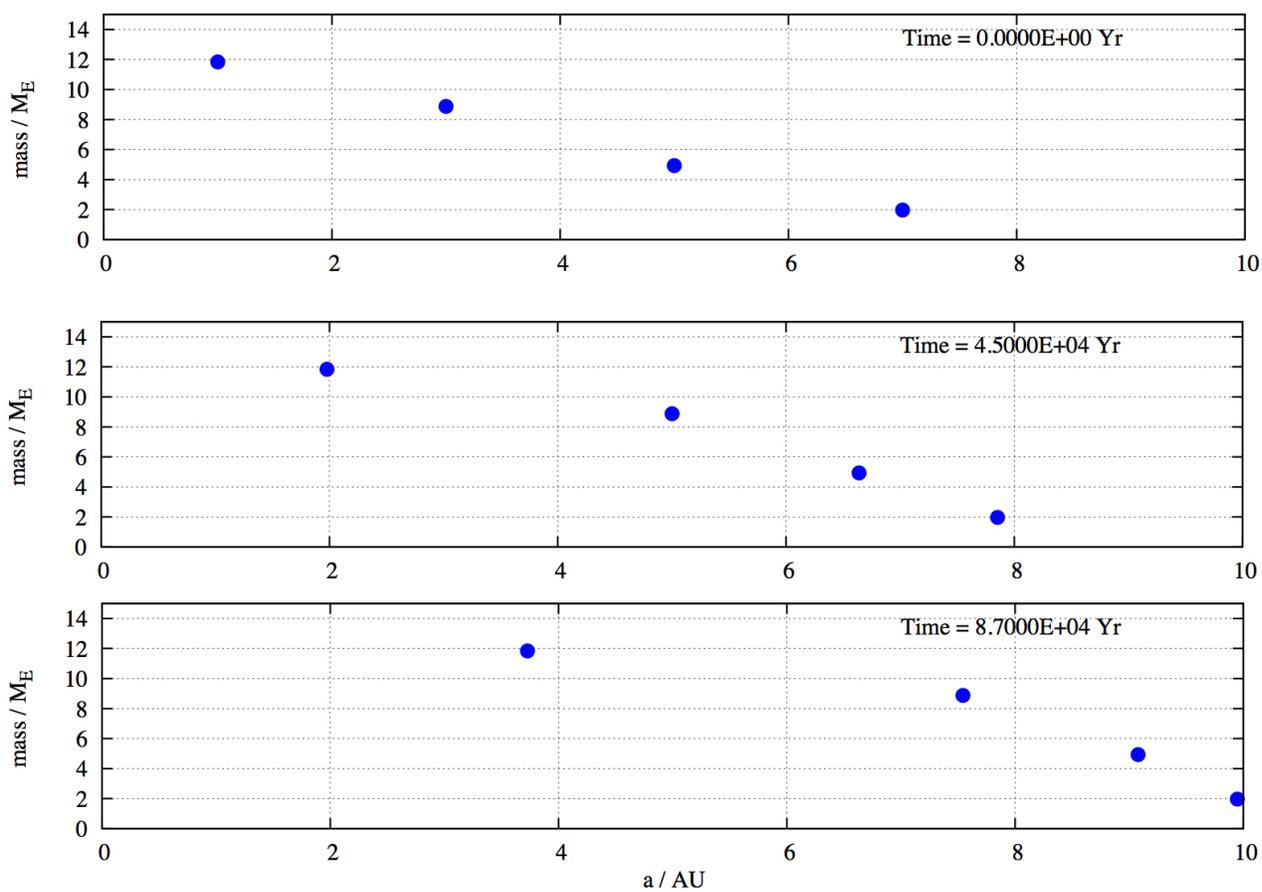
Figure 3: *Outwards migration of the planets in an adiabatic gas disk with α = 0. On the x-axis is the semimajor axis a in AU of each planet and on the y-axis their masses in $M_{\oplus}$.*

3.2   Simulations

The simulations here presented were done assuming that the gas disk is adiabatic up to 0.5 AU and isothermal from 0.5 AU and beyond. As noticed in the test section, the migration in the disk will be outwards in the adiabatic part and inwards in the isothermal part if the parameter α is close to zero. It is expected, therefore that stable system of planets can be formed even with the gas present.

The first set of initial system is exactly the one from the test section. It was observed the evolution of the system for α equals 0 (see Figure 4) and 0.1 (see Figure 5).
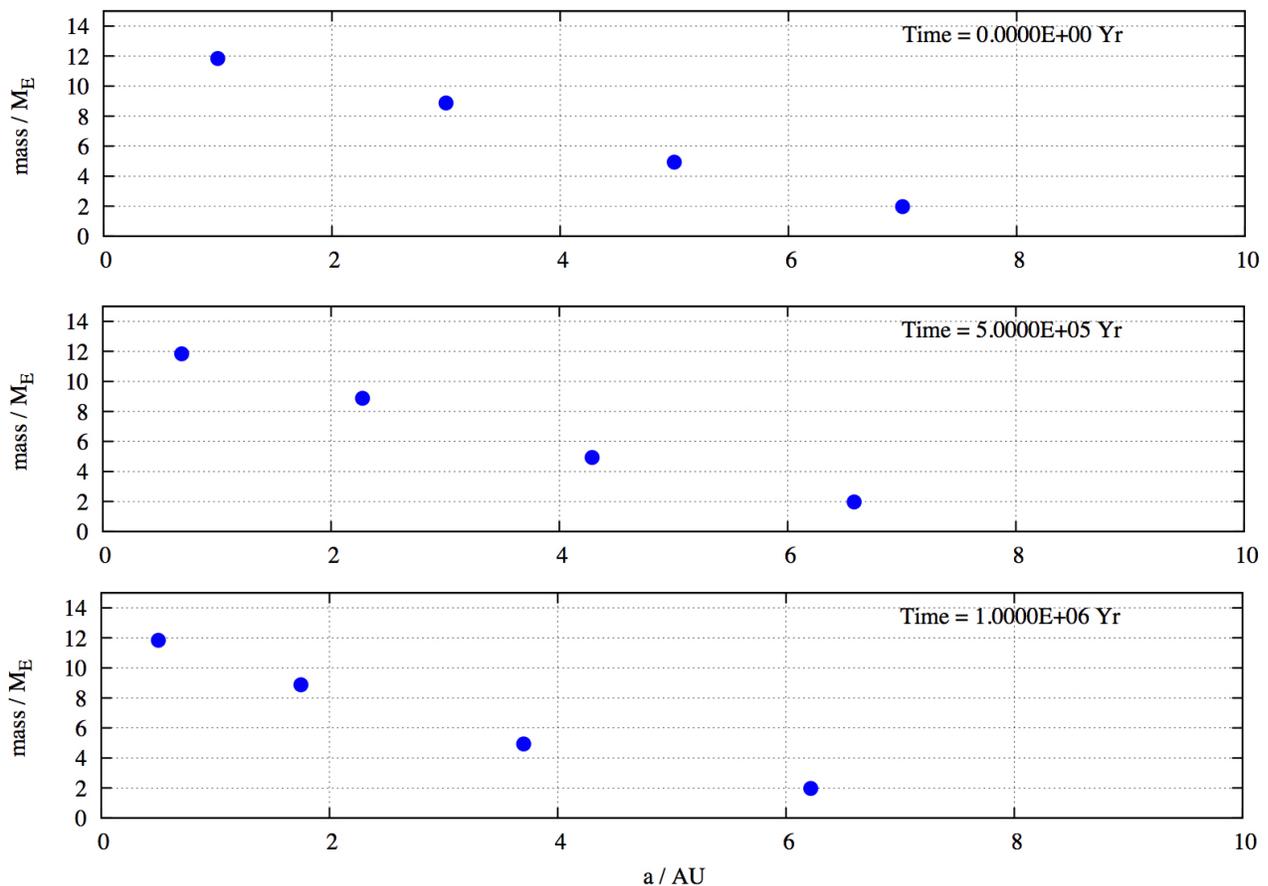

Figure 4: *The stability of this system is not guaranteed, the system did not reach a final state due to the slow rate of migration.*

It was noticed that the system with α equals 0.1 evolved faster as predicted by formulas (5) and (7). Therefore, the value $\alpha = 0.1$ was used for further investigation.

In order to get a general picture of the whole process 10 planets of mass randomly distributed around $3M_\oplus$ were distributed from 1 to 7 AU (see Figure 6).
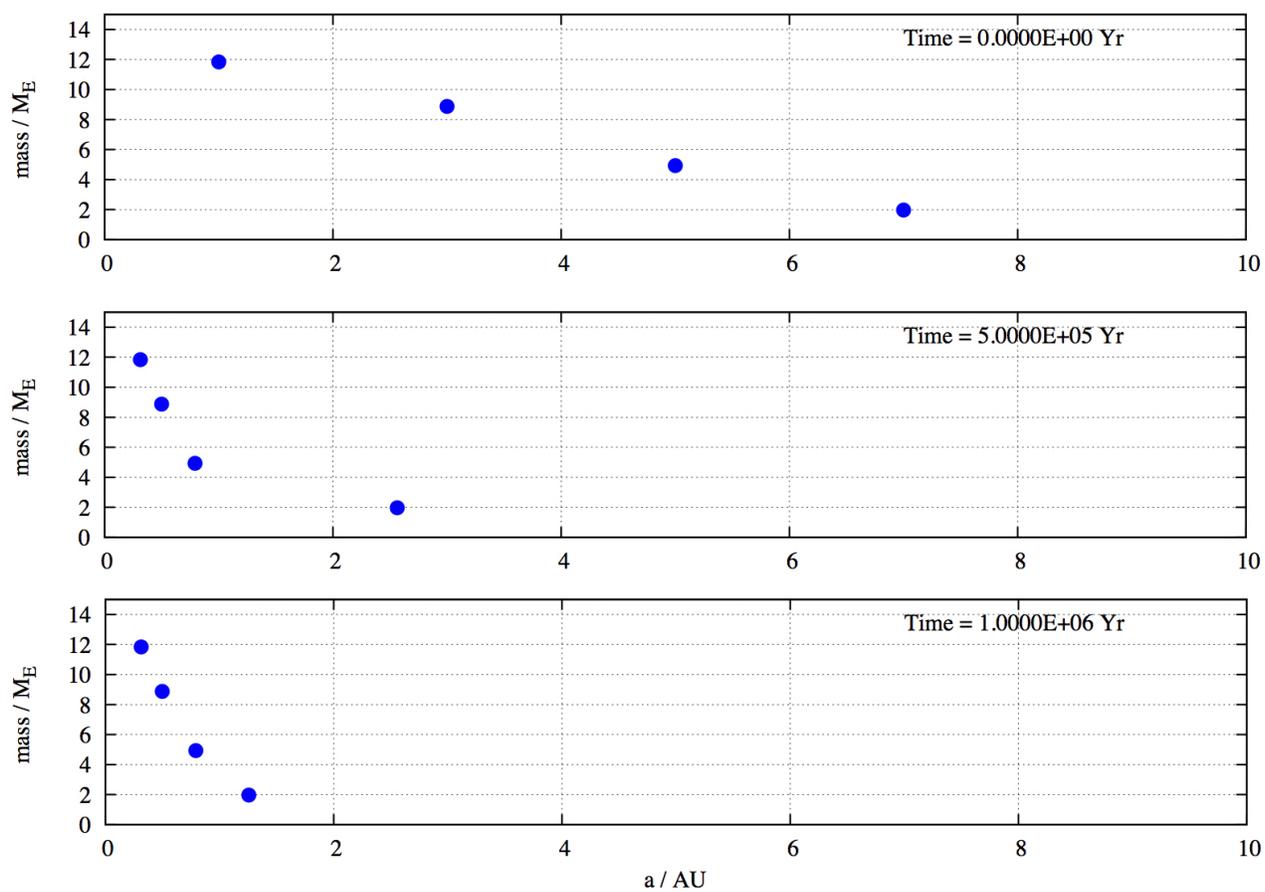
Figure 5: *In this simulation a stable formation of planets were observed. The configuration in the last panel did not seem to evolve.*
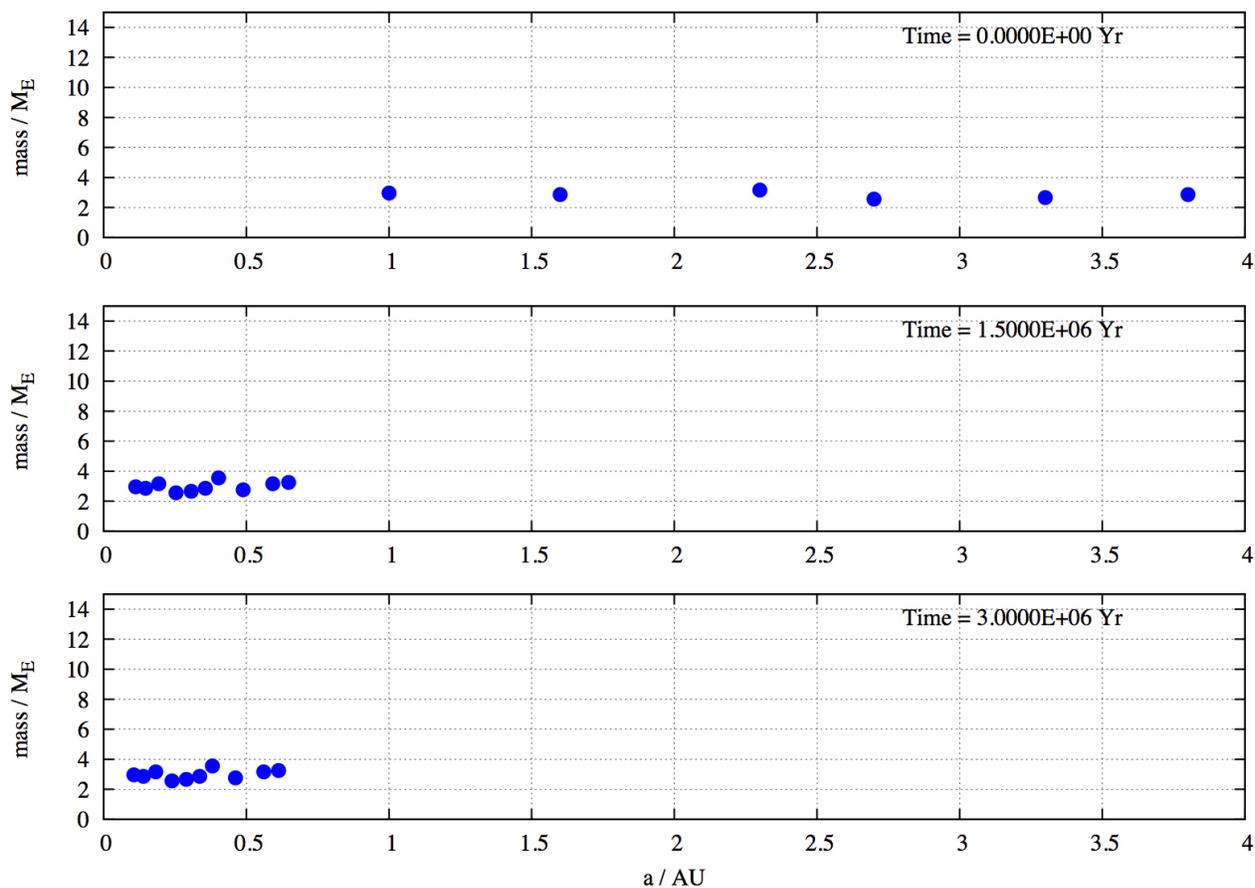
Figure 6: *The system again seems to have reached a final stage and no planetary accretion was observed.*

One can see that even though the planets got packed in much less spaced orbits they did not merge with each other. To test whether this system would also be possible in a longer and shorter lasting gas disk two new simulations were run: one with the gas disk decay time being 10^8 yrs (see Figure 7) and the other being 10^6 yrs (see Figure 8).
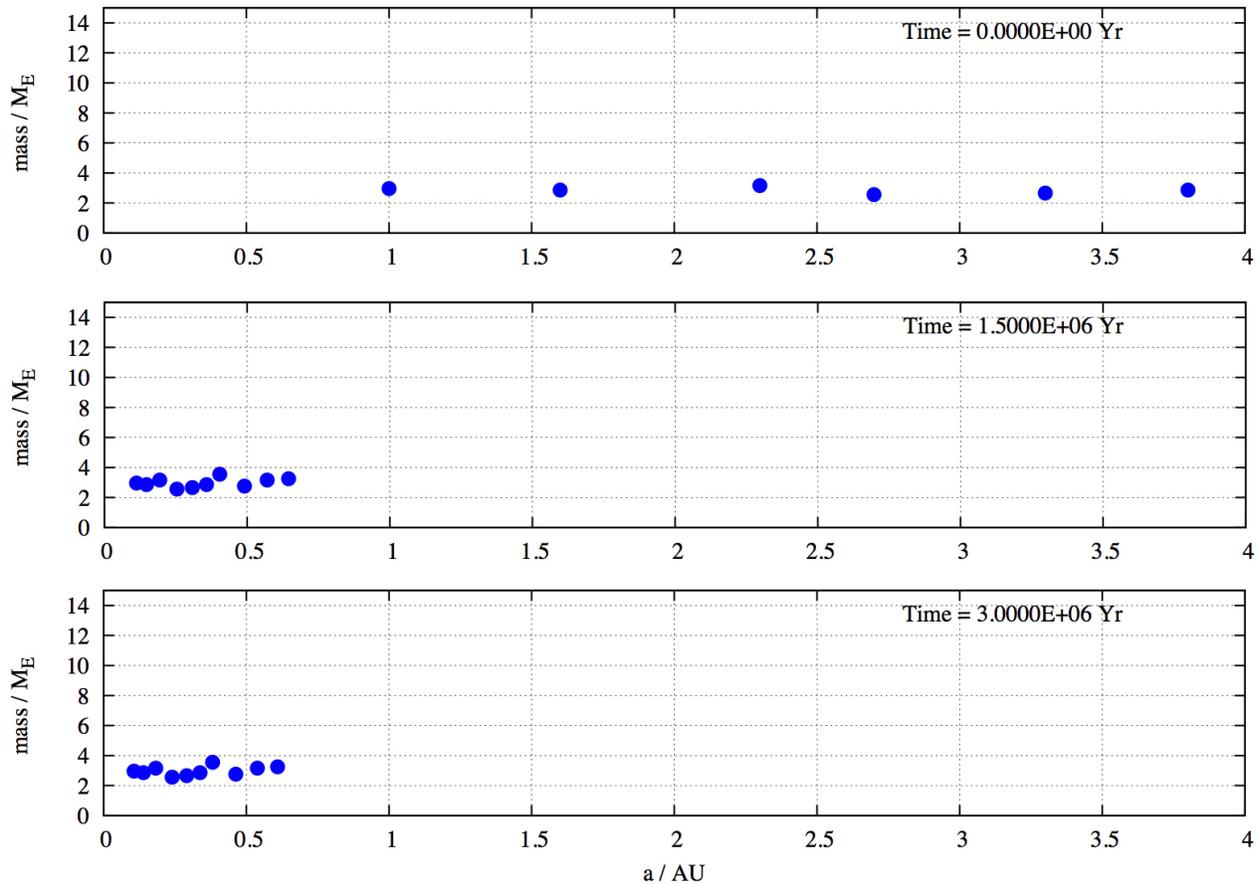

Figure 7: *Gas disk decay time: 10^8 yrs. The system again seems to have reached a final stage and no planetary accretion was observed.*

From what we can see in the figures above the disk decay time was not crucial for the final formation of the planetary system.

4 Discussion

It was possible to form packed systems localized in small radii through Type I Migration with a power law of $\alpha = 0.1$ for the surface density if the planets are already present in the gas disk further out and if
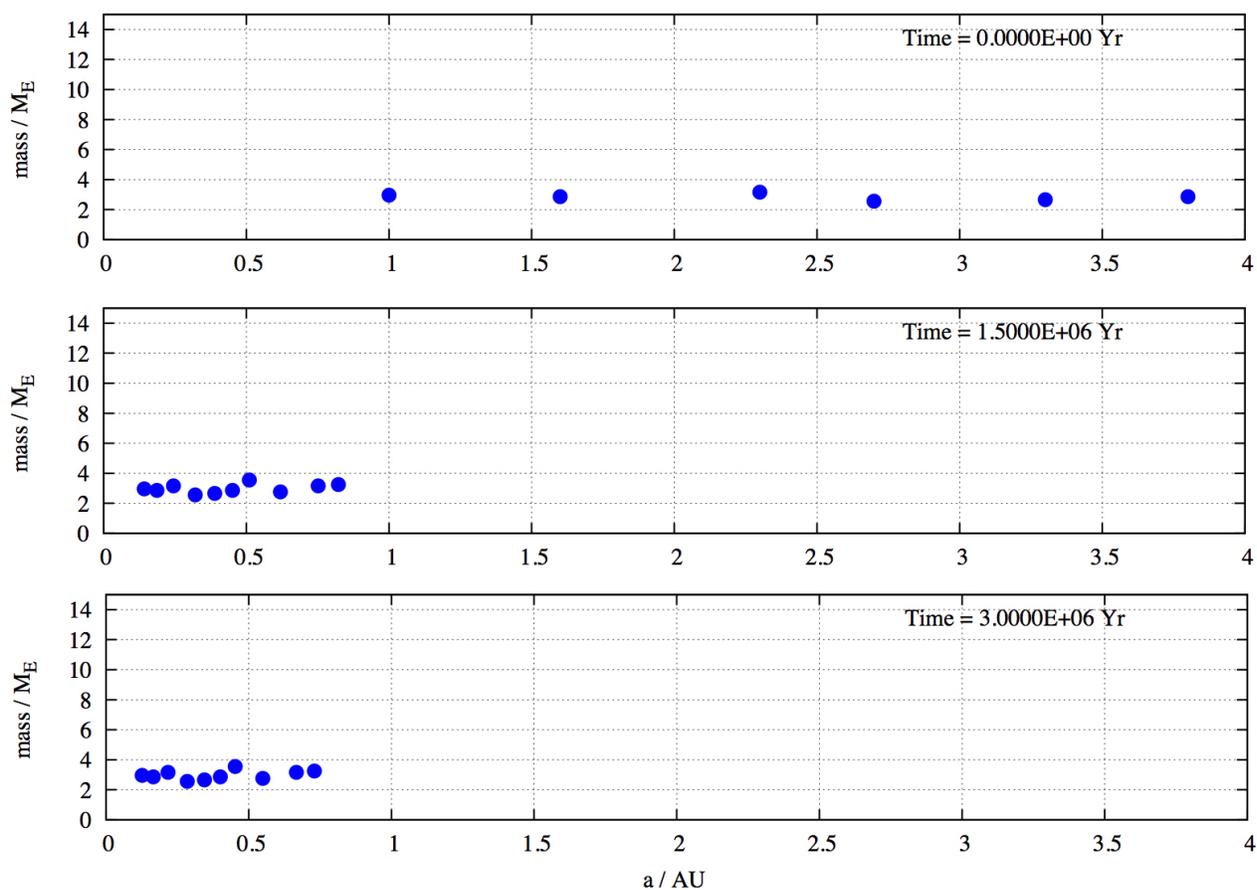
Figure 8: *Gas disk decay time:* 10^6 *yrs. The system again seems to have reached a final stage and no planetary accretion was observed.*

the thermodynamical characteristics of the disk change in a certain radius, that in this paper was chosen to be 0.5 AU.

The choice of α being 0.1 was made in order to get outwards migration in the adiabatic portion of the disk and inwards migration in the isothermal part. Creating, therefore, a planetary trap.

The idea that the gas would be adiabatic in inner regions and isothermal and outer regions comes from the fact that a dense region of the gas would trap better the heat of the system and therefore make this region 'more adiabatic', and inversely for the isothermal case.

But the value 0.5 AU itself was chosen more to try to form tightly packed inner planetary systems rather than to reflect any kind of thermodynamical result or observation. Therefore, even though the simulations were successful in creating packed planetary systems further research about the adiabatic-isothermal threshold in a gas disk is necessary.

Another result that can be observed from the last simulations (i.e., Figures 6, 7 and 8) was that the planets did not merge forming bigger planets as the ones found in STIPs formations. This was due to the slow rate of migration that allowed the planets to get in resonance with each other (see Table 3 for the found resonances).

Table 3: *Resonances found for the system in Figure 6. The period ratios are between one planet and the one to its immediate right as in Figure 6.*

Planet	a (AU)	Period (yrs)	Period Ratios	Closest Resonance	Relative Error (%)
1	0.126	0.045	0.667	2/3	0.02
2	0.166	0.067	0.666	2/3	0.09
3	0.217	0.101	0.666	2/3	0.11
4	0.285	0.152	0.749	3/4	0.12
5	0.345	0.203	0.799	4/5	0.08
6	0.401	0.254	0.831	5/6	0.24
7	0.453	0.305	0.750	3/4	0.02
8	0.549	0.407	0.748	3/4	0.22
9	0.666	0.544	0.875	7/8	0.02
10	0.728	0.621	-	-	-

5   Conclusion

It was shown in the section 3.1 that the implementation of the ideas presented by *Paardekooper et al. (2010)* were done in a correct manner. Furthermore, it was shown that in some cases the resulting systems in the simulation resembled STIPs using only the effects due the gas disk transition from adiabatic to isothermal.

It is of extreme importance to notice that the results presented in this report were obtained by associating only one value for the power law for the density through out the entire disk. This is indeed a limitation of

the simulations presented here, but it is also a really interesting aspect because it shows that systems like the ones found in the section 3 can be formed by a same global power law in surface density.

Therefore, it was noticed that planetary disks really close to be homogeneous in surface density can generate planet traps only by thermodynamical transitions. In this present report it was assumed that this transition is a characteristics of the disk itself and is independent of the planet masses. But further research could study cases in which the the radius of thermodynamical transition depends on the mass of the planets or how it is related to the disk itself.

The simulations produced planets in resonance with each other, which is not true in most STIPs. The simulations also did not produce accretion of planets. A more diverse choice of parameters like initial surface density at 1 AU and scale height at 1 AU could be varied to verify if accretion of planets would happen or not.

Acknowledgments

References

[1] Paardekooper S.-J. et al. (2010) Mon. Not. R. Astron. Soc., 401, 1950.

[2] Duncan, M. J., Levison, H. F. & Lee, M.H. (1998). A.J., 116, 2067.

[3] Baruteau et al. (2014) in: Protoplanetary Disks and Stars VI, University of Arizona Press, 667.