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Question-Answer Semantics

[Semântica de Perguntas e Respostas]

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Abstract: The present paper deals with a special kind of many-valued semantics: Question-Answer Semantics, in which logical values are products of questions about semantic predicates ('true', 'false', etc.) and corresponding answers ('yes', 'no', etc.). It purports to clarify some difficulties related to what is meant by basic speech-acts like assertion and denial. Deep disagreements occur whenever any two speakers disagree about the meaning of the words they use. In the special case of truth-values, Frege took these as the referents of propositions that are classes of sentences accepted ('true') or rejected ('false'). Starting from this usual depiction of truth and falsehood, a general algebraic framework $\mathbf{AR}_m^n$ is proposed in order to systematize the use of truth-values within a dialectical perspective of logic. A special attention is paid to two pseudo-speakers radically opposed, namely: Heraclitus, and Nagarjuna, according to whom truth-values refer to everything and nothing, respectively. In the end, a dialectical (pseudo-Hegelian) negation is sketched as a very peculiar function, namely: as an ontological object-forming operator, which is similar to the arithmetic successor-forming operator $\mathcal{S}_{n+1}(x)$ applied to integers and also matches with a generalized theory of truth-values as Millian or Kripkean proper names.
Keywords: Affirmation. Bivalent answerhood. Aufhebung. Negation. Truth-values.

Resumo: O presente artigo trata de um tipo especial de semântica de múltiplos valores: Semântica de Perguntas e Respostas, na qual valores lógicos são produtos de perguntas sobre predicados semânticos ('verdadeiro', 'falso', etc.) e respostas correspondentes ('sim', 'não' etc.). Pretende esclarecer algumas dificuldades relacionadas ao que se entende por atos básicos de fala, como afirmação e negação. Discordâncias profundas ocorrem sempre que dois falantes discordam acerca do significado das palavras que usam. No caso especial dos valores de verdade, Frege os tomou como os referentes de proposições que são classes de sentenças aceitas ('verdadeiras') ou rejeitadas ('falsas'). Partindo dessa representação comum da verdade e da falsidade, é proposta uma estrutura algébrica geral $\mathbf{AR}_m^n$, a fim de sistematizar o uso de valores de verdade em uma perspectiva dialética da lógica. Uma atenção especial é dada a dois pseudo-falantes radicalmente opostos, a saber: Heráclito e Nagarjuna, segundo os quais os valores de verdade se referem, respectivamente, a tudo e nada. No final, uma negação dialética (pseudo-hegeliana) é esboçada como uma função muito peculiar, a saber: como um operador ontológico gerador de objetos, semelhante ao operador aritmético gerador de sucessores $\mathcal{S}_{n+1}(x)$ aplicado a números inteiros e também combina com uma teoria generalizada de valores de verdade como nomes próprios milianos ou kripkeanos.
Palavras-chave: Afirmação. Aufhebung. Negação. Respostas bivalentes. Valores de verdade.

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1. Introduction: Beyond Yes and No

> Thought is saying no, and it is to itself
> that thought says no.
> (Alain, *Remarks on Religion*, 1924)

A general ambition of the present paper is to implement the above quotation, by showing how meaningful it is. This statement is composed of two parts, the first of which is to be qualified by the second part. Our thesis is that, by analogy with paraconsistency, negation is no more nor less essential to rational thinking than consistency is essential to logical thinking: despite their considerable role in both domains, they are not indispensable after all. At the same time, negating the role of logical negation is still a process of negation, so that a crucial diculty is to see in what level of discourse negation is referred to. Our method to make this point will consist in analyzing thought in general and logic in particular in the light of one and the same process: dialogue.

Let us consider any two participants of a dialogue $\mathcal{D} = \langle \mathbf{Q}, \mathbf{A} \rangle$ between, say, Quinn et Alan. Quinn asks only with yes- and no-answers to Alan about an arbitrary proposition φ, in order to get informations about it. How many questions can be asked by Quinn about φ, and what answers can be expected from Alan? A normal account should run as follows: φ is true or false according to Alan, given that φ is expressed by a declarative statement. Let the sentential content of φ be 'It will rain tomorrow'. If the question is

Quinn – Will it rain tomorrow?

then Alan is supposed to reply

Alan – Yes.

to state his opinion that φ is true. Then Alan thinks that it will rain tomorrow; if the question is

Quinn – Will it not rain tomorrow?

then Alan is supposed to reply

Alan – No.

to state his opinion that φ is not true, i.e. false. Hence Alan thinks that it will rain tomorrow.

At the same time, Alan may fall short of arguments to be entitled to give a definite answer. If it is the case, then it should be perfectly normal to hear Alan replying

Alan – I do not know.

thereby alluding to the fact that he is not able to answer either positively or negatively to whether or not it will rain tomorrow. Does this not mean that the proposition φ is neither true nor false? Not at all, so long as a clear-cut distinction is made between what propositions are and what is known about them.

However, a somehow curious situation might be the one in which Alan replies

Alan – Well, yes and no.

How can a proposition be true and false without altering the meaning of truth and falsehood? A way to account for this first oddity consists in introducing viewpoints in the basic semantics we are going to use to make sense of dialogues. Thus, 'yes and no' might mean that φ may be true from one viewpoint and false from another one, thereby reducing the surprising reply to a case of facile relativism. For example, Alan may have seen the local weather broadcast on TV and heard there that rain is announced in his city for the coming day. So Alan is justified to think that φ is true. At the same time, his farmer cousin told him that rain will not fall due to the behavior of birds and insects in his own farm. Hence Alan is also justified to think that φ is false. Deciding who is more reliable between a weather broadcast and an experienced farmer is not our point, however; Alan is merely entitled to answer 'yes-and-no' in the present case for want of any conclusive reason at hand, and this does not imply anything about whether it will eventually rain or not.

Now the game could be pushed further, soon leading to even more awkward dialogical situations. What happens if, for example, Alan replies

Alan – Neither yes-and-no nor neither-yes-nor-no?

It seems highly implausible for anybody to face such a dubious reply in daily life, and a semanticist of natural language might take this empirical data as a sucient reason to discard its relevance. Now here is the point at which things become interesting to us, actually. The issue concerns the limits of rationality in a dialogue: are there impossible replies in every dialogical situation, such that no agent could be considered as a rational agent by stating them? In order to tackle this meta-question, we are going to propose a framework intended to set forth a formal semantics and its main components, that is, truth-values, thus generalizing a question-answer semantics introduced in Schang (2015) and paralleling the extended debate around new versions of the Liar Paradox with Shramko & Wansing (2006)'s theory of generalized truth-values. Before that, it is to be noted that the following dialogical framework differs from the logical trend set out in Lorenzen & Lorenz (1978) and called 'dialogical logic'. For there are clear-cut differences between the latter and the concept of 'dialogue' as depicted throughout the present paper. Firstly, dialogic logic present any dialogue as a disagreement between a proponent P and an opponent O, whilst our present dialogue includes a neutral Questioner and an Answerer. Although the questioner may raise some doubts about how rational the answers may be, we will see that this situation comes from other reasons. Secondly, the difference in dialogical logic between two sorts of rules for dialogical games, namely: particle rules (to specify the meaning of logical constants) and structural rules (to organize the course of the dialogue between P and O) need not be stated hereby for want of any conflictual relation between **Q** and **A**. Thirdly, the present semantic framework will propose several kinds of statements for any given sentential question whereas dialogical logic usually relies upon simple armations made by P and denials made by O. Although Rückert (2004) illustrated a many-valued dialogical logic, our aim is to extend this move by introducing *non-bivalent answers* in addition to the non-bivalent truth-values.

2. Generalizing the Question-Answer Game

Let $\mathbf{AR}^n_m$ be any such set of questions-answers related to an arbitrary proposition φ. This framework consists of a finite number of answers to a finite number of corresponding questions. A primitive situation is the one where Alan gives no reply to Quinn: he remains silent, thus according no indication about the value of φ. One can depict this framework as a set of questions and answers formulated by Quinn and Alan, respectively.

Q – φ?
A – … .
Q – Not φ?
A – … .

Quinn may ask as many questions as she pleases, this will not change the number of corresponding answers as long as Alan remains silent. Silence is what it is, namely: a plain absence of value assigned to φ. Given that the value of a proposition finds its expression in an answer, we say that the proposition φ has no value in the above dialogue. It could be objected to it that a big confusion results from this process, i.e. a confusion between the truth-value of a proposition and a belief expressed about it. Indeed, φ is true or false whatever Alan may believe about it. There are at least two reasons not to take such a critics into account, however: the nature of truth-values is not taken for granted, if not by some hypothetical metaphysicians of logic. But also, truth-values proceed as sets of answers whose circumscribing rules are still unexplained. We follow in this respect the reply addressed by Wansing & Belnap (2010) to Dubois (2008), according to which a truth-value does not express a belief but an *information*.

A broader framework is the two-valued or *bivalent* one, in which every proposition is either true or false. In such a case, Alan is always in position to give an affirmative or negative answer to Quinn. And even in the case in which Alan does not feel secure with her sources at hand, he will take a decision by arguing for either the truth or the falsity of φ.

The general rationale underlying $\mathbf{AR}^n_m = \{\mathcal{L}, \mathbf{Q}(\varphi), \mathbf{A}(\varphi), \mathcal{V}^n_m\}$ includes a formal language $\mathcal{L}$ with a finite set of formulas φ (atomic or molecular) and logical constants, two kinds of sentential functions $\mathbf{Q}(\varphi)$ and $\mathbf{A}(\varphi)$, and a domain of valuation $\mathcal{V}^n_m$. The question function $\mathbf{Q}(\varphi) = \langle \mathbf{q}_1(\varphi), \ldots, \mathbf{q}_n(\varphi) \rangle$ applies to an arbitrary formula φ. It consists of a number of *sentential questions*, i.e. questions n related to the *semantic predicates* assigned to φ: 'true' or 'false', in the mainstream case of bivalence; but it may also be 'both-true-and-false', 'neither-true-nor-false', and the like. The answer function $\mathbf{A}(\varphi) = \langle \mathbf{a}_1(\varphi), \ldots, \mathbf{a}_n(\varphi) \rangle$ assigns m kinds of *sentential answers*, i.e. direct answers related to each of the preceding questions about the semantic value(s) of φ. These answers are 'yes' and 'no', in the usual cases. But they may also be 'yes-and-no' or 'neither-yes-nor-no', in other unusual cases which are to be explained later on. The number of questions and related answers available in a definite dialogical context gives rise to a domain of values $\mathcal{V}^n_m$ including a total number of $m^n = 2^2 = 4$ truth-values.

It is important to see that the 'truth-values' of the dialogical framework $\mathbf{AR}^n_m$ do not merely correspond to the usual semantic predicates like 'true', 'false', and the like; rather, they are answers related to these semantic predicate(s) assigned to a given formula. For example, stating that the formula φ is true means that the answerer of a given dialogical game says 'yes' to the first ordered question whether φ is true. In symbols: $\mathbf{a}_1(\varphi) = 1$. In the contrary case, saying that φ is false means that the answerer says 'yes' to the second ordered question whether φ is false. In symbols: $\mathbf{a}_2(\varphi) = 1$. It results from this dialogical semantics that the truth-bearers of it are not propositions but, rather, statements: a proposition is taken to be true if and only if one given speaker states something about it during a given dialogue.

It may be objected to this semantics that it complicates the usual truth-functional semantics irrelevantly, by introducing a useless distinction between propositions and statements and even relativizing the semantic predicates in a dangerous way. Apart from philosophical considerations about the nature of truth-values, the following wants to show that such a dialogical and algebraic semantics is in position to augment both the expressive and explanatory power of formal languages. Consider for instance a situation in which the speaker assumes that one answer to one question suces to characterize her statement about the atomic proposition p. Then she will answer either 'yes' or 'no' to the question whether p is true, without any further ado:

(1) 'Yes, I state that it will rain tomorrow.' (in symbols: $\mathbf{a}_1(p) = 1$)
or
(2) 'No, I do not state that it will rain tomorrow.' (in symbols: $\mathbf{a}_1(p) = 0$)

Now the agent can also combine answers to more than only one question, to make her position more explicit:

(1*) 'Yes, I state that it will rain tomorrow; and no, I do not state that it will not rain tomorrow.' (in symbols: $\mathbf{a}_1(p) = 1, \mathbf{a}_2(p) = 0$))
or
(2*) 'No, I do not state that it will rain tomorrow; and yes, I state that it will not rain tomorrow.' (in symbols: $\mathbf{a}_1(p) = 0, \mathbf{a}_2(p) = 1$)).

Why bother with (1*)–(2*)? In order to clarify the logical relations of dependence or independence that are supposed to stand behind the agents' answers. For the case (2) is still ambiguous. Does (2) mean the same as (2*)? The speaker may deny p without arming anything after all, as in the case in which she does not feel herself in position to state arguably and prefer to suspend her judgment. The aim of the general framework $\mathbf{AR}^n_m$ is to disambiguate such situations and to explain, for instance, in what dialogical games saying 'no' to the question whether φ is true is or is not the same as saying 'yes' to the question whether φ is false. It also purports to clarify the way in which concepts like affirmation and negation are used in this respect, insofar as some muddle generally occurs about what these concepts are supposed to mean. Are affirmation and negation two opposed speech-acts made by speakers, or are they properties of the formula expressed by speakers? The famous Principle of Bivalence (PBV) stands at the core of this issue, and we assume in the following that a clear-cut distinction between truth-values and speech-acts is essential towards a semantic clarification.

(PBV) For every formula φ, either φ is true or φ is false:

$$\mathbf{a}_1(\varphi) = 1 \text{ if, and only if, } \mathbf{a}_2(\varphi) = 0.$$

A systematic treatment of such dialogical situations requires to take into consideration games that stand beyond, but also beneath the mainstream dialogues where (PBV) holds. The variety of dialogical games is determined by the kinds of questions and answers at hand.

A way to make sense of the m available answers in an arbitrary domain of values $\mathcal{V}^n_m$ echoes to and extends earlier works strictly devoted to semantic predicates, especially Shramko & Wansing (2006)'s process of generalized truth-values. Let Y be a basic answer, 'yes'. It can be viewed as a singleton $\{Y\}$, that is, a set of answers $\mathcal{S}_m = \mathcal{S}_1$ including

$m = 1$ element. In the vein of constructive values, any given set $\mathcal{S}_m$ can be partitioned into posets $\mathcal{P}(\mathcal{S}_m)$. A partition of $\mathcal{S}_1$ leads to the two-element set $\mathcal{S}_2 = \{Y, \overline{Y}\}$, where $\{\overline{Y}\}$ means the failure of yes-answer and can be equated with the 'no'-answer N. Thus $\mathcal{S}_2 = \{Y, N\}$. A partition of $\mathcal{S}_2$ leads to the four-element set $\mathcal{S}_4 = \{Y\overline{Y}, Y, \overline{Y}, \overline{Y\overline{Y}}\}$, where $Y\overline{Y}$ can be read as 'both yes and no' and $\overline{Y\overline{Y}}$ as 'neither yes nor no'. And so on, indefinitely. A central problem is to make intuitive sense of the complex answer standing beyond 'yes' and 'no', assuming that the latter is already a complex element of the form $\overline{Y}$. Let us try to do this in the following illustration of dialogical semantics, by motivating the increasing number of the n questions and m answers. The basic answer value Y and its complement $\overline{Y}$ will be symbolized by the Boolean values 1 and 0, for sake of simplicity. As for the complex answers combining yes- and no-answers, both their interpretation and symbolization will be explained in the following.

3. Beyond Bivalent Answerhood

There are two categories of *trivial* semantics in $\mathbf{AR}^n_m$, those in which the dialogue leads to no relevant information because it is impossible to discriminate what the answerer accepts and rejects.
The first category is the semantics in which no answer is available at all, so that the corresponding dialogue is not a real one.

0-valent semantics
$\mathbf{AR}^n_m = \mathbf{AR}^n_0 = \mathbf{AR}_0$
$\mathbf{Q}(\varphi) = \langle \mathbf{q}_1(\varphi), \ldots, \mathbf{q}^n(\varphi)\rangle$
$\mathbf{A}(\varphi) = \mathbf{a}_1(\varphi)$, with $\mathbf{a}_1 = \emptyset$

Note that, in the above case, the absence of answer means the absence of any corresponding kind of answer: $\overline{Y}$ is already a proper answer, so that it differs from a pure absence of answer just as the empty set differs as a plain set $\emptyset$ or an element of a set $\{\emptyset\}$ –compare with $\overline{Y} = N$ in $\mathcal{S}_2$.
The second category corresponds to any dialogical situation including only one available answer.

1-valent semantics
$\mathbf{AR}^n_m = \mathbf{AR}^n_1 = \mathbf{AR}^1$
$\mathbf{Q}(\varphi) = \langle \mathbf{q}_1(\varphi), \ldots, \mathbf{q}_n(\varphi)\rangle$
$\mathbf{A}(\varphi) = \mathbf{a}_1(\varphi)$, with $\mathbf{a}_1 \mapsto \{1\}$
$\mathbf{A}(\varphi) \in \{\langle 1\rangle\}$

There may be an indefinite number n of such univalent semantics, that is, so many instances as there can be different kinds of questions about a formula. At the same time, only one kind of answer, 'yes', is available in these primitive semantics including the basic yes-answer: $\mathbf{a}_1(\varphi) = 1$. In other words, any semantic including only one, negative no-answer will be treated as a fragment of the next game semantics rather than a proper one-valued semantics.

2-valent semantics
$\mathbf{AR}^n_m = \mathbf{AR}^1_2 = \mathbf{AR}_2$
$\mathbf{Q}(\varphi) = \langle \mathbf{q}_1(\varphi)\rangle$
$\mathbf{A}(\varphi) = \mathbf{a}_1(\varphi)$, with $\mathbf{a}_1 \mapsto \{1{,}0\}$
$\mathbf{A}(\varphi) \in \{\langle 1\rangle, \langle 0\rangle\}$

$\mathbf{AR}^1_2$ is the dialogical semantics characteristic by any logical system obeying (PBV), that is, all these in which any formula φ is taken to be true or false: nothing more beyond $\mathcal{S}_2$, and nothing less beneath $\mathcal{S}_2$. Such an application of (PBV) means that $\mathbf{a}_1(\varphi) = 1$ if and only if $\mathbf{a}_2(\varphi) = 0$, as is the case in $\mathbf{AR}^1_2$: every yes-answer to the question whether φ is true (or false) entails a no-answer to the question whether φ is false. Moreover, it is worthwhile to see that such a characterization of (PBV) never appeals to the logical constant of *sentential* negation, which is part and parcel of a proposition φ by yielding the negative formula $\neg\varphi$. In other words, (PBV) uniquely concerns the semantic predicates of truth and falsity. At the same time, (PBV) can also be entertained dialogically as a yes-no principle ruling the correct use of sentential answers. Thus,

(PBV*) Every sentential question $\mathbf{q}_i(\varphi)$ must be answered either positively or negatively:

either $\mathbf{a}_i(\varphi) = 1$, or $\mathbf{a}_i(\varphi) = 0$.

It turns out that (PBV*) differs from (PBV), however. For imagine a speaker that answers positively both to the question whether φ is true and φ is false. Then such a speaker obeys (PBV*) but infringes (PBV) by stating that φ is true and false at once. The question of how such a situation can make sense is still to be explained.

4-valent semantics
$\mathbf{AR}^n_m = \mathbf{AR}^2_2 = \mathbf{AR}^4$
$\mathbf{Q}(\varphi) = \langle \mathbf{q}_1(\varphi), \mathbf{q}_2(\varphi)\rangle$
$\mathbf{A}(\varphi) = \langle \mathbf{a}_1(\varphi), \mathbf{a}_2(\varphi)\rangle$, with $\mathbf{a}_i(\varphi) \mapsto \{1{,}0\}$
$\mathbf{A}(\varphi) \in \{\langle 1{,}1\rangle, \langle 1{,}0\rangle, \langle 0{,}1\rangle, \langle 0{,}0\rangle\}$

This semantics echoes with Belnap (1977)'s logic FDE, given the translation $B = \langle 1, 1\rangle$, $T = \langle 1, 0\rangle$, $F = \langle 0, 1\rangle$, and $N = \langle 0, 0\rangle$. It clearly appears that such a speaker still obeys (PBV*) whilst violating (PBV). Belnap's model of computer networks also helps to make sense of such non-bivalent answers, in accordance with an *epistemic* interpretation of truth-values and by dealing with semantic predicates as informations given by a single computer: any two computers of a single network may deliver mutually inconsistent informations about a given sentence φ whilst the agent has no reason to favor either one. In such a case, there is evidence both for the truth and the falsity of φ and the speaker is entitled to answer positively to both the question whether φ is told true and whether φ is told false. Oppositely, there may be a situation in which no information at all is available about φ; in such a case, the speaker is entitled to answer negatively to the same paired questions $\mathbf{q}_1(\varphi)$ and

$\mathbf{q}_2(\varphi)$.

8-valent semantics
$\mathbf{AR}^n_m = \mathbf{AR}^3_2 = \mathbf{AR}_8$
$\mathbf{Q}(\varphi) = \langle \mathbf{q}_1(\varphi), \mathbf{q}_2(\varphi), \mathbf{q}_3(\varphi)\rangle$
$\mathbf{A}(\varphi) = \langle \mathbf{a}_1(\varphi), \mathbf{a}_2(\varphi), \mathbf{a}_3(\varphi)\rangle$,
with $\mathbf{a}_i(\varphi) \mapsto \{1{,}0\}$
$\mathbf{A}(\varphi) \in \{\langle 1,1,1\rangle, \langle 1,1,0\rangle, \langle 1,0,1\rangle, \langle 0,1,1\rangle,$
$\langle 1,0,0\rangle, \langle 0,1,0\rangle, \langle 0,0,1\rangle, \langle 0,0,0\rangle\}$

Here is a more peculiar semantics that is the powerset of a three-valued system such that $\mathbf{AR}_8 = \mathcal{P}(\mathbf{AR}_3)$, $\mathbf{AR}_3$ being a fragment of the previous $\mathbf{AR}_4$. Thus, $\mathbf{AR}_8$ is also a fragment of the next full set of $n = 4$ questions $\mathbf{AR}^4_2 = \mathbf{AR}_{16}$. In $\mathbf{AR}_8$, the additional sentential question $\mathbf{q}_3(\varphi)$ can be read in two distinct ways: as whether φ is both true and false, or as φ is neither true nor false.

Now what about the case in which the speaker answers positively to it? This seems absurd, insofar as the same speaker already answered positively to the question whether φ is true and whether φ is false. If so, what can the difference be between φ's being told true, told false, and told both? Actually, a crucial distinction is to be made hereby between successive and simultaneous valuations. A *successive* valuation means that φ is told true only and then false only, whereas a *simultaneous* valuation means that φ is told both-true-and-false at one and the same situation. The latter may be accounted in terms of nested networks. Let us consider three computer networks c_1, c_2, c_3 related to each other in a nested network $C = \{c_1, c_2, c_3\}$. In c_1 there is evidence for φ, so that φ is told true therein; in c_2 there is evidence against φ, so that φ is told false therein. And in c_3 there is evidence both for and against φ, so that φ is told both-true-and-false therein. The whole network system C thereby results in a valuation system in which the speaker is entitled to hold φ true, false, and both at once. Another instance of dialogical semantics $\mathbf{AR}_8$ will be the one in which $\mathbf{q}_3(\varphi)$ is about whether φ is neither true and false.

The process of nesting computer networks can be used increasingly, in order to account for any powerset $\mathcal{P}(\mathbf{AR}^n_m)$ of any given dialogical semantics $\mathbf{AR}^n_m$. In all such cases, the number of available answers $n = 2$ is constant, thereby obeying (PBV*) and matching both with Belnap (1977)'s semantic pattern of single computer network and its extension by Shramko & Wansing (2006). Now a trickier dialogical situation is the following, where more than $n = 2$ possible answers may occur.

9-valent semantics
$\mathbf{AR}^n_m = \mathbf{AR}^2_3 = \mathbf{AR}_9$
$\mathbf{Q}(\varphi) = \langle \mathbf{q}_1(\varphi), \mathbf{q}_2(\varphi)\rangle$
$\mathbf{A}(\varphi) = \langle \mathbf{a}_1(\varphi), \mathbf{a}_2(\varphi)\rangle$, with $\mathbf{a}_i(\varphi) \mapsto$ $\{1, {}^1\!/_2, 0\}$
$\mathbf{A}(\varphi) \in \{\langle 1,1\rangle, \langle 1,{}^1\!/_2\rangle, \langle 1,0\rangle, \langle {}^1\!/_2,1\rangle, \langle {}^1\!/_2,{}^1\!/_2\rangle,$
$\langle {}^1\!/_2,0\rangle, \langle 0,1\rangle, \langle 0,{}^1\!/_2\rangle, \langle 0,0\rangle\}$

A primary task consists in interpreting the 'non-bivalent' answer ${}^1\!/_2$, which goes beyond the domains ruled by (PBV*). The model of computer networks that has been introduced earlier does not rely upon either a probabilistic or modal interpretation of truth-values. Therefore, 1/2 cannot be read as 'maybe', 'possibly' and the further answers like, e.g., 1/4 will not be read as 'rather no than yes', either. Rather, let us consider any extension of the 'bivalent' or pure yes-no answers as combinations of such answers in nested networks. The above set of 9-valued semantics includes two categories of question-answer games, depending upon the 'glutty' or 'gappy' interpretation to be assigned to the third answer.

In the first interpretation, 1/2 = {1,0} is to be read 'yes and no' to a sentential question whether, e.g., φ is true: $\mathbf{q}_1(\varphi)$. We specify this 'glutty' semantic game by a positive lower case symbol, $\mathbf{AR}^2_{3+}$. The domain of answer values is thus to be reformulated in $\mathbf{AR}^2_{3+}$ as follows: $\mathbf{a}_i(\varphi) \in \{1,\{1,0\},0\}$. Let us consider for example some computer network c_1 where there is evidence for and no evidence against φ, together with another related network c_2 in which there is evidence both for and against φ (see Figure 1). The non-bivalent answer 'yes and no' does make sense in this nested network, insofar as the no-answer of c_1 and the yes-answer of c_2 about φ are concatenated into a synthetic yes-and-no-answer in C; this situation relevantly differs from the situation in which a sentence is said to be both true and false, for φ is merely told true without being told false in c_1. At the same time, it cannot be said in such a nested network that φ is both true and untrue: such a combination does not make sense by depriving a sentence of its semantic predicated that has been stated initially. For this reason, an extension of the dialogic games beyond the realm of (PBV*) helps to augment the expressive power of $\mathbf{AR}^n_m$ by accounting more complex answers in terms of nested networks.

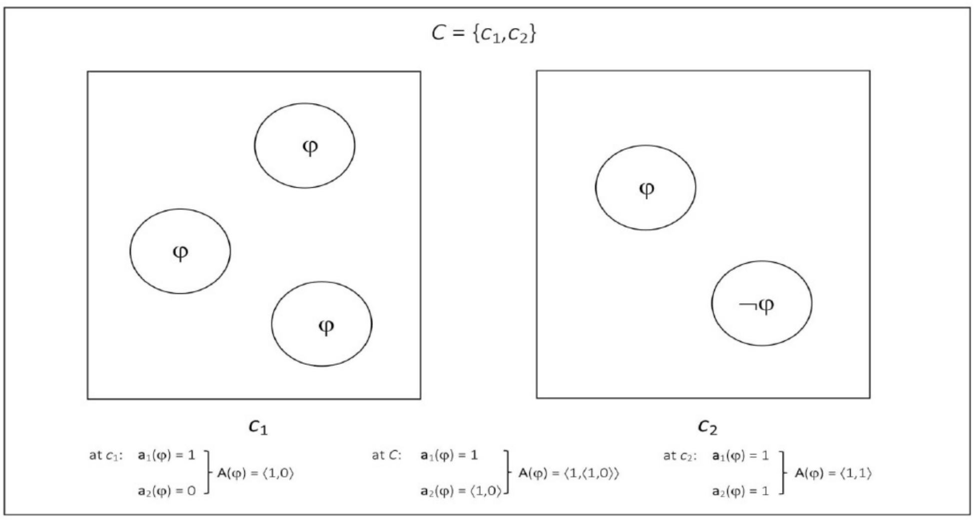

Figure 1: A model for $\mathbf{AR}^2_{3+}$.

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In the second interpretation, $1/2 = \overline{\{1,0\}}$ is to be read 'neither yes nor no' and we can specify this 'gappy' semantic game by a negative lower case symbol, $\mathbf{AR}^2_{3-}$. The corresponding domain of answer values in $\mathbf{AR}^2_{3-}$ is $\mathbf{a}_i(\varphi) \in \{1,\{1,0\},0\}$. This seems much more difficult to make sense than the 'glutty' case. For how can an answerer account for a situation in which there is no evidence either for or against the truth of a sentence? A way to make sense of this 'gappy' answer is to entertain a situation in which either the answerer has no full access to the computer network and, thus, cannot answer to the sentential question for want of exhaustive computer data.

A final issue is what can be called an 'Equivalence Problem' between the semantic games of $\mathbf{AR}^n_m$. It can be formulated in two main questions, namely: Are there equivalences between the different question-answer games, so that it would be useless to go further on with the number of questions and answers? What is the point of adding answers to questions in any semantic game, from a semantic point of view? Let us attempt to answer to these two questions according to their order of appearance.

Indeed, there seems to be no difference in dialogical meaning between some games like, e.g., $\mathbf{AR}^3_2$ and $\mathbf{AR}^2_3$; at the same time, there is a difference in the number of corresponding truth-values in these two games –8 and 9, respectively. More generally, there is no one-one mapping between both questions-answer games such that, for any $m, n > 1$ and $m \neq n$, $\mathbf{AR}^m_n \neq \mathbf{AR}^m_n$. Actually, there is a deep difference between semantic games with non-bivalent questions and non-bivalent answers: according to the above interpretation of the 'glutty' answers as *indeterminate* answers –for want of available computer data, such a lacking situation has no counterpart in the 'glutty' questions whenever the latter occur in *determinate* computer networks. It results in the following set of equivalence between the answer values of $\mathbf{AR}_8$ and $\mathbf{AR}_9$, respectively: $\langle 1,0,0\rangle = \langle 1,0\rangle ; \langle 0,1,0\rangle = \langle 0,1\rangle ; \langle 1,1,0\rangle = \langle 1,1\rangle$, whilst all the other answers differ from each other. The ultimate valuation $\langle 0,0,0\rangle$ might appear as a situation in which nothing can be answered positively about the sentence and, thus, might be conceivably taken as a counterpart of $\langle 1/2, 1/2\rangle = \langle \overline{\{1,0\}}, \overline{\{1,0\}}\rangle$. Yet, any negative answer 0 entails a determinate situation and thereby differs from the indeterminate answer $1/2 = \langle\{1,0\}\rangle$. Rather, the triple no-answer should amount to a determinate situation in which the sentential questions are inappropriate to specify the network at hand –the third question $\mathbf{q}_3(\varphi)$ should be replaced by the semantic predicate 'both true and false' in place of 'neither true nor false', for example.

Furthermore, the addition of new questions helps to make more finegrained distinctions between distinctive meanings of the semantic predicates. Consider the 'gappy' semantic predicate, 'neither true nor false'. The answer of indeterminacy embedded into $\mathbf{AR}_9, \mathbf{a}_i(\varphi) = 1/2$, can be equated with Kleene's third truth-value, 'unknown'; whereas the answer of indeterminacy embedded into $\mathbf{AR}_8, \mathbf{a}_3(\varphi) = 1$, can be equated with Łukasiewicz's third truth-value, 'undecided'. By this way, our question-answer game throws more light between usual many-valued systems and brings a more precise criterion for individuating logical systems through their domains of values: $Ł_3$ should be characterized by the determinate semantic game $\mathbf{AR}_2^3$, where $\mathbf{a}_i(\varphi) = 1/2$ is the gappy sentential answer; whilst K_3 should be characterized by the indeterminate semantic game $\mathbf{AR}_3^2 = \mathbf{AR}_9$, where $\mathbf{q}_3(\varphi)$ is about a gappy sentential question. Another such criterion of individuation for logical systems is about their characteristic sets of logical constants in a logical system; we tackle this point later on.

The general rationale of our game semantics can be afforded for any interpretation of the questions and answers.

m^n-valued semantics

$\mathbf{AR}_m^n$

$\mathbf{Q}(\varphi) = \langle \mathbf{q}_1(\varphi), \ldots, \mathbf{q}_n(\varphi) \rangle$

$\mathbf{A}(\varphi) = \langle \mathbf{a}_1(\varphi), \ldots, \mathbf{a}_n(\varphi) \rangle$, with $\mathbf{a}_i(\varphi) \mapsto \{1, 1-1/(n-1), \ldots, 1/(n-1), 0\}$

$\mathbf{A}(\varphi) \in \{\langle 1, \ldots, 1 \rangle, \ldots, \langle 0, \ldots, 0 \rangle\}$ (each $\mathbf{a}_i(\varphi)$ being an element of the n-tuple $\mathbf{A}(\varphi)$.)

Once the formal machinery $\mathbf{AR}_m^n$ is settled, we are in position to account for a number of disagreements as deaf dialogues between speakers, that is, as resulting from disagreements between their question-answer backgrounds. This may also throw a new light upon the way in which some allegedly 'irrational' behaviors correspond to so-called 'impossible' answer values and actually resort to alternative rationalities.

4. Impossible answers

In their theory of generalized truth-values, Shramko & Wansing (2006) extended a former notion introduced by Priest (1984), namely: *hyper-contradiction*, meaning that higher levels of contradiction may appear within rational discourse. For example, a proposition is hyper-contradictory if it is both true-and-false and merely true. In the line of these kinds of structured valuations, Shramko & Wansing went on positing the existence of other combinations than mere truth and mere falsity: both-true-and-false and neither-true-nor-false (symbol: $\{B, N\}$), true and both-true-and-false) (symbol: $\{T, B\}$), and so on, by virtue of a combination of elements within a finite set of truth-values. These values are called 'impossible'. Does this mean that such a process is nothing but an 'algebraic de-

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vice uninterpreted', as famously said by Quine against any extension of logic beyond bivalence? Following Aristotle's Principle of Non-Contradiction (PNC),

It is impossible for anyone to believe that a same thing is and is not, as said by Heraclitus according to some. For it is not necessary to agree with what has been said (by Heraclitus).[1]

This amounts to say that no proposition and its negation are true at once. Let φ be an arbitrary proposition, and $\neg\varphi$ be its sentential negation. Contradiction is expressed by the conjunction $\varphi \wedge \neg\varphi$, and a semantic translation of this form results in 'φ is true and φ is not true', i.e., 'φ is true and φ is false'. The logical constants of $\mathbf{AR}^n_m$ need to be explained now to make this point clearer, since (PNC) refers to sentential negation and conjunction. These can be characterized in the same way, irrespective of the number of questions and answers in a given dialogical game.

Let $\mathbf{A}(\varphi) = \langle \mathbf{a}_1(\varphi), \ldots, \mathbf{a}_n(\varphi)\rangle$ be the truth-value of any statement made by a answerer about φ. Then the value of its sentential negation $\neg\varphi$ can be characterized in two ways, depending upon the number n of sentential questions: either as

$$\mathbf{A}(\neg\varphi) = \langle \mathbf{a}_n(\varphi)\rangle$$

when $n = 1$, as in the semantic game $\mathbf{AR}^1_2$ and its characteristic truth-table

φ	$\neg\varphi$
1	0
0	1

or as

$$\mathbf{A}(\neg\varphi) = \langle \mathbf{a}_n(\varphi), \ldots, \mathbf{a}_1(\varphi)\rangle$$

when $n > 1$. An illustration is given by the semantic game $\mathbf{AR}^3_2$, where the ordered values are simplified in the form $\mathbf{A}(\varphi) = abc$.

φ	$\neg\varphi$
111	111
110	011
101	101
011	110
100	001
010	010
001	100
000	000

The former kind of sentential negation is an inversion-operator on answers, such that $\neg 1 = 0$. It proceeds from the application of (PBV), according to which any yes-answer to the question whether φ is true is a no-answer to the question whether φ is false, i.e., whether $\neg\varphi$ is true. The latter kind of sentential negation is a permutation-operator on answers, such that it reverts the ordering of the single sentential answers $\mathbf{a}_i(\varphi) \in \mathbf{A}(\varphi)$. It does not proceed from (PBV) anymore but, rather, from a more general principle

[1] Aristotle, *Metaphysics*, Book 3, 1005 23-26.

that does hold for every game semantics:

(F = Neg) Every answer to the question whether φ is false is the same as the answer to the question whether $\neg\varphi$ is true.

Taking the aforementioned example again (see Figure 1), let $\mathbf{A}(\varphi) = \langle 1, 1/2 \rangle = \langle 1, \{1,0\} \rangle$ in the semantic game $\mathbf{AR}^2_{3+}$ (where 1/2 is read 'both yes and no'). Then the speaker says 'yes' to the question whether φ is true and 'yes and no' to the question whether φ is false. Let $C = \{c_1, c_2\}$ the corresponding nested network such that there is evidence for and no evidence against φ in c_1, evidence both for and against φ in c_2. Therefore, there is evidence against and no evidence for $\neg\varphi$ in c_1, evidence both against and for φ in c_2 –which amounts to the same as having evidence both for and against $\neg\varphi$ in c_2. Hence $\mathbf{A}(\neg\varphi) = \langle \{1,0\}, 1 \rangle$. We are going to apply (F = Neg) in the following, in order to see what may go wrong in dialogues where **Q** and **A** do not assume the same domain of answer values.

The conjunction $\varphi \wedge \psi$ still needs be defined beforehand. It assumes an ordering relation between each of the paired single answers $\mathbf{a}_i(\varphi), \mathbf{a}_i(\psi)$ included into the ordered answers $\mathbf{A}(\varphi), \mathbf{A}(\psi)$, such that $1 > 0$. Let $ord(\mathbf{a}_i(\varphi), \mathbf{a}_i(\psi))$ be an ordering function applied to the paired answer values of φ and ψ. Then conjunction can be defined as follows in any semantic game $\mathbf{AR}^n_m$:

$$\mathbf{A}(\varphi \wedge \psi) = \langle ord(\mathbf{a}_1(\varphi), \mathbf{a}_1(\varphi)), \ldots, ord(\mathbf{a}_n(\psi), \mathbf{a}_n(\psi)) \rangle$$

It turns out that the meaning of *ord* depends upon the semantic predicate at hand in a given sentential question: true, false, true-and-false, and the like. Take the semantic game $\mathbf{AR}^3_2$, dealing with the three semantic predicates T in $\mathbf{q}_1$, F in $\mathbf{q}_2$, and B (or N) in $\mathbf{q}_3$. When the answer is about T, $ord(\mathbf{a}_i(\varphi), \mathbf{a}_i(\psi))$ takes the minimal value of the conjuncts because any doubt about whether φ or ψ is true makes the whole conjunction $\varphi \wedge \psi$ doubtful itself. Thus

$$\mathbf{a}_1(\varphi \wedge \psi) = min(\varphi, \psi)$$

When the answer is about F, the contrary holds because the least evidence against either φ or ψ is enough to answer positively to the question whether $\varphi \wedge \psi$ is false. Thus

$$\mathbf{a}_2(\varphi \wedge \psi) = max(\varphi, \psi)$$

When the answer is about B (or N), finally, the answer will depend upon the meaning of 1/2: 'both yes and no', or

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'neither yes nor no'. In the first 'glutty' case, then any failure of evidence for φ or is sufficient to answer negatively to $\varphi \wedge \psi$. Thus

$$\mathbf{a}_3(\varphi \wedge \psi) = min(\varphi, \psi) \text{ in } \mathbf{AR}_2^{3+}.$$

In the second 'gappy' case, the indeterminate situation of either φ or ψ contaminates the whole and makes $\varphi \wedge \psi$ indeterminate as well. Thus

$$\mathbf{a}_3(\varphi \wedge \psi) = max(\varphi, \psi) \text{ in } \mathbf{AR}_2^{3-}$$

It results in two kinds of characteristic truth-tables for $\varphi \wedge \psi$ in $\mathbf{AR}_2^3$, depending upon the meaning of the answer value 1/2:

- in $\mathbf{AR}_2^{3+}$: $\mathbf{A}(\varphi \wedge \psi) = \langle min(\varphi, \psi), max(\varphi, \psi), min(\varphi, \psi)\rangle$
  Example: if $\mathbf{A}(\varphi) = \langle 1,1,0\rangle$ and $\mathbf{A}(\psi) = \langle 0,1,1\rangle$, then $\mathbf{A}(\varphi \wedge \psi) = \langle 0,1,0\rangle$
- in $\mathbf{AR}_2^{3-}$: $\mathbf{A}(\varphi \wedge \psi) = \langle min(\varphi, \psi), max(\varphi, \psi), max(\varphi, \psi)\rangle$
  Example: if $\mathbf{A}(\varphi) = \langle 1,1,0\rangle$ and $\mathbf{A}(\psi) = \langle 0,1,1\rangle$, then $\mathbf{A}(\varphi \wedge \psi) = \langle 0,1,1\rangle$

A third relevant logical constant to be defined in $\mathbf{AR}_m^n$ is disjunction. Although the latter is not concerned with (PNC), it can be defined as the *dual* of conjunction; this means that

$$\mathbf{A}(\varphi \vee \psi) = \langle ord(\mathbf{a}_1(\varphi), \mathbf{a}_1(\psi)), \ldots, ord(\mathbf{a}_n(\varphi), \mathbf{a}_n(\psi))\rangle$$

such that

$ord(\mathbf{a}_i(\varphi), \mathbf{a}_i(\psi)) = max(\varphi, \psi)$ for $\vee$ whenever $ord(\mathbf{a}_i(\varphi), \mathbf{a}_i(\psi)) = min(\varphi, \psi)$ for $\wedge$
$ord(\mathbf{a}_i(\varphi), \mathbf{a}_i(\psi)) = min(\varphi, \psi)$ for $\vee$ whenever $ord(\mathbf{a}_i(\varphi), \mathbf{a}_i(\psi)) = max(\varphi, \psi)$ for $\wedge$

Conditional could be also characterized in $\mathbf{AR}_m^n$; now it will be addressed into the present paper, due to its ambiguity from a many-valued point of view and its irrelevance for the coming discussion.[2]

[2]For an account of conditional in a particular case of $\mathbf{AR}_m^n$, namely: $\mathbf{AR}_2^2 = \mathbf{AR}_4$, see e.g. Schang (2017).

Let us turn back to (PNC) and Aristotle's logic. Why should it impossible to think 'contradictorily', according to him? The answer is: by virtue of the bivalent framework assumed by him in his logical framework $\mathbf{AR}^1_2$. Indeed, whoever states therein that φ is true thereby commits in thinking that φ is not false and conversely, so that there is no reply such that a speaker could state the truth of φ and $\neg\varphi$ simultaneously. Nothing prevents a priori Heraclitus from choosing a broader framework inside $\mathbf{AR}^n_m$, however. Nothing is said thus far about the semantic game that should characterize the dialogical behavior of Heraclitus, either –this point will be raised in the final section. We merely state by now that his behavior is not irrational, unless Aristotle is in position to show that only $\mathbf{AR}^1_2$ makes sense for any rational speaker. The Stagirite attempted to do so by means of the so-called *elenctic* argument: although there exists no deductive argument to demonstrate the universal validity of PNC, it is often stated that whoever accepts a contradiction is inevitably led to absurd consequences and, thereby, eventually rejects what she accepted initially.

Let us illustrate such a demonstration by imagining a dialogue between Aristotle and Heraclitus, accordingly:

Aristotle – Is it the case that φ, that is, $\neg\varphi$ is true?

Heraclitus – Yes, φ is true.

Aristotle – Hence$\neg\varphi$ is false, isn't it?

Heraclitus – No.

Aristotle – You mean that both φ and $\neg\varphi$ are true?

Heraclitus – Yes.

One first result is of relevance and argues against (PBV), in the above dialogue: accepting the falsity of φ may not be the same as rejecting the truth of φ for every speaker, whilst accepting the falsity of φ is the same as accepting the truth of $\neg\varphi$ for any speaker. Indeed, Heraclitus accepts $\neg\varphi$ without rejecting its opposite φ, so that φ and $\neg\varphi$ are not exclusive from each other in his viewpoint. That which is really 'opposed' from Aristotle's viewpoint is not so from Heraclitus', and the phrase 'true contradiction' makes sense only if these two perspectives are conflated.

Here is the step at which Aristotle should introduce his elenctic argument, which is expected to play the role of knock-down argument against the defenders of true contradictions.

Aristotle – Well, let us suppose that φ and $\neg\varphi$ can be true together. Then your position is indefensible, because you should reject the negation of what you just accepted.

Heraclitus – That is to say?

Aristotle – If you accept φ and $\neg\varphi$ at once, then you accept $(\varphi \wedge \neg\varphi)$. And if such is the case, then you cannot accept $\neg(\varphi \wedge \neg\varphi)$. Therefore, you end up with stating now that which you just rejected a few minutes ago. I am right, and you are wrong.

Heraclitus ? Why so?!

Aristotle – Because that is how language and thought are made, and you cannot object anything to this necessity. For this reason, you cannot accept and reject one and the same proposition at once. Consequently, you must end with conceding (PNC) whenever you concede that a proposition like $(\varphi \wedge \neg\varphi)$ cannot be accepted and rejected at once.

Heraclitus – I agree with the first part of your conclusion. But not with the second one.

Aristotle – You cannot proceed in that way!

Heraclitus – Yes I can, and I am going to show it. I have told you that φ and $\neg\varphi$ are true together, and I see no difference between this affirmation and the affirmation that $(\varphi \wedge \neg\varphi)$ is true as well, in accordance to the meaning of conjunction. However, you are mistaken by assuming that I must reject $\neg(\varphi \wedge \neg\varphi)$ for the reason that I just accepted $(\varphi \wedge \neg\varphi)$. It is normal to proceed in that way so long as you assume that a proposition and its negation are contradictories. You are free to do so, but nothing prevents me from doing so in turn. I do not proceed in that way, actually, and that is why I accept both $(\varphi \wedge \neg\varphi)$ and its negation $\neg(\varphi \wedge \neg\varphi)$. To sum up, accepting a formula is not the same as rejecting its negation. Or not for anyone, at the very least, contrary to what you seem to take it for granted.

Aristotle – What you just said does not make sense. You cannot accept a contradiction as you just did it, because whoever proceeds in that way does not say anything meaningful. This is just noise, there is nothing meaningful in these words.

Heraclitus – I do not accept a 'contradiction' by accepting both φ and $\neg\varphi$, once again. This is a contradiction *for you*, not for me.

Aristotle – You play with words, and it is worthless to go on discussing with you.

Heraclitus – As you please. I do not intend to *contradict* you.

Aristotle – ... .

Translated into our question-answer framework, the disagreement between Aristotle and Heraclitus comes from the fact that the former identifies $\mathbf{a}_2(\varphi) = 1$ with $\mathbf{a}_1(\varphi) = 0$, insofar as he does not conceive any other way of thinking φ in his unique and restricted framework $\mathbf{AR}^1_2 = \mathbf{AR}_2$ obeying (PBV). Admittedly, there is no difference for both Aristotle and Heraclitus between 'φ' and 'φ is true', on the one hand, and '$\neg\varphi$' and 'φ is false', on the one hand. But there is one strong difference between 'being told false' and 'being rejected' for Heraclitus, whereas there is none for Aristotle. The main point is that, according to Aristotle, every rational speaker cannot but reject what is taken to be false. Falsehood is a value to be rejected, by definition. But, once again, a confusion arises between distinctive standards of rationality inside crossed dialogues: that which is 'false' for Aristotle is not so with the same sense of the word for Heraclitus, and any reference to truth-values may lead to misunderstandings whenever a dialogue includes speakers assuming different 16 semantic frameworks. It is as if one refers to lines with a non-Euclidean geometer, given that these do not have for the latter the same features as for a Euclidean geometer. In the same vein, Heraclitus agrees that the negation of a true proposition is false by definition; the point is that he will not *reject* it after all. Both are acceptable, according to him, and there is no contradictory relation between a true proposition and its negation in the formula $(\varphi \wedge \neg\varphi)$. If such is the case, which semantic game is assumed by Heraclitus, and what does it mean by the universal truth of everything? Does it really think so, by passing?

A similar difficulty arises whenever any two speakers do not share the same assumptions in their arguments, and the logical problem is about whether there are universal principles with which everyone must comply as a rational agent. For example, Priest (2010) tackled the issue of the Tetralemma to deal with the limits of thought. In this issue borrowing from Indian philosophy, a speaker, Nagarjuna, is blamed for saying nothing in particular by answering negatively to whatever is asked to him. Let us see what could result from a dialogue between him and Aristotle.

Aristotle – Do you hold φ to be true?
Nagarjuna – No.
Aristotle – Alright. Then you hold φ to be false?
Nagarjuna – No, either.
Aristotle – Again?! Listen, this morning I tried to discuss with Heraclitus. He told me that he accepted not only φ or φ but both at once, which does not make sense. Are you doing the same by rejecting φ and $\neg\varphi$ at once? I mean, are you alluding that these propositions are not only true, despite their being true together?
Nagarjuna – No more.
Aristotle – I will end up with knowing what you are thinking, at any rate! It is this: neither φ nor$\neg\varphi$ are true or false, because these are neither true nor false. That's it?
Nagarjuna – No.
Aristotle – Enough, I am done with it. Once again.

A usual version of the Tetralemma is symbolized by the formal language of bivalent logic, although we already insisted on how misleading it may be to identify φ and $\neg\varphi$ with the syntactic expressions of truth and falsehood.

(a) φ
(b) $\neg\varphi$

(c) $\varphi \wedge \neg\varphi$
(d) $\neg(\varphi \wedge \neg\varphi)$

The contrary situation seems to occur here above with respect to the first dialogue, insofar as the speaker rejects whatever was accepted by Heraclitus. To make sense of this context, most of the commentators opted for an extension of the number of truth-values beyond the four elements –true, false, both, and none. The result is a logical threshold beyond which the speaker ends with accepting or rejecting what was rejected or accepted systematically at the preceding level, by virtue of a Law of Excluded $(n + 1)$th stating that every proposition must have at least one of the n truth-values occurring in a given finite set. The Law of Excluded Third is just a variant of the bivalent framework in which $n = 2$; Nagarjuna seems to assume a different framework, given his four successive rejections.

Priest (2010) proposed a 5-valued semantics for such a speaker, which amounts to something like the following framework:

5-valued semantics
$\mathbf{AR}^n_m = \mathbf{AR}^1_5 = \mathbf{AR}_5$
$\mathbf{Q}(\varphi) = \langle \mathbf{q}_1(\varphi)\rangle$
$\mathbf{A}(\varphi) = \langle \mathbf{a}_1(\varphi)\rangle$, with $\mathbf{a}_i(\varphi) \mapsto \{1, 3/4, 2/4, 1/4, 0\}$
$\mathbf{A}(\varphi) \in \{\langle 1\rangle, \langle 3/4\rangle, \langle 2/4\rangle, \langle 1/4\rangle, \langle 0\rangle\}$

Priest (2010) interpreted Nagarjuna's attitude as an attitude of *silence*. But, it may seem queer to consider silence as a proper truth-value if every value corresponds to an answer given by a speaker to a preceding question. On the contrary, the preceding framework $\mathbf{AR}^n_0 = \mathbf{AR}_0$ appears to be more appropriate to account for silence. Silence is not a truth-value in this sense, it appears rather like a failure of value for the related sentence. Moreover, should Nagarjuna ever answer 'no' in the aforementioned four questions, and why could he not express his rejection once more beyond the step (d) of the dialogue?

In a nutshell, a similar problem occurs with speakers like Heraclitus and Nagarjuna: the former seems to accept everything, whilst the latter seems to accept nothing. If such is the case, then there is no sense in proceeding so from a *logical* point of view. For whoever thinks in a logical way makes a minimal *partition* between what is accepted and what is rejected. Aristotle always ends with the last word with paraconsistent systems, insofar as the strategy of extending the number of truth-values beyond mere truth and mere falsehood does not prevent him from eventually imposing a general bipartition between two sets of semantic predicates. Such is the rationale behind Suszko's reduction thesis [17], against the 'mad idea' of many-valuedness: bivalence corresponds in his view to the framework in which truth and falsehood occur as ultimate sets of semantic predicates, and any other truth-value can be reduced to any of the two accepted general values –either the so-called 'designated', accepted values, or the rejected values –the so-called 'non-designated' values, inside the class of the *m* answers.

Here is where rationality and logic may take separate ways, however. Just as alternative models have been designed for non-Euclidean geometries, an alternative model $\mathcal{V}_m^n$ might be promoted for non-Aristotelian and, even, non-Suszkian logics. Even beyond such a logical achievement, we want to show that an alternative model can be designed in order to make sense of an extension of truth-values without collapsing again into the logical pattern of Suszko's bipartition. This consists in replacing the process of partition by a process of *expansion*.

5. From logic to dialectics through dialogue

An entirely different account of truth-values can be given, by reference to the dialectical process thesis-antithesis-synthesis. Without contending to present a genuine explication of what Hegel said explicitly about logic, the following wants to show two things with respect to Hegel's expected import in this domain: firstly, how our game semantics is able to make sense formally of Hegel's concept of *Aufhebung*, in a way that deeply departs from the usual account of negation as a mapping over a closed domain of truth-values; and secondly, to what extent a charitable interpretation of dialectical negation is in position to disentangle the general misunderstanding between Aristotle and Heraclitus, or between Aristotle and Nagarjuna.

For one thing, let us consider the process of thesis-antithesis-synthesis. There is at least one reason to pay attention to it, namely: 'antithesis' is closely related to the logical concept of contradiction. Thus, let p be an arbitrary atomic proposition. In the prime step of thesis, p is affirmed by a speaker who takes it to be true. In the second step of antithesis, p is denied. This is symbolized by a *sentential* negation on p, which means that p is taken to be false as well. This affirmation is contrary to the first one, but it also expresses an affirmation after all. In the third step of synthesis, p and its negation $\neg p$ are both affirmed. Here is the step at which most of the analyses take an end, due to the allegedly logical 'impossibility' of affirming both the truth of a proposition p and the truth of its negation $\neg p$, that is, both the truth and falsity of p. What should happen if we go on further, following Heraclitus? A second level of dialectics should be applied to the ultimate affirmation of the preceding level, together with the similar processes of antithesis and synthesis. Whilst a speaker belonging to the framework $\mathbf{AR}_2^1 = \mathbf{AR}_2$

would assimilate the preceding negated to the Law of Excluded Third: $p \vee \neg p$ (due to the 'Morganian' behavior of bivalent negation), the dialectician speaker does not stop there and goes on by making a further synthesis of the opposed affirmations.

What happens afterwards, and what going on with such a process for? Here is the main idea of the present paper: the trouble with Heraclitus does not lie in his dialogical behavior but, rather, in the motivation that underlies it. This motivation is not the one that logicians are expected to have as a matter of fact, when they refer to truth-values and the construction of models intended to prove the validity or invalidity of formulas. Our point is to show hereby that Heraclitus does not affirm or deny the truth-value of propositions; rather, his point is to *create* truth-values, through a dialectical process of expanding model. This requires some minimal explication, before tackling its various upshots.

For this purpose, the process dialectics will be applied to three different case studies: propositions, truth-values, and integers. By applying the dialectical process of thesis-antithesis-synthesis three times successively, it results in a combination of simple objects through a sequence of affirmations, negations, and aggregations of these:

Sentential dialectics

Level 1

Thesis	p
Antithesis	$\neg p$
Synthesis	$p \wedge \neg p$

Level 2

Thesis	$p \wedge \neg p$
Antithesis	$\neg(p \wedge \neg p)$
Synthesis	$(p \wedge \neg p) \wedge \neg(p \wedge \neg p)$

Level 3

Thesis	$(p \wedge \neg p) \wedge \neg(p \wedge \neg p)$
Antithesis	$\neg((p \wedge \neg p) \wedge \neg(p \wedge \neg p))$
Synthesis	$((p \wedge \neg p) \wedge \neg(p \wedge \neg p)) \wedge \neg((p \wedge \neg p) \wedge \neg(p \wedge \neg p))$

The sentential reading of dialectics does not seem appropriate, however. On the one hand, the above three levels are already equivalent with each other in the semantic game $\mathbf{AR}_2^2 = \mathbf{AR}_4$, whenever p is taken to be both true and false. For then $\mathbf{A}(p) = \mathbf{A}(\neg p) = \mathbf{A}(p \wedge \neg p) = \langle 1, 1\rangle$, and the same holds for any kind of sentence φ. Therefore, $\mathbf{A}(\varphi) = \mathbf{A}(\neg p)$ whenever $\mathbf{A}(\varphi) = \langle 1, 1\rangle$. On the second hand, the increasing complexity of the semantic games does not rely upon the structural complexity of sentences. Rather, it concerns the increasing complexity of their semantic predicates: T, F, B, N, T, B, and so on; or the increasing complexity of their answer values: 'yes', 'no', 'yes and no', 'neither yes nor no', 'yes-and-no and neither-yes-nor-no', and so on. We suspect Heraclitus to assume a semantic game of the type $\mathbf{AR}_1^n = \mathbf{AR}_1$, where only yes-answers apply to an indefinitely increasing set of semantic predicates in n sentential questions.

In this second level of semantic predicates, negation proceeds by *complementation* and is symbolized by a horizontal bar above the semantical predicate of truth. Such a non-sentential negation of truth (or 'non-truth') refers to whatever is not true within the whole set of semantic predicates. This corresponds uniquely to the predicate of falsity in the bivalent framework $\mathbf{AR}_2^1 = \mathbf{AR}_2$; but it need not be the case in any higher-order framework of the type $\mathbf{AR}_m^n$ whenever $m, n \geq 2$.

Semantical dialectics

Level 1

Thesis	$\top$
Antithesis	$\overline{\top}$
Synthesis	$\top\overline{\top}$

Level 2

Thesis	$\top\overline{\top}$
Antithesis	$\overline{\top\overline{\top}}$
Synthesis	$\top\overline{\top}\,\overline{\top\overline{\top}}$

Level 3

Thesis	$\top\overline{\top}\,\overline{\top\overline{\top}}$
Antithesis	$\overline{\top\overline{\top}\,\overline{\top\overline{\top}}}$
Synthesis	$\top\overline{\top}\,\overline{\top\overline{\top}}\,\overline{\top\overline{\top}\,\overline{\top\overline{\top}}}$

The main difference between speakers like Aristotle and Heraclitus lies in their interpretation of double negation, whenever speakers reject what they just rejected at an earlier stage of discussion. Whilst Aristotle would state that whoever rejects p must affirm its 'contradictory' $\neg p$, Heraclitus would not do so; rather, he would go on negating the propositions negated repeatedly. It is clear that, in the light of the above explication, every level of discussion leads to an even more complex statement, and the structured statements of this kind cannot be simplified without assuming a limitation of the framework $\mathbf{AR}^n_m$ between any two speakers. Rather than proceeding in this way, a third comparison can be made between propositions, truth-values, and integers.

Just as truth-values are viewed like sets of propositions, integers are sets of elements sharing a common property. Starting from an integer symbolized by 1, any negation creates a difference by adding a supplementary integer, the latter being then combined to the preceding integer through the process of synthesis. This results in a third integer that is not identical either to the first or to the second one.

Here is how natural integers can be constructed dialectically, following a previous analogy made between logic and arithmetics in Schang (2016). Referring to the parlance of computer science, the bits 1 and 0 are used to represent this increasing process of numbering. The latter was depicted by Leibniz, followed by a mapping between the *Book of Change*, or Chinese *Yi King*, and the numerical binary system.

Arithmetical dialectics

Level 1

Thesis	1
Antithesis	10 = 2
Synthesis	11 = 3

Level 2

Thesis	11 = 3
Antithesis	100 = 4
Synthesis	101 = 5

Level 3

Thesis	101 = 5
Antithesis	110 = 6

Synthesis 111 = 7

Leibniz's analogy may be also helpful to account for the meaning of truth and falsehood from a dialectical viewpoint, strictly based on yes- and no-answers. At least five statements may result from it. Firstly, the basic bits 1 and 0 may be taken to be symbols of Being and Non-Being. Secondly, each antithesis occurs as a combination of Being and Non-Being that extends the domain of values, rather than splitting it into distinctive parts. Thirdly, the primary occurrence of truth (symbol: 1) in the above dialectical process means that every thing both participates of and departs from it through the introduction of Non-Being (symbol: 0). Fourthly, the conspicuous occurrence of Being entails that no proposition is completely devoid of truth; consequently, 'falsehood' is merely another name to designate whatever is not absolutely true at the first level of dialectics –at Level 1, and thereby includes a part of Non-Being. Fifthly, falsehood is not the complementary of truth at every step of the dialectics; complementation and falsehood are on a par only at the first step of dialectics, wherein the entire domain of values includes two elements only –truth, and falsehood.

Here lies the main and thorough difference between the 'logic' of dialectics, as expounded here above, and what is ordinarily meant by a logical system: dialectical values behave like *singular terms* within an expanding model, whereas logical values are sets of propositions splitting a basic common property. The expanding model is what is meant by an 'expanding semantics', in opposition to the 'partitioning semantics' that usually depicts negation as a process of dichotomy.

Let us consider the latter way of conceiving negation, resulting in Lewis Carroll's diagrams of symbolic logic and mentioned by Priest (2010) to account for the so-called Logic of *Catuskoti* –or Tetralemma, by means of the third diagram and its four elements. Let 1 and 0 be the usual symbols for affirmative and negative answers to questions, respectively. Then partition can be viewed as a particular case of $\mathbf{AR}^n_m$, i.e. an increasing game semantics $\mathbf{AR}^n_2$ restricted to $m = 2$. It turns out that the framework $\mathbf{AR}^n_m$ leads to results largely different from those of a partition semantics like $\mathbf{AR}^n_2$ (see Figure 2).

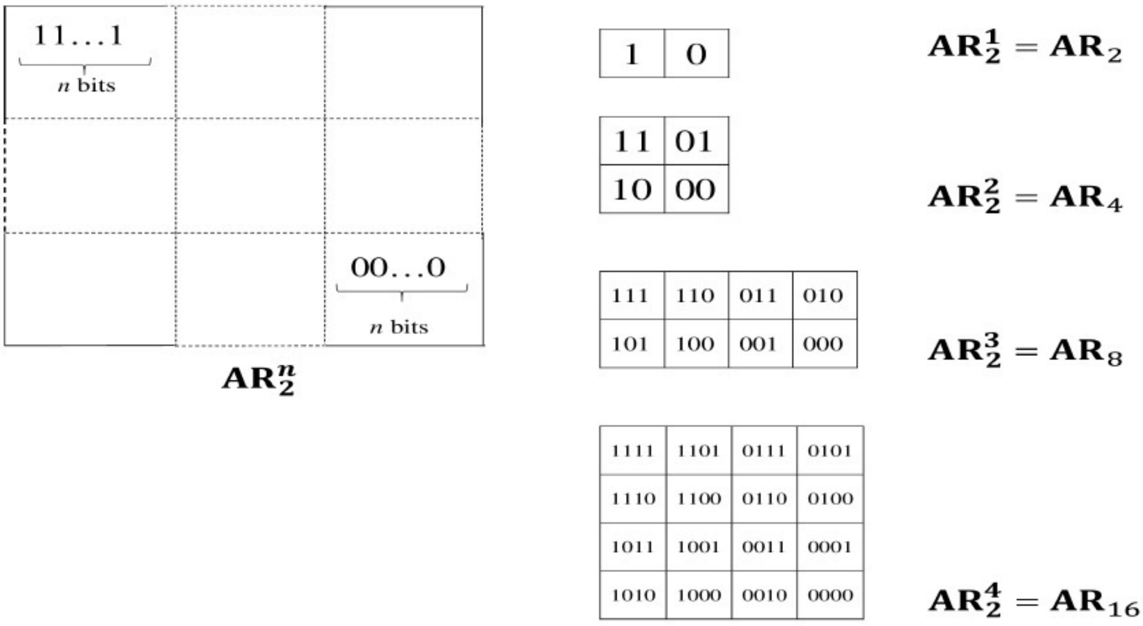

Figure 2: Partition semantics in $\mathbf{AR}_2^n$.

On the one hand, let us note that the transition from each diagram to its successor is made by means of an increasing number of n questions (about semantic predicates) and a constant amount of $m = 2$ answers –yes, or no. Thus, a primary diagram would correspond to a framework $\mathbf{AR}_2^0 = \mathbf{AR}_1$, where no asked question makes sense within a uniform domain of omnipresent truth. The above first particular diagram embeds bivalence, where one relevant question can be asked about the unique property in the domain of $\mathbf{AR}_2^1 = \mathbf{AR}_2$. The second diagram is reminiscent of the four-valued algebra of Belnap's logical system FDE, which includes $n = 2$ questions and $m = 2$ possible answers in $\mathbf{AR}_2^2 = \mathbf{AR}_4$. The third last diagram $\mathbf{AR}_2^3 = \mathbf{AR}_8$ helps to see how the preceding diagrams differ from each other, that is, by adding one question successively and thereby doubling the number of semantic predicates within a sequence of type $\mathbf{AR}_2^{n+1}$. And so on, for an increasing set of n sentential questions.

On the the other hand, such a construction of truth-values by dichotomy does not reproduce the dialectical process thesis-antithesis-synthesis faithfully: one does not see how to go from one level of judgement to another one in these diagrams, and the introduction of new sets lets aside the additional mechanism we described previously. Dialectics proceeds by adding one property, just as in the diagrams. Nevertheless, the essential difference lies in the number of available answers from a dialectical viewpoint: only $n = 1$ answer holds there, as was the case for Heraclitus and Nagarjuna in their fictional dialogue with Aristotle. In other words, diagrams proceed by dichotomy in the increasing sequence of frameworks $\mathbf{AR}_2^{n+1}$, whereas dialectics proceeds by expansion in the increasing set of frameworks $\mathbf{AR}_{n+1}^1 = \mathbf{AR}_{n+1}$. A confusion between both models occurs at $n+1 = 2$, which corresponds to the case in which only one semantic predicate, viz. truth T, is available within the domain of truth-values.

A last, but not least note on these results concerns the dialectical status of 'contradiction' in the light of $\mathbf{AR}_m^n$.

Contradiction really occurs inside the diagrammatic process of dichotomy, insofar as each new box partially or completely excludes the other ones by rejecting at least one of their elements. In the dialectical process of expansion, however, the negation at hand does *not* exclude but, rather, produces a new object through the process of antithesis.

Moreover, each of the above boxes does not express a mere *addition* or union of single answers to sentential questions. For example, the box 110 of the third particular diagram means 'true or neither-true-nor-false', by junction of the 3-tuples 100 and 010 within a fixed set of 3-tuples: $100 \cup 010 = 110$. By contrast, dialectics proceeds as an *aggregation* of successive n-tuples. Unlike the Boolean operations, the synthesis 111 does constitute neither the union $\cup$ nor the intersection $\cap$ of 110 and 001; assuming such Boolean operations at the basis of dialectical negation is like considering the integer 13 as the sum 1 + 3. Now this is precisely what logicians do frequently, when they intend to condemn Hegel's logic of contradiction by applying the operation of intersection as, e.g., in $100 \cap 011 = 000$. Contradiction is neither exclusive nor inclusive, in the *dynamic* perspective of dialectics where falsehood merely designates a later stage of truth: neither intersection nor union do make sense hereby, although they easily render the ideas of exclusive and inclusive contradiction.

So there is no contradiction for Heraclitus, we said. That is: there is no pure Non-Being from his viewpoint, symbolized by complete sequences of 0 in the above diagrams. Actually, the operation liable to make sense of dialectical negation is the operation of aggregation or addition in a domain of integers. In other words, dialectical negation proceeds like a successor-forming operator $\mathcal{S}_{n+1}^x$ applied to natural integers x such that $\mathcal{S}(n) = n+1$.

6. Ontological negation

We argued earlier that Heraclitus subscribed to a semantic game of the kind $\mathbf{AR}_1^n = \mathbf{AR}_1$, accepting everything and never going beyond the realm of the yes-answers. This can be explained by the kind of model that characterizes the way Heraclitus takes things to be. In other words, there is a genuinely *ontological* motivation behind Heraclitus' se-

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mantics. Let us see to what extent there cannot be any no-answer in his viewpoint, i.e. why nothing can be excluded from his set of beliefs.

Dialectics went through the present work under two different senses. For one thing, as the adequate description of formal semantics within increasing algebras, i.e., increasing domains of values resulting from a question-answer process $\mathbf{AR}^n_m$. Then the word has been used in the sense of a threefold process of thesis-anthesis-synthesis, as associated typically to Hegel and which corresponds to its dialectical negation or *Aufhebung*. Far from being impossible, the values coming from this process of 'sursumption' (by contrast with the Kantian 'subsumption') proceed by *exceeding* a given set of elements whilst *conserving* them in the form of even more structured and aggregated values; it is to be recalled here the analogy with the method consisting in adding one bit, in the binary interpretation system of dialectics. Identity and difference between objects are conflated by this way; so is the case for propositions and individuals, so long as the successive levels of discourse are not taken into account to distinguish the forms of cumulative of identity and the primary object (the 'Absolute'). These values are not truth-values in the logical sense of the word, however, if one considers these as exclusive subsets that help to define the relation of logical consequence thanks to a minimal demarcation line between designated and non-designated values in a given domain.

An essential precondition to make the allegedly 'impossible values' possible is replacing the initial issue introduced by speakers in a dialogue: logicians deal with consequence as a relation of preservation, whereas Heraclitus or Nagarjuna have nothing to preserve and always have ontology in mind when they talk about truth and falsehood. Admittedly, there is a link between truth-values and integers, especially in the area of many-valued logics where truth-values count more than $n = 2$ elements. But again, neither Bernays (1926) nor Suszko (1977) would endow these values with another sense than being various elements belonging to either of the two necessary and sufficient sets –the set of designated and non-designated values, in order to prove the independence of axioms among a given set of propositions. We argued that dialectics is quite another story: values are single elements within a theory, i.e. a set of increasing propositions, insofar as the expanding domain of values is on a par with the expanding set of propositions in a corresponding language.

Borrowing from the theory of naming in philosophy of language, we argue that truth-values (or semantic predicates) can be viewed as kinds of *proper names*.

In the Fregean sense of proper names, or in the Russellian theory of definite descriptions, a name corresponds to a number of objects that are truly predicated of it. This entails that more than one individual can be given the same proper name in this broad sense of the word, just as the True may be assigned to a large class of propositions. Therefore, the normal course of logic is to deal with truth-values as Fregean proper names; but there is another way of considering proper names, and we argue that that one is able to make sense of a 'dialectical logic'. In the Millian or Kripkean approach to proper names, indeed, these ones correspond to radically singular terms that cannot be assigned to more than one individual. In the same vein, the dialectical process of expansion is such that every semantic predicate characterizes a new unique individual. The cardinality of these truth-values may thus vary according to the underlying theory of logic and language –it depends upon whether the theory assumes bivalence or not. Now the debate we articulated around Heraclitus, Nagarjuna or the pseudo-Hegel does not deal with this logical issue of dealing with consequence; it is about ontology, so that truth-values are not properties of propositions but, rather, names of individual objects.

If such is the case, then some ontological commitment reappears through the issue of 'impossible values'. That is, each logical system intends to provide rules of discourse through a given language. Language is a set of propositions and, supposing that the corresponding speech-acts are limited to affirmative acts related to what there is, the logician deals with truth-values and methods for preserving some of these (whether they be unique, or not) among a set of propositions, from the premises until the conclusion. Now what is the point of preserving that which cannot be lost, whenever everything or nothing is taken to be true according to some peculiar speakers? Here is the most serious reason to consider dialectical negation as a non-logical negation, preferably to the account of an ontological negation mapping into a unique object that does not exist before that very process of mapping. No homomorphism proceeds from a domain of values $\mathcal{V}_m^n$ into itself with dialectical negation, the latter being characterized as a model-expanding operator dn that has nothing to do with the usual sentential operator of logic:

$$dn(\mathcal{V}_m^n) = \mathcal{V}_m^n + 1$$

Thus, such a higher-order and non-sentential negation is a mapping $\mathcal{V}_m^n$ into $\mathcal{V}_m^n + 1$ whose function extends the domain of values itself, irrespective of the question whether truth is preserved between the first and the second object –this is always the case with Heraclitus, so that such a question is trivial from his point of view.

To give a schematic account, every thing proceeds from the absolute Being in the Heraclitean (or Hegelian) monist world, whereas every thing seems to proceed from the absolute Non-Being in the illusory world of Nagarjuna (or the Buddhist philosophers). Consequently, any proposition should participate of truth for the former, of falsity for the former. It is as if nothing cannot not be true, according to Heraclitus; whereas nothing cannot not be false, according to Nagarjuna. Everything must be accepted, according to Heraclitus; everything must be rejected, according to Nagarjuna. Starting from this logical and ontological framework where propositions are merely truth-values accepted or rejected by speakers, the *dual* behavior of our two abnormal speakers echoes with Bahm (1958), according to whom the Jain and Buddhist logics are dual and stand for opposite propositional attitudes. However, our own point is that one should not characterize such speakers within one and the same semantic game $\mathbf{AR}^n_1 = \mathbf{AR}_1$. This holds only for Heraclitus, whereas a proper semantic game for Nagarjuna should rather be $\mathbf{AR}^n_0 = \mathbf{AR}_0$. His unique 'answer' is a non-answer of silence that comes from the vacuous state of the world in which nothing is to be properly answered, even negatively. For this reason, we depart from Priest (2010) by viewing only one way of interpreting the semantic predicate of truth according to Heraclitus and Nagarjuna: as a relative truth with the former speaker, hence his positive answer to any sentential question; as an absolute truth with the latter speaker, hence his stance of silence for want of any thing to be answered either positively or negatively.

As a conclusion, does it eventually make sense to question speakers like Heraclitus and Nagarjuna? It does not, so long as the underlying concepts of truth and falsehood are to be understood in the sole light of a logical process of *dichotomy* between accepted and rejected statements. It does, so long as the quest for alternative ways of thinking how things are is at stake. We need to enlarge the language game to make sense of abnormal answers, for this purpose, and we take the semantic framework $\mathbf{AR}^n_m$ to be in position to do so.

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