Classical Mechanics in School Physics: A Study of Particular Cases

Angel Akio Tateishi*

Universidade Tecnológica Federal do Paraná, Departamento Acadêmico de Física, Pato Branco, PR, Brazil

Apocryphal quotes attributed to Albert Einstein often lead physics educators to constantly verify sources and correct misconceptions about what Einstein said. This circumstance has inspired a similar reflection on the content taught in introductory basic physics courses. Indeed, how often are physics topics taught merely through the presentation of "formulas," without examining or discussing their limitations, the underlying idealizations, or the reasons for their use? Focusing on teaching the concept of force, this work presents a didactic review of broader contexts that lead to some well-known equations in physics formula sheets. In other words, starting from more general contexts, it will be shown that certain expressions are valid only in particular cases. The goal is to draw attention to the possibility that such particular cases are being taught as if they were the only possible description of the complexity of physical reality, rather than as physical models with their respective limitations.

Keywords: Classical Mechanics; Alternative Approach; Vector Quantities; Formalism; Idealized Models.

I. INTRODUCTION

Among the many quotes attributed to Albert Einstein, one that is widely circulated in various contexts is:

> *Everything should be as simple as possible, but not simpler.*

According to Alice Calaprice [1], this quote is one whose attribution to Einstein is subtly debatable, and it appears to be an edited or simplified version of part of a lecture Einstein gave in 1933:

> *It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience.*

Although longer, this version is more detailed and less susceptible to misinterpretation because it is properly contextualized, and the authorship and reference are reliable and verifiable. The first quote is simpler, but its meaning and interpretation become more complete and less ambiguous when the original source is known.

How many "formulas" have been taught, memorized, and quoted for so long that their origin, limitations, or the reason for using them are no longer subjects of curiosity or scrutiny? For example, the famous $F = ma$, the kinematic equations, the acceleration of circular motion $a_c = -v^2/r$, and the definitions of work $W = Fd\cos\theta$ and torque $\tau = Fr\sin\theta$ all seem to follow the idea that everything should be as simple as possible.

In general, physics is taught according to the presumed level of mastery of mathematical language at each stage of education. Without differential, integral, and vector calculus; without linear algebra; without ordinary and partial differential equations, teaching can become limited to the simplest expressions, which describe only particular cases of more general contexts.

However, simplifying things does not necessarily mean making them easier or oversimplified. Note that the phrase *"to make the irreducible basic elements as simple and as few as possible"* is accompanied by *"without having to forgo the adequate representation of a single piece of experiential data"*. This is why Newton's laws and Maxwell's equations are so important—not because they simplify the description of physical phenomena or our understanding of the world, but because, starting from a few equations, by interpreting what is encoded in the mathematical language, we can apply and adapt them to the study of various systems.

Suppose we forget that physics is taught through mathematical models (which have limitations). In that case, we may end up, for example, confusing or limiting the concept of force to a formula used merely to substitute numbers or to the 'arrows' representing pushes or pulls on blocks. In other words, expressions for particular cases can become the only known description of physical systems, since it is often not mentioned that physics also applies to more complicated or realistic cases.

In general, physics undergraduate program graduates have likely encountered mechanics courses—classical, an-

---

* angeltateishi@utfpr.edu.br

alytical, statistical, and quantum—from which they can establish basic physics within a broader context, mathematically and in applications. However, many physics instructors may come from other academic backgrounds. Therefore, as a teaching aid for physics education, this work proposes a concise and didactic review addressing some classical mechanics concepts and their relations to what is taught about mechanics at the basic level.

In this context, we will consider the equations $F = ma$, $v(t) = v_0 + at$, $v = r\omega$, $a_c = -v^2/r$, $W = Fd\cos\theta$, and $\tau = Fr\sin\theta$ as our statement *"Everything should be as simple as possible, but not simpler"*, and we will look for the 'more general statements' from which simplified versions were derived. We will show how a broader view of these expressions enables a better understanding of their limitations and the possibilities for more general cases. At the same time, fundamental concepts related to vectors and calculus are addressed in a more interpretative and accessible manner. This text was developed based on the author's pedagogical approach to these topics in the classroom. For this reason, this work is not characterized as the result of research on teaching, but rather as a reflection of the practice of teaching physics. It always seeks a balance between presenting concepts simply and maintaining the rigor of mathematical language.

Therefore, Section II will address Newton's second law, $F = ma$; Section III will discuss the traditional definition of kinematics; Section IV will examine what is implicit in the equations describing curvilinear motion in two dimensions; Section V will present topics typically covered in textbooks following Newton's laws, placing impulse, work, and torque within a broader context: the different ways of multiplying vectors and their associated concepts. Finally, Section VI will be dedicated to the discussion and conclusion.

II. NEWTON'S SECOND LAW

The concept of force is one of the pillars of physics [2]. Although it has a long history, even before Newton [3], force is commonly associated solely with the expression

$$\sum F = ma, \tag{1}$$

or with arrows pushing or pulling blocks in physics exercises. Mathematically, Eq. (1) describes particular cases in which translational motion is restricted to a single direction. With only one spatial coordinate, vector notation is avoided. Force and acceleration are vector quantities; therefore, for motion in two or three dimensions, we write:

$$\sum \vec{F} = m\vec{a}. \tag{2}$$

Recall that acceleration represents how the velocity vector changes over time, that is, $\vec{a} = \frac{d\vec{v}}{dt}$. A vector has both magnitude and direction, and these two properties can vary with time[**?**]. If neither the magnitude nor the direction of the velocity vector $\vec{v}$ changes over time, then the derivative $\frac{d\vec{v}}{dt} = \vec{0}$. The 'zero vector' notation reminds us that when there is a vector on one side of the equation, there must be a vector on the other side. The expression cannot be considered an equation if this does not occur. Moreover, $\vec{0}$ indicates that all vector components must be zero.

That said, we can interpret the equation

$$\sum \vec{F} = \vec{0} = m\vec{a}. \tag{3}$$

For $m\vec{a} = \vec{0}$ to hold, either the mass is zero ($m = 0$) or the acceleration is zero ($\vec{a} = \vec{0}$). It makes no physical sense for the mass to be zero, since it is through mass that interaction occurs, and it represents inertial mass (no mass, no inertia). Therefore, the only possibility is that the acceleration is zero. For this to happen, the velocity vector must be constant in magnitude and direction. A constant direction means a straight trajectory, and a constant magnitude means uniform motion. In other words, an object tends to remain in uniform straight-line motion unless a nonzero resultant force acts on it, changing its velocity vector.

It is important to remember that acceleration and velocity are vectors, as these terms are often interpreted according to common sense. For example, in the case of vehicles, accelerating means increasing the magnitude of velocity (as shown by the speedometer). Turning while maintaining a constant speed or braking is typically not perceived as acceleration in common understanding. In physics, if the velocity vector changes direction over time, the motion is characterized as accelerated. Curvilinear trajectories are a consequence of accelerated motion.

Another particularity of Eq. (2) is its validity only when the mass m is constant. In its most general form, Newton's second law is described as the time variation of linear momentum $\vec{p} = m\vec{v}$,

$$\sum \vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}). \tag{4}$$

Using the product rule for derivatives, we have

$$\sum \vec{F} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}, \tag{5}$$

where it can be seen that, in addition to the variation in velocity, we must consider the influence of the mass variation over time, as illustrated in Fig.(1). If the mass m is constant in time, then $\frac{dm}{dt} = 0$ and Eq. (2) is recovered. The other particular case would be if the velocity vector is constant but the mass varies with time. This is not trivial, since we would have uniform straight-line motion ($\vec{v}$ constant) in which the force always adds or removes mass from the system to maintain the same velocity.

Treating the mass as constant makes sense, especially when we are already considering that the entire mass of
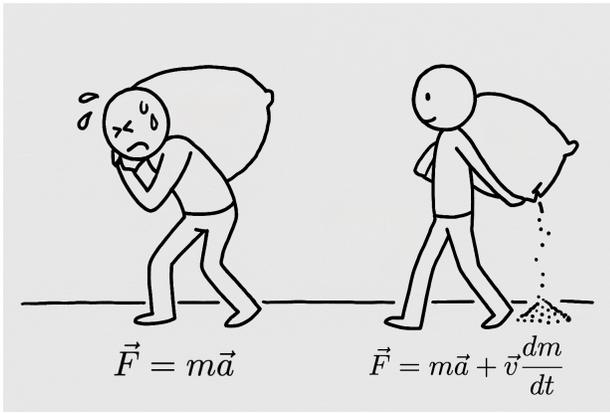
Figure 1. When 'F=ma' isn't enough: carrying a leaking sack means dealing with variable mass. On the left, the case with constant mass — $\vec{F} = m\vec{a}$. On the right, variable mass system, $\vec{F} = m\vec{a} + \vec{v}\dfrac{dm}{dt}$, where the second term accounts for the momentum variation due to mass leakage.

the object is concentrated at a single point in space, that is, using the *particle* approximation. A point has zero dimension, meaning it has no width, height, or depth. Thus, in terms of the degrees of freedom of an object, the forces acting on a particle can only cause translational motion. Discussing deformations and rotations caused by forces only makes sense if we consider that the object has an extension in space, which a point does not have. This approximation of treating an object as a particle is important because it is always possible to calculate the center of mass of an object and thereby simplify the study of translational motion[?], by considering that all forces act directly at the center of mass. However, forces can also cause rotation. In this case, the object's shape, how its mass is distributed in volume, the location where the force is applied, and the axis of rotation cannot be disregarded. To differentiate between translational and rotational motions and their respective velocities and accelerations, we use the adjective *linear* for translational motions (linear velocity $\vec{v}$, linear acceleration $\vec{a}$); and the adjective *angular* for rotational motions (angular velocity $\vec{\omega}$, angular acceleration $\vec{\alpha}$).

Here we are referring only to solid objects; when analyzing fluids, which do not have a defined shape, it is preferable to use the volumetric mass density $\rho = \frac{m}{V}$ instead of the total mass. Another characteristic is that the fluid velocity can vary not only with time but also with space. In this case, acceleration is commonly described using the material derivative symbol D/Dt, that is,

$$\vec{a}(\vec{r}, t) = \frac{D}{Dt}\vec{v}(\vec{r}, t) = \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v}. \qquad (6)$$

Where $\frac{\partial \vec{v}}{\partial t}$ represents the variation of the velocity vector field concerning time, and $(\vec{v} \cdot \nabla)\vec{v}$ represents the variation of the velocity vector field concerning the spatial coordinates.

III. KINEMATICS

In general, kinematics is described as the branch of physics that describes the motion of bodies without concerning itself with the causes of the motion [4]. However, this definition may seem contradictory, as kinematics often deals only with cases where the cause of motion is a constant force.

Suppose a one-dimensional case in which a single constant force F_c acts on a particle of mass m. Newton's second law for this case is

$$F_c = ma = m\frac{dv}{dt}. \qquad (7)$$

Since Eq. (7) involves an equals sign and derivatives, we call it a differential equation. The derivative is called ordinary because the velocity function depends only on time. Therefore, we have an *ordinary differential equation*. We can rewrite Eq. (7) as follows:

$$dv = \left(\frac{F_c}{m}\right) dt. \qquad (8)$$

Read "dv" as a *small variation[?] in velocity*, which is directly proportional to a *small variation in time dt*. That is, the longer the time interval considered, the greater the change in velocity. In addition, dv is proportional to the force F_c (the greater the force, the greater the change in velocity), and inversely proportional to the mass[?] m (the greater the mass, the smaller the change in velocity, and vice versa).

Suppose we sum all the infinitesimal variations dv from an initial velocity at time $t = 0$, v_0, to some velocity at time t. In that case, we obtain the total variation in velocity $v(t)$ over that time interval. This is equivalent to summing all the dt variations multiplied by the term F_c/m. When summing discrete variables, we use the symbol $\sum$; when summing continuous variables, we use a different symbol: $\int$. Although integrals may trigger traumatic memories from undergraduate studies, they merely symbolize the sum of continuous quantities. What has just been described is represented by

$$\int_{v_0}^{v(t)} dv = \int_0^t \left(\frac{F_c}{m}\right) dt. \qquad (9)$$

Since the mass m and the force F_c are constants, the value of F_c/m is also constant. Because $\int_0^t dt$ sums only contributions of terms that depend on time, the factor F_c/m can be taken outside the integral:

$$\int_{v_0}^{v(t)} dv = \left(\frac{F_c}{m}\right) \int_0^t dt. \qquad (10)$$

Solving these integrals yields

$$v(t) = v_0 + \left(\frac{F_c}{m}\right) t. \qquad (11)$$

Note that on the left-hand side we have velocity (length/time), so on the right-hand side, each term, v_0 and $(F_c/m)t$, must also have the dimensions of velocity. Since t is time, F_c/m must necessarily have physical dimensions of length divided by time squared, also known as acceleration. Therefore, we can define

$$\frac{F_c}{m} = a_c. \tag{12}$$

Using the above definition in Eq. (11), we obtain one of the standard equations of kinematics:

$$v(t) = v_0 + a_c t. \tag{13}$$

From this, we can obtain the function that describes the position x at any time t. It is enough to consider the definition of velocity as the rate of change of position with respect to time, that is,

$$v(t) = \frac{dx}{dt} = v_0 + a_c t. \tag{14}$$

Rewriting this in terms of infinitesimal variations,

$$dx = (v_0 + a_c t)dt, \tag{15}$$

we observe that an infinitesimal variation in position dx is proportional to a small variation in time dt. Moreover, dx also depends on the instantaneous velocity at each time t, which is proportional to the term $v_0 + a_c t$. The sum of all such variations on both sides of Eq. (15) is represented by the integrals

$$\int_{x_0}^{x(t)} dx = \int_0^t (v_0 + a_c t)dt, \tag{16}$$

whose final result is the well-known position equation:

$$x(t) = x_0 + v_0 t + \frac{1}{2}a_c t^2. \tag{17}$$

The position at any time t is the sum of the initial position x_0 with the distance traveled due to the initial velocity ($v_0 t$) and due to the change in velocity over time $(\frac{1}{2}a_c t^2)$.

The results of Eqs. (11) and (17) are valid for any constant force. For example, suppose the reference axis is vertical and the acting force is the weight force; in that case, we have $F_c = -mg$. Substituting this into Eq. (12),

$$\frac{-mg}{m} = a_c = -g. \tag{18}$$

In this way, we recover the equations of free fall: $v(t) = v_0 - gt$ and $y(t) = y_0 + v_0 t - (1/2)gt^2$.

The force may depend on time, for example, $F(t) = F_0 t$ (a force that increases linearly with time), or a force that decreases exponentially, $F(t) = F_0 e^{-bt}$. In such cases, the integral on the right-hand side of Eq. (9) would be different, and we would obtain different expressions for the velocity and position as functions of time. We

also encounter cases where multiple forces act simultaneously, such as in free fall with air resistance. The more interactions we consider, the more complex the resulting differential equations become. However, the versatility of Newton's second law lies in the fact that, if we know the forces acting on an object and we know how to solve differential equations, we can determine the functions that describe the behavior of velocity and position.

Therefore, a more appropriate definition is that kinematics is the branch of physics that does not deal with motion caused by time-dependent forces and that is limited to Cartesian coordinates.

IV.   NEWTON'S SECOND LAW FOR CURVILINEAR MOTION

In Cartesian coordinates, the unit and orthogonal vectors (unit vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$) that form the vector basis are fixed. In polar coordinates, which are used to describe curvilinear motion, the positions x and y are expressed in terms of the radius r and the angle θ. However, the vector basis is no longer fixed but follows the particle along its trajectory. As illustrated in Fig.2 This facilitates the description of curvilinear motion[?], but one consequence is that the unit vectors ($\hat{r}$ and $\hat{\theta}$) also vary with time. The derivatives of the position and velocity vectors become more complex compared to the Cartesian case. Thus, we can express the linear velocity and linear acceleration vectors regarding angular velocity and acceleration. In general, the description of a physical problem can be simplified by an appropriate choice of coordinate system.
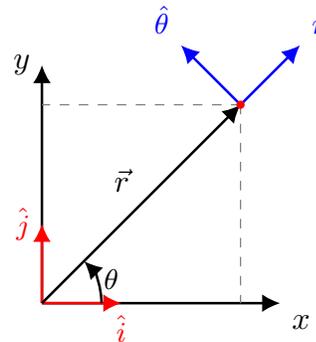

Figure 2. Representation of the polar coordinate system superimposed on the Cartesian system. The unit vectors $\hat{i}$ and $\hat{j}$ (in red) represent the fixed Cartesian basis. The position vector $\vec{r}$ points from the origin to the particle (red dot). The unit vectors $\hat{r}$ and $\hat{\theta}$ (in blue) form the moving basis associated with the polar system, where $\hat{r}$ points in the radial direction and $\hat{\theta}$ is tangent to the trajectory, forming an angle θ with the x-axis.

The velocity in polar coordinates is given by [5]:

$$\vec{v}(t) = \left(\frac{dr}{dt}\right)\hat{r} + r\left(\frac{d\theta}{dt}\right)\hat{\theta}. \tag{19}$$

Here, dr/dt describes how the radius of the trajectory changes with time. The angle variation with respect to time is defined as the angular velocity, $d\theta/dt = \omega$.

The velocity has both a radial and a tangential component. To better understand this, let us consider two particular cases. First, suppose that the angle θ does not vary with time, i.e., the angle is constant and the object does not rotate. In this case, the only possible motion is in the radial direction, meaning the radius r of the trajectory changes (the object either moves closer to or farther from the center). Second, suppose that the radius is constant. In this case, we have circular motion since the angle θ variation always occurs tangentially to the radius.

With this understanding, we can rewrite $\vec{v}(t)$ as follows:

$$\vec{v}(t) = \left(\frac{dr}{dt}\right)\hat{r} + (r\omega)\hat{\theta}. \tag{20}$$

From this, acceleration (always defined as $\vec{a}(t) = d\vec{v}/dt$) can be calculated. The final result [5] may not appear very friendly:

$$\vec{a}(t) = \left[\frac{d^2r}{dt^2} - r\omega^2\right]\hat{r} + \left[r\alpha + 2\left(\frac{dr}{dt}\right)\omega\right]\hat{\theta}, \tag{21}$$

where $\alpha = d\omega/dt$ is the angular acceleration, that is, the rate of change of angular velocity with respect to time.

Again, we have radial and tangential components of acceleration: two terms in the radial direction and two in the tangential direction. The negative sign in the radial component indicates that the acceleration points toward the center. This general expression describes the acceleration for any curvilinear motion in which both the radius and the angle vary with time.

Things become simpler when we consider particular cases. The most common assumption is that the radius r is constant, i.e., the motion is circular. Mathematically, this implies that $dr/dt = 0$. With this, Eqs. (20) and (21) become:

$$\vec{v}(t) = 0\hat{r} + (r\omega)\hat{\theta}, \tag{22}$$

and

$$\vec{a}(t) = \left(-r\omega^2\right)\hat{r} + (r\alpha)\hat{\theta}. \tag{23}$$

Another particular case we can consider is when the angular velocity ω is constant. This means the particle always takes the same time to complete a full revolution. In this way, we arrive at the equations of uniform circular motion (since r is constant) (and ω is constant, thus $\alpha = 0$):

$$\vec{v}(t) = (r\omega)\hat{\theta}, \tag{24}$$

and

$$\vec{a}(t) = \left(-r\omega^2\right)\hat{r}. \tag{25}$$

With only one component in both velocity and acceleration, we can always avoid vector notation by considering only the magnitudes of these vectors. This yields the well-known relations:

$$v = r\omega, \tag{26}$$

and

$$a = -r\omega^2 = -r\left(\frac{v}{r}\right)^2 = -\frac{v^2}{r}. \tag{27}$$

Finally, Newton's second law for a curvilinear motion with constant radius and constant angular velocity is

$$\vec{F} = \frac{mv^2}{r}(-\hat{r}). \tag{28}$$

In this context, as trivial as it may seem, it is important to remember that every curvilinear motion is accelerated—that is, there is variation in the velocity vector. In this case, the force responsible for the acceleration prevents the object from following uniform linear motion; in other words, it prevents the velocity vector $\vec{v}$ from remaining constant. This tendency of the force to sustain the curvilinear (or, in this particular case, circular) motion—constantly causing the particle to change direction—is represented by the negative sign in the unit vector $\hat{r}$.

V.   IMPULSE, WORK, AND TORQUE

Let us begin with a few simple questions regarding the action of forces on an object or system: How long does the force act, and how does it influence the time variation of linear momentum? What is the relationship between the displacement vector of the object and the direction of the applied force? When should we be concerned about the location on the object where the force is applied, and what kinds of motion can this cause? These questions are addressed by the topics usually introduced after the applications of Newton's second law—namely, impulse and collisions; work and energy; torque and rotations.

Summarizing or recalling these ideas is possible because force is mathematically described as a vector. The starting point is to remember that a vector can be multiplied in three different ways:

- A vector multiplied by a scalar, $a\vec{A}$, where a is any scalar. The vector retains its direction and only undergoes a scaling transformation (change in magnitude).

- A vector multiplied by another vector, resulting in a scalar: $\vec{A}\cdot\vec{B} = \text{scalar}$, which is naturally called the *dot product*. It can be interpreted as the projection of one vector onto another.

- A vector multiplied by another vector, resulting in a third vector: $\vec{A} \times \vec{B} = \vec{C}$, where $\vec{C}$ is a vector perpendicular to the plane formed by vectors $\vec{A}$ and $\vec{B}$ and indicates that a rotation in this plane would go from $\vec{A}$ to $\vec{B}$. In brief, this is the definition of the *cross product* that I use in class.

Since force is a vector quantity, it can also be multiplied in three different ways:

- Force multiplied by a scalar—in this case, time t. This gives us a quantity that accounts for how long the force acted. It answers the question: *Does it matter how long the force acts on the object?* This leads to the definition of *impulse* $\vec{I}$:

$$\vec{I} = \int_{t_i}^{t_f} \vec{F}\, dt = \int_{\vec{v}_i}^{\vec{v}_f} d(m\vec{v}). \tag{29}$$

- Force multiplied by a displacement vector using the dot product. This gives us a quantity that considers the distance over which the force acted and how it contributed to that displacement. This is the definition of *work* W:

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r}\,. \tag{30}$$

- Force multiplied by a displacement vector using the cross product. This gives us the plane and the direction of a rotation caused by the force $\vec{F}$, where the displacement vector $\vec{r}$ starts at a reference point (typically the location of the axis of rotation) and ends at the point where the force $\vec{F}$ is applied. Suppose we consider a distribution of force $d\vec{F}$ instead of discrete forces. In that case, we obtain the definition of *torque* (or the *moment* of a force distribution, where "moment" is interpreted as an average over the distribution):

$$\vec{\tau} = \int \vec{r} \times d\vec{F}. \tag{31}$$

A summary of the above is presented in Fig. 3.

From Eq. (29), we derive the content related to collisions and the conservation of linear momentum. From Eq. (30), we obtain the concepts of energy (kinetic and potential), as well as the important principle of energy conservation. Finally, from Eq. (31), we can develop the theory of rigid body rotation[?].

Eqs. (29) and (30) can be simplified if we consider the particular case of constant forces. However, it is important to note that the adjective *constant* depends on the integration variable. The force is considered constant in the definition of impulse, Eq. (29), if it does not depend on time. In contrast, in the definition of work, Eq. (30), the force is constant if it does not depend on any spatial variable. Not coincidentally, according to Noether's
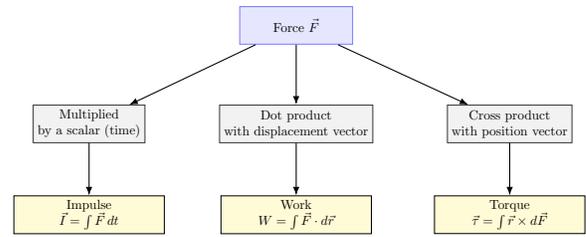

Figure 3. Diagram illustrating how different mathematical operations applied to the Force vector result in fundamental physical concepts studied in introductory Physics courses.

theorem [6], the conservation of linear momentum arises from invariance under spatial translations, while energy conservation is characterized by invariance under time translations.

Thus, taking into account this subtlety in the definition of what it means for a force to be constant in each context, the force can be taken outside the respective integrals:

$$\vec{I} = \vec{F} \int_{t_i}^{t_f} dt = \vec{F}(t_f - t_i) = \vec{F}\Delta t, \tag{32}$$

and

$$W = \vec{F} \cdot \int_{\vec{r}_i}^{\vec{r}_f} d\vec{r} = \vec{F} \cdot \Delta \vec{r}\,. \tag{33}$$

Note that the dot product is still present, since the integral of $d\vec{r}$ remains a vector, as we have defined through $\Delta \vec{r} = \vec{r}_f - \vec{r}_i$ (the total displacement $\vec{d}$). The next simplification would be to avoid this vector notation for the dot product. The concept behind the dot product is the "shadow" or projection that one vector casts onto the other. If the two vectors are parallel (i.e., the angle between them is 0 degrees), the projection is maximal; if the vectors are perpendicular (angle of 90 degrees), there is no projection.

Now, what familiar function reaches a maximum[?] at $\theta = 0$ and becomes zero when $\theta = \pi/2$? The answer is that work can be represented as:

$$W = Fd\cos\theta. \tag{34}$$

Eq. (31) can be simplified if we consider that the forces are discrete and constant in time, rather than distributed over a region. In this case, we replace the integral symbol with the summation symbol Σ, that is,

$$\vec{\tau} = \sum_{i=1}^{N} \vec{r}_i \times \vec{F}_i. \tag{35}$$

If only one force is acting, the summation will have only one term. The resulting torque is a vector perpendicular to the plane of rotation of the object, and it also

indicates the direction of rotation within that plane. In general, torque is a vector with three components, each perpendicular to one of the three coordinate planes (xy, yz, and zx). However, vector notation can be avoided if rotation is restricted to a single plane. If the plane is the xy-plane, rotation will either occur from x to y or from y to x, and in both cases the torque will have only a z-component (which is perpendicular to the xy-plane). It can therefore be positive or negative, indicating the direction of the rotation.

Nevertheless, this simplification comes at the cost of disregarding the whole mathematical meaning of the cross product and its connection to the definition of rotational planes. If the rotation is restricted to a single plane, torque can also be defined using the angle between $\vec{r}$ and $\vec{F}$. Recall that these two vectors must define a plane. If they are parallel ($\theta = 0$), they do not define a plane (or define one whose parallelogram area is zero). The area of the plane is maximized when the vectors are perpendicular ($\theta = \pi/2$), in which case the area is rF.

Which function do we know that is zero at $\theta = 0$ and maximal at $\theta = \pi/2$? By answering this question, we obtain the definition:

$$\tau = Fr\sin\theta, \tag{36}$$

in which r is called the lever arm.

Therefore, a rarely mentioned disadvantage is that the particular case represented by Eq. (36) omits the fact that torque is a vector[?], thus making it harder to understand what torque actually is—being colloquially confused with force itself, rather than seen as the description of rotational motion caused by a force.

These concepts of scalar and vector products are necessary to understand mathematically, for instance, why it becomes explicit that the magnetic force[?], $\vec{F}_{mag} = q(\vec{v} \times \vec{B})$, does not perform work. From the cross product definition, it is easy to infer that the magnetic force is perpendicular to the plane formed by the velocity vector and the magnetic field. The work done by the magnetic force is given by

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}_{mag} \cdot d\vec{r} = \int_{\vec{r}_i}^{\vec{r}_f} q(\vec{v} \times \vec{B}) \cdot d\vec{r}. \tag{37}$$

Recalling that the displacement $d\vec{r}$ and the velocity $\vec{v}$ of the electric charge are related by $\vec{v} = d\vec{r}/dt$, we can rewrite the above equation as:

$$W = \int_{\vec{r}_i}^{\vec{r}_f} q(\vec{v} \times \vec{B}) \cdot \vec{v}\, dt. \tag{38}$$

Therefore, knowing that the result of $\vec{v} \times \vec{B}$ is perpendicular to the velocity vector $\vec{v}$ and that the scalar product quantifies the projection of one vector onto another, we conclude that, due to this perpendicularity, the work done by the magnetic force is zero. The magnetic force is always at a right angle ($\pi/2$) to the displacement vector.

VI.   DISCUSSION AND CONCLUSION

If one had to simplify the definition of force into a single word that still encompassed the full breadth of the concept, that word would be *interaction.* Moreover, the concept of force has been the protagonist of this work, and it is a vector quantity.

Even when the topic was kinematics, we saw that the equations studied in kinematics are examples of motion resulting from forces that are constant in time. If the interaction involved were not constant, the equations of motion would have to be modified accordingly. When all forces acting on an object are constant in time, it suffices to sum them vectorially, $\sum \vec{F}$. This net force is directly proportional to the object's mass m and can produce changes in the velocity vector that describes the object's motion, that is, $\sum \vec{F} = m\vec{a}$. As we have seen, Newton's second law uses this form to assume that mass m is constant in time. In such a case, a constant acceleration results from constant forces. From this acceleration, we can derive the velocity function; from the velocity, we obtain the position function as a function of time. All these functions are consistent with the kinematic equations.

Implicitly, we relate the net force to the translational motion of an object. However, forces can cause other types of motion besides translation, such as rotation and deformation. If these rotational and deformational effects are negligible, we may treat the object as a particle and assume that all forces act at its center of mass. In the classroom, it is important to emphasize that forces can induce other motions beyond translation and that objects can simultaneously undergo multiple types of motion.

Of course, it is possible to approach curvilinear motion by approximating the object as a particle. In this case, vectors and vector bases become essential. The equation $\sum \vec{F} = m\vec{a}$ is general because it can be adapted to different coordinate systems. The variables x, y, and z are treated as independent in Cartesian coordinates. However, in the case of circular motion in the xy-plane, there is a well-defined relationship between the values of x and y. In such situations, polar coordinates are more convenient. As we showed in Section IV, the final expressions for the velocity and acceleration vectors in polar coordinates are more complex than in the Cartesian case. We derived the expressions commonly used in basic physics education from particular cases with constant radius and constant angular velocity.

Although it may be challenging to introduce different types of coordinate systems at the introductory level, it is possible to mention that other coordinate systems beyond Cartesian exist[?]. It may be easier, for example, to explain the acceleration in uniform circular motion, $a = -\frac{v^2}{r}$, by saying that it is an adaptation of Newton's second law to polar coordinates. In this context, it is important to highlight the limitations of such models in the classroom. These equations only apply when the path is circular and the angular velocity is constant. The "ex-

ceptions" to these particular conditions are, in fact, more common in everyday life.

Emphasizing that forces are vectors also provides a more structured perspective or a mnemonic for recalling key concepts in mechanics. Since force is a vector, it can be combined in three distinct ways: with a scalar, with another vector resulting in a scalar, and with another vector resulting in a vector. Time (a scalar quantity) and displacement (a vector quantity) are the quantities commonly combined with force. In the first case, we obtain the concept of *impulse*—force multiplied by the time it acts. In the second, we have the definition of *work*—how much the force contributes to motion in the direction of the displacement. In the third, we have a way to describe rotations caused by a force acting on an extended object (i.e., not modeled as a point particle).

Presenting the overall structure of a topic can be helpful, as it provides learners with a roadmap of what will be covered and how the concepts are interconnected. Highlighting each concept's limitations and specific assumptions is also essential, as it helps develop critical thinking when applying and evaluating physical models.

Finally, we propose interpreting the term *basic physics* in light of Einstein's quotes mentioned in the introduction. As applied to physics, the adjective *basic* should not imply that the subject must be simple, simplified, or easy. Instead, we suggest interpreting *basic* as referring to that which provides a foundation, fundamental principles upon which all the rest is built.

[1] CALAPRICE, Alice. *The ultimate quotable Einstein*. 2. ed. Oxford: Princeton University Press, 2011. ISBN 9780691138176.

[2] JAMMER, Max. *Conceitos de força: um estudo sobre os fundamentos da dinâmica*. Rio de Janeiro: Contraponto Editora, 2011. ISBN 9788578660345.

[3] DIAS, Penha Maria Cardoso. F = ma?!! O nascimento da lei dinâmica. *Revista Brasileira de Ensino de Física*, v. 28, n. 2, p. 205–234, 2006.

[4] LUZ, Antônio M. R. da; ÁLVARES, Beatriz Alvarenga; GUIMARÃES, Carla da Costa. *Física: contexto aplicações: ensino médio*. 2. ed. São Paulo: Scipione, 2016. ISBN 9788526299191.

[5] TAYLOR, John R. *Mecânica Clássica*. Porto Alegre: Bookman, 2013. ISBN 978-85-8260-088-7.

[6] ACEVEDO, O. A.; PIMENTEL, B. M. Teoremas de Nöther: um estudo. *Revista Brasileira de Ensino de Física*, v. 45, p. e20230091, 2023.