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Lorentz violation, Bhabha scattering and finite temperature

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In this paper an introduction to Lorentz violation has been done. Corrections due to Lorentz violation for the cross section of the Bhabha scattering has been calculated at zero and finite temperature. The finite temperature corrections are calculated using the Thermo Field Dynamics (TFD) formalism.

Keywords: Thermo Field Dynamics (TFD), Bhabha scattering

I. INTRODUCTION

The Standard Model (SM) is a theory that describes all the elementary particles and its interactions. Lorentz and CPT symmetry have been the foundation of the SM. Although successfully confirmed, the SM is not a fundamental theory since gravity is not included. A fundamental theory that unifies SM and gravity would emerge at energies approaching Planck scale ($\approx 10^{19}$ GeV). Tiny violations of Lorentz and CPT symmetries could emerge in models unifying gravity with quantum physics such as string theory [1]. This would involve finding new physics. For study such violations a new theory has been developed that is called the Standard Model Extension (SME).

The SME [2, 3] contains the Standard Model, General Relativity and all possible operators that break Lorentz symmetry. In addition, the SME is divided into two parts: (i) the minimal version which has operators with dimensions $d \leq 4$ and preserves conventional quantization, hermiticity, gauge invariance, power counting renormalizability, and positivity of the energy and (ii) the non-minimal version which also includes operators of higher dimensions. The structure of the SME is one way to investigate the Lorentz violation. Another interesting way is to modify the interaction vertex between fermions and photons, i.e., a new non-minimal coupling term added to the covariant derivative. The non-minimal coupling term may be CPT-odd or CPT-even.

The main objective of this work is to study corrections for the Bhabha scattering [4] due to Lorentz violation. It is a process usually used in test of experiments at high energy accelerators [5–8]. The cross section for this scattering in the context of non-minimal coupling term at zero and finite temperature is calculated. Lorentz symmetry violation is expected to be small at very high energies, i.e., Planck energies ($\sim 10^{19}$GeV). But this is not valid in all cases. It is likely that Lorentz violation operators with dimension $d > 4$ will be relevant in searches involving very high energies [9]. Although Bhabha scattering, at high energy in colliders like LEP, are still at zero temperature, there is certainly no investigation at extremely high energy with non-zero temperature. Even though these are small numbers still it is important to calculate the role of temperature in Bhabha Scattering. These estimates will give us a reasonable idea of the role of SME at finite temperatures.

There are three different, but equivalent, formalisms to introduce temperature effects in a quantum field theory. (i) The Matsubara formalism [10] which is based on a substitution of time, t, by a complex time, $i\tau$. (ii) The closed time path formalism [11] that is a real time formalism at finite temperature. (iii) The Thermo Field Dynamics (TFD) formalism [12–16]. TFD formalism is a real time finite temperature formalism. Here this formalism is considered. TFD has as basic ingredients: (i) the doubling of the original Fock space and (ii) the Bogoliubov transformation. This doubling consists of Fock space composed of the original, S and the tilde space $\tilde{S}$. The map between the tilde $\tilde{A}_i$ and non-tilde A_i operators is defined by the following tilde (or dual) conjugation rules:

$$(A_iA_j)^\sim = \tilde{A}_i\tilde{A}_j, \quad (\tilde{A}_i)^\sim = -\xi A_i, \qquad (1)$$
$$(A_i^\dagger)^\sim = \tilde{A}_i{}^\dagger, \quad (cA_i + A_j)^\sim = c^*\tilde{A}_i + \tilde{A}_j,$$

with $\xi = -1$ for bosons and $\xi = +1$ for fermions. The physical variables are described by non-tilde operators. The Bogoliubov transformation is a rotation involving these two spaces. As a consequence the propagator is written in two parts: $T = 0$ and $T \neq 0$ components.

This paper is organized as follows. In section II, the Lagrangian of the Standard Model Extension is briefly

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discussed. The non-minimal Lorentz-violating coupling is considered. In section III, the Bhabha scattering at zero temperature with Lorentz violation is determined. In Section IV, a brief introduction to the TFD formalism is presented. In section V, the cross section for Bhabha scattering with Lorentz violation at finite temperature is calculated. In section VI, some concluding remarks are presented.

II. INTRODUCTION TO THE STANDARD MODEL EXTENSION

Here a brief introduction to the Lagrangian of the SME is presented. The SME is a field-theory based test framework in which to explore Lorentz violation. The framework consists of known physics in the form of the standard model plus general relativity action along with all possible Lorentz-violating terms that can be constructed from the associated fields. The mechanisms by which coefficients for Lorentz violation could arise in the SME can be divided into two categories: explicit Lorentz-symmetry breaking and spontaneous Lorentz-symmetry breaking. The full SME includes an infinite number of operators of ever-increasing mass dimension. A Lagrangian that describes the full SME is given as

$$\mathcal{L}_{SME} = \mathcal{L}_{SM} + \mathcal{L}_{GR} + \mathcal{L}_{LV} \tag{2}$$

where $\mathcal{L}_{SM}$ describes the usual SM fields, $\mathcal{L}_{GR}$ describes the usual General Relativity (GR) fields and $\mathcal{L}_{LV}$ describes all possible Lorentz-violating terms constructed from SM and GR fields and background coefficients.

As an example, let's consider the following Lorentz-violating terms occurring in the fermion and photon sectors.

A. Dirac field of the SME

The Lagrangian for the fermion sector of the extended quantum electrodynamics of the SME is

$$\mathcal{L} = \bar{\psi}\left(i\Gamma^\mu \overset{\leftrightarrow}{\partial}_\mu - M\right)\psi, \tag{3}$$

where

$$\Gamma^\mu = \gamma^\mu + (c^{\mu\nu} + d^{\mu\nu}\gamma_5)\gamma_\nu + e^\mu + if^\mu\gamma_5 + \frac{1}{2}g^{\kappa\mu\nu}\sigma_{\kappa\nu}, \tag{4}$$

$$M = m + (a^\mu + b^\mu\gamma_5)\gamma_\mu + \frac{1}{2}H^{\mu\nu}\sigma_{\mu\nu}. \tag{5}$$

The parameters in Γ^μ are dimensionless while the ones in M have dimension of mass. γ^μ, γ_5 and $\sigma_{\kappa\nu}$ denote the Dirac matrices. The coefficients for Lorentz violation are a^μ, b^μ, $c^{\mu\nu}$, $d^{\mu\nu}$, e^μ, f^μ, $g^{\kappa\mu\nu}$ and $H^{\mu\nu}$.

B. The pure-photon sector of the minimal-SME

The lagrangian of the pure photon sector of the SME is given by

$$\mathcal{L} = \mathcal{L}_{EM} + \mathcal{L}_{CPT-odd} + \mathcal{L}_{CPT-even}, \tag{6}$$

where

$$\mathcal{L}_{EM} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{7}$$

is the Maxwell lagrangian,

$$\mathcal{L}_{CPT-odd} = \frac{1}{2}(k_{AF})^\kappa \epsilon_{\kappa\lambda\mu\nu}A^\lambda F^{\mu\nu} \tag{8}$$

is the CPT-odd pure-photon term, where the coupling coefficient $(k_{AF})^\kappa$ is real and has dimension of mass and $\epsilon_{\kappa\lambda\mu\nu}$ is the totally antisymmetric Levi-Civita tensor. The CPT-even Lorentz-violating term is given by

$$\mathcal{L}_{CPT-even} = -\frac{1}{4}(k_F)_{\kappa\lambda\mu\nu}F^{\kappa\lambda}F^{\mu\nu}, \tag{9}$$

where the coupling $(k_F)_{\kappa\lambda\mu\nu}$ is real, dimensionless and has the same symmetry of a Riemann tensor

$$(k_F)_{\kappa\lambda\mu\nu} = -(k_F)_{\lambda\kappa\mu\nu}, \quad (k_F)_{\kappa\lambda\mu\nu} = -(k_F)_{\kappa\lambda\nu\mu}, \quad (k_F)_{\kappa\lambda\mu\nu} = (k_F)_{\mu\nu\kappa\lambda}, \tag{10}$$

plus a double null trace which yields 19 independent components.

C. Non-minimal Lorentz-violating term

Besides studies of the structure of the SME, other ideas were proposed to examine Lorentz violating operators in this broad framework. An alternative procedure is to modify just the SME interaction part via a non-minimal coupling. The new interaction breaks the Lorentz and CPT symmetries. This coupling will modify the standard Lagrangian to

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \frac{1}{2\xi}\left(\partial_\mu A^\mu\right)^2, \tag{11}$$

where the covariant derivative includes a non-minimal coupling term, i.e.,

$$D_\mu = \partial_\mu + ieA_\mu + igb^\nu \tilde{F}_{\mu\nu} \tag{12}$$

with e, g and b^μ being the electron charge, a coupling constant and a constant four vector, respectively. $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\rho}F^{\alpha\rho}$ is the dual electromagnetic tensor. There are other possible non-minimal coupling terms that exhibit Lorentz violation [17]. In this paper a Lorentz violating CPT-odd term is chosen to study Bhabha scattering. This non-minimal coupling leads to the new interaction. The interaction lagrangian is given by

$$\mathcal{L}_I = -e\bar{\psi}\gamma^\mu\psi A_\mu - gb^\nu\bar{\psi}\gamma^\mu\psi\partial^\alpha A^\rho\epsilon_{\mu\nu\alpha\rho}. \tag{13}$$

The first term describes the usual QED vertex and the second term is a new vertex that implies violation of Lorentz symmetries due to the four vector b^ν, which specifies a preferred direction in the space-time.

III. BHABHA SCATTERING WITH LORENTZ VIOLATION

In this section the main results obtained in [18] are briefly reviewed. Our interest is to calculate the cross section for the process, $e^-(p_1)e^+(p_2) \to e^-(p_3)e^+(p_4)$, in the presence of Lorentz-violating terms at zero temperature. This process has the Feynman diagrams given in FIG. 1. These vertices in the diagrams are represented

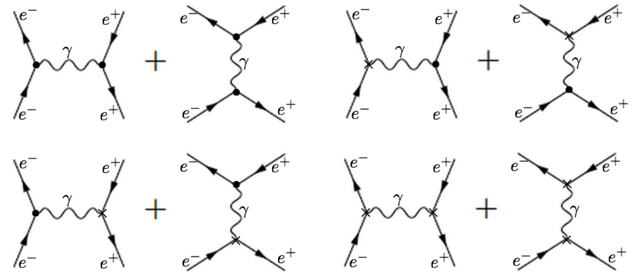

Figura 1: Exchange and annihilation diagrams with different vertices.

as

$$\bullet \to V^\mu = -ie\gamma^\mu \tag{14}$$

and

$$\times \to gV_\rho = -gb^\nu\gamma^\mu q^\alpha\epsilon_{\mu\nu\alpha\rho}, \tag{15}$$

where q^α is the momentum operator.

In order to calculate the cross section for this process the S-matriz element is written as

$$\mathcal{M} = \mathcal{M}_0 + \mathcal{M}_1 + \mathcal{M}_2, \tag{16}$$

where $\mathcal{M}_0$ is the matrix element in conventional QED, $\mathcal{M}_1$ is linear in Lorentz-violating parameter and $\mathcal{M}_2$ is quadratic in Lorentz-violating parameter. Using the Lagrangian (11) these matrix elements are given explicitly as

$$\mathcal{M}_0 = e^2\left[\frac{\bar{u}(p_2)\gamma^\alpha u(p_1)\bar{v}(p_3)\gamma_\alpha v(p_4)}{(p_1-p_2)^2} - \frac{\bar{u}(p_2)\gamma^\alpha v(p_4)\bar{v}(p_3)\gamma_\alpha u(p_1)}{(p_1+p_3)^2}\right], \tag{17}$$

$$\begin{aligned}\mathcal{M}_1 = 2egb^\nu\epsilon_{\mu\nu\sigma\rho}\Big[&\frac{(p_1-p_2)^\sigma\bar{u}(p_2)\gamma^\rho u(p_1)\bar{v}(p_3)\gamma^\mu v(p_4)}{(p_1-p_2)^2}\\ &- \frac{(p_1+p_3)^\sigma\bar{u}(p_2)\gamma^\rho v(p_4)\bar{v}(p_3)\gamma^\mu u(p_1)}{(p_1+p_3)^2}\Big],\end{aligned} \tag{18}$$

$$\begin{aligned}\mathcal{M}_2 = g^2b^\gamma b^\sigma g^{\kappa\lambda}\epsilon_{\epsilon\sigma\tau\lambda}\epsilon_{\omega\gamma\sigma\kappa}\Big[&\frac{(p_1-p_2)^\sigma(p_1-p_2)^\tau\bar{u}(p_2)\gamma^\omega u(p_1)\bar{v}(p_3)\gamma^\epsilon v(p_4)}{(p_1-p_2)^2}\\ &- \frac{(p_1+p_3)^\sigma(p_1+p_3)^\tau\bar{u}(p_2)\gamma^\epsilon v(p_4)\bar{v}(p_3)\gamma^\omega u(p_1)}{(p_1+p_3)^2}\Big].\end{aligned} \tag{19}$$

Then the differential cross section, taking an average over the spin of the incoming particles and summing over the spin of the outgoing particles, is defined as

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{1}{64\pi^2E_{\rm CM}^2}\cdot\frac{1}{4}\sum_{\rm spins}|\mathcal{M}|^2, \tag{20}$$

where $E_{\rm CM}$ is the center of mass energy.

Let's consider the center of mass frame,

$$\begin{aligned}p_1 &= (E,\vec{p}), & p_2 &= (E,-\vec{p}),\\ p_3 &= (E,\vec{p'}) \quad \text{and} & p_4 &= (E,-\vec{p'}),\end{aligned} \tag{21}$$

where $|\vec{p}|^2 = |\vec{p'}|^2 = E^2$, $\vec{p}\cdot\vec{p'} = E^2\cos\theta$ and $s = (2E)^2 = E_{CM}^2$, and the relations

$$\begin{aligned}\sum u(p_1)\bar{u}(p_1) &= \not{p}_1 + m,\\ \sum v(p_1)\bar{v}(p_1) &= \not{p}_1 - m.\end{aligned} \tag{22}$$

In addition

$$\bar{v}(p_2)\gamma_\alpha u(p_1)\bar{u}(p_1)\gamma^\alpha v(p_2) = \mathrm{tr}\left[\gamma_\alpha u(p_1)\bar{u}(p_1)\gamma^\alpha v(p_2)\bar{v}(p_2)\right], \tag{23}$$

has been used. Henceforth the electron mass is ignored since all the momenta are large compared to the electron mass.

Considering $b^\nu = (b_0, 0)$, a time-like four vector, the differential cross section, eq. (20), is

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{e^4(\cos 2\theta + 7)^2}{256\pi^2 E_{CM}^2(\cos\theta - 1)^2} + \frac{b_0^2\, e^2 g^2 \sin^2\left(\frac{\theta}{2}\right)}{256\pi^2(\cos\theta - 1)^2}(-65\cos\theta + 6\cos 2\theta + \cos 3\theta + 122), \quad (24)$$

with the first term being the usual QED differential cross section at lowest order and the second term contain the contributions due to Lorentz violation. Using experimental data for this scattering, that are readily available, an upper bound to Lorentz violating parameter has been determined. Some details about this calculation has been done in [18].

The next step is to determine the thermal corrections to the Bhabha scattering. The temperature effects will be introduced using the TFD formalism.

IV. THERMO FIELD DYNAMICS - TFD

In the TFD formalism the thermal average of any operator A is equal to its temperature dependent vacuum expectation value, i.e., $\langle A\rangle = \langle 0(\beta)|A|0(\beta)\rangle$, where $|0(\beta)\rangle$ is a thermal vacuum, $\beta = \frac{1}{k_B T}$ with T being the temperature and k_B is the Boltzmann constant. Such construction leads to the doubling the degrees of freedom in a Hilbert space accompanied by the temperature dependent Bogoliubov transformation. This doubling is defined by the tilde conjugation rules, associating each operator in S to two operators in S_T, where the expanded space is $S_T = S \otimes \tilde{S}$, with S being the standard Fock space and $\tilde{S}$ the tilde space. The thermal quantities are introduced through the Bogoliubov transformation which applies a rotation in the tilde and non-tilde variables. Let's consider the Bogoliubov transformation for fermions and bosons separately.

A. Fermions

Considering $c_p^\dagger$ and c_p being creation and annihilation operators respectively, the Bogoliubov transformations for fermions are

$$c_p = \mathsf{u}(\beta)c_p(\beta) + \mathsf{v}(\beta)\tilde{c}_p^\dagger(\beta), \quad (25)$$
$$c_p^\dagger = \mathsf{u}(\beta)c_p^\dagger(\beta) + \mathsf{v}(\beta)\tilde{c}_p(\beta), \quad (26)$$
$$\tilde{c}_p = \mathsf{u}(\beta)\tilde{c}_p(\beta) - \mathsf{v}(\beta)c_p^\dagger(\beta), \quad (27)$$
$$\tilde{c}_p^\dagger = \mathsf{u}(\beta)\tilde{c}_p^\dagger(\beta) - \mathsf{v}(\beta)c_p(\beta), \quad (28)$$

where $\mathsf{u}(\beta) = \cos\theta(\beta)$ and $\mathsf{v}(\beta) = \sin\theta(\beta)$.

B. Bosons

By considering $a_p^\dagger$ and a_p being the creation and annihilation operators respectively, the Bogoliubov transformation are

$$a_p = \mathsf{u}'(\beta)a_p(\beta) + \mathsf{v}'(\beta)\tilde{a}_p^\dagger(\beta), \quad (29)$$
$$a_p^\dagger = \mathsf{u}'(\beta)a_p^\dagger(\beta) + \mathsf{v}'(\beta)\tilde{a}_p(\beta), \quad (30)$$
$$\tilde{a}_p = \mathsf{u}'(\beta)\tilde{a}_p(\beta) + \mathsf{v}'(\beta)a_p^\dagger(\beta), \quad (31)$$
$$\tilde{a}_p^\dagger = \mathsf{u}'(\beta)\tilde{a}_p^\dagger(\beta) + \mathsf{v}'(\beta)a_p(\beta), \quad (32)$$

where $\mathsf{u}'(\beta) = \cosh\theta(\beta)$ and $\mathsf{v}'(\beta) = \sinh\theta(\beta)$.

An important note, the commutation or anti-commutation relations for creation and annihilation operators are similar to those at zero temperature.

V. BHABHA SCATTERING AT FINITE TEMPERATURE

In this section the Bhabha scattering with Lorentz violation at finite temperature is studied. The Lorentz-violating parameter is introduced via non-minimal coupling term in the covariant derivative. In such study the Lagrangian (11) and eq. (12) will be used.

The cross section at finite temperature is defined as

$$\left(\frac{d\sigma}{d\Omega}\right)_\beta = \frac{1}{64\pi^2 E_{\rm CM}^2}\cdot\frac{1}{4}\sum_{\rm spins}|\mathcal{M}(\beta)|^2, \quad (33)$$

where $\mathcal{M}(\beta)$ is the S-matrix element at finite temperature, which is calculated as

$$\mathcal{M}(\beta) = \langle f,\beta|\hat{S}^{(2)}|i,\beta\rangle, \quad (34)$$

with $\hat{S}^{(2)}$ being the second order term of the $\hat{S}$-matrix. The $\hat{S}$-matrix is defined as

$$\hat{S} = \sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int dx_1 dx_2\cdots dx_n \mathbb{T}\left[\hat{H}_I(x_1)\hat{H}_I(x_2)\cdots\hat{H}_I(x_n)\right], \quad (35)$$

where $\mathbb{T}$ is the time ordering operator and $\hat{H}_I(x) = H_I(x) - \tilde{H}_I(x)$ describes the interaction. The thermal states are

$$|i,\beta\rangle = c_{p_1}^\dagger(\beta)d_{p_2}^\dagger(\beta)|0(\beta)\rangle,$$
$$|f,\beta\rangle = c_{p_3}^\dagger(\beta)d_{p_4}^\dagger(\beta)|0(\beta)\rangle, \quad (36)$$

with $c^\dagger_{p_j}(\beta)$ and $d^\dagger_{p_j}(\beta)$ being creation operators. The transition amplitude becomes

$$\begin{aligned}\mathcal{M}(\beta) &= \frac{(-i)^2}{2!}\int d^4x\,d^4y\langle f,\beta|(\mathcal{L}_I\mathcal{L}_I-\tilde{\mathcal{L}}_I\tilde{\mathcal{L}}_I)|i,\beta\rangle\\ &= \Big(\mathcal{M}_0(\beta)+\mathcal{M}_b(\beta)+\mathcal{M}_{bb}(\beta)\Big)-\Big(\tilde{\mathcal{M}}_0(\beta)+\tilde{\mathcal{M}}_b(\beta)+\tilde{\mathcal{M}}_{bb}(\beta)\Big),\end{aligned}\tag{37}$$

where $\mathcal{M}_0(\beta)$ is the matrix element in conventional QED and $\mathcal{M}_b(\beta)$ and $\mathcal{M}_{bb}(\beta)$ are the linear and quadratic matrix elements for the Lorentz violating interaction, respectively. These matrix elements are given as

$$\mathcal{M}_0(\beta) = -\frac{e^2}{2}\int d^4x\,d^4y\,\langle f,\beta|\bar\psi(x)\gamma^\mu\psi(x)\bar\psi(y)\gamma^\nu\psi(y)A_\mu(x)A_\nu(y)|i,\beta\rangle,\tag{38}$$

$$\mathcal{M}_b(\beta) = -egb^\nu\epsilon_{\mu\nu\sigma\rho}\int d^4x\,d^4y\langle f,\beta|\bar\psi(x)\gamma^\omega\psi(x)\bar\psi(y)\gamma^\mu\psi(y)A_\omega(x)\partial^\sigma A^\rho(y)|i,\beta\rangle,\tag{39}$$

$$\mathcal{M}_{bb}(\beta) = -\frac{1}{2}g^2b^\nu b^\rho\epsilon_{\mu\nu\alpha\sigma}\epsilon_{\omega\rho\delta\gamma}\int d^4x\,d^4y\langle f,\beta|\bar\psi(x)\gamma^\mu\psi(x)\bar\psi(y)\gamma^\omega\psi(y)\partial^\alpha A^\sigma(x)\partial^\delta A^\gamma(y)|i,\beta\rangle.\tag{40}$$

There are similar equations for matrix elements that include tilde operators.

By taking that the fermion field is written as

$$\psi(x)=\int dp\,N_p\left[c_pu(p)e^{-ipx}+d^\dagger_pv(p)e^{ipx}\right],\tag{41}$$

where N_p is the normalization constant, and the photon propagator at finite temperature, that is defined as

$$\langle 0(\beta)|\mathbb{T}A_\mu(x)A_\nu(y)|0(\beta)\rangle = i\int\frac{d^4q}{(2\pi)^4}e^{-iq(x-y)}\Delta_{\mu\nu}(q,\beta),\tag{42}$$

with $\Delta_{\mu\nu}(q,\beta)=\Delta^{(0)}_{\mu\nu}(q)+\Delta^{(\beta)}_{\mu\nu}(q)$, where $\Delta^{(0)}_{\mu\nu}(q)$ and $\Delta^{(\beta)}_{\mu\nu}(q)$ are zero and finite temperature parts respectively. Explicitly

$$\Delta^{(0)}_{\mu\nu}(q)=\frac{\eta_{\mu\nu}}{q^2}\begin{pmatrix}1&0\\0&-1\end{pmatrix},\tag{43}$$
$$\Delta^{(\beta)}_{\mu\nu}(q)=-\frac{2\pi i\delta(q^2)}{e^{\beta q_0}-1}\begin{pmatrix}1&e^{\beta q_0/2}\\e^{\beta q_0/2}&1\end{pmatrix}\eta_{\mu\nu}.$$

Then the Lorentz invariant transition amplitude, in the center of mass frame, becomes

$$\begin{aligned}\mathcal{M}_0(\beta) &= -ie^2\Big[\bar u(p_1)\gamma^\mu u(p_3)\bar v(p_4)\gamma_\mu v(p_2)\Delta'(p_3-p_1)-\bar u(p_2)\gamma^\nu v(p_1)\\ &\quad\times\bar v(p_3)\gamma_\nu u(p_4)\Delta'(p_1+p_2)\Big]\tanh^2\Big(\frac{\beta E_{CM}}{2}\Big).\end{aligned}\tag{44}$$

In a similar way the linear term in the Lorentz violating parameter becomes

$$\begin{aligned}\mathcal{M}_b(\beta) &= 2egb^\nu\epsilon_{\mu\nu\sigma\rho}\Big[(p_3-p_1)^\sigma\bar u(p_1)\gamma^\rho u(p_3)\bar v(p_4)\gamma^\mu v(p_2)\Delta'(p_3-p_1)\\ &\quad+(p_1+p_2)^\sigma\bar u(p_2)\gamma^\rho v(p_1)\bar v(p_3)\gamma^\mu u(p_4)\Delta'(p_1+p_2)\Big]\tanh^2\Big(\frac{\beta E_{CM}}{2}\Big),\end{aligned}\tag{45}$$

and the quadratic term in the Lorentz violating coefficients is

$$\mathcal{M}_{bb}(\beta) = ig^2 b^\nu b^\rho \eta^{\sigma\gamma}\epsilon_{\mu\nu\alpha\sigma}\epsilon_{\omega\rho\delta\gamma}\Big[q_1^\alpha q_1^\delta \bar{u}(p_1)\gamma^\mu u(p_3)\bar{v}(p_4)\gamma^\omega v(p_2) \times \Delta'(p_3-p_1) - q_2^\alpha q_2^\delta \bar{u}(p_2)\gamma^\mu v(p_1)\bar{v}(p_3)\gamma^\omega u(p_4)\Delta'(p_1+p_2)\Big]\tanh^2\Big(\frac{\beta E_{CM}}{2}\Big), \quad (46)$$

where $q_1 = p_3 - p_1$ and $q_2 = p_1 + p_2$. The results for the transition amplitudes obtained in [18] are recovered in the limit $T \to 0$, which implies $\tanh^2(\beta E_{CM}/2) \to 1$ and $(e^{\beta E_{CM}} - 1)^{-1} \to 0$.

In order to calculate the differential cross section, let's use the eqs. (22) and (23) and to consider $b^\nu = (b_0, 0)$, a time-like four vector. Then the differential cross section, eq. (33), at finite temperature is

$$\left(\frac{d\sigma}{d\Omega}\right)_\beta = \Bigg[\frac{e^4(\cos 2\theta + 7)^2}{256\pi^2 E_{CM}^2(\cos\theta - 1)^2} + \frac{b_0^2\, e^2 g^2 \sin^2\left(\frac{\theta}{2}\right)}{256\pi^2(\cos\theta - 1)^2}(-65\cos\theta + 6\cos 2\theta + \cos 3\theta + 122)\Bigg]\tanh^4\left(\frac{\beta E_{CM}}{2}\right) + \frac{1}{256\pi^2 E_{CM}^2}\Big[\Pi_1(\beta) + b_0^2\,\Pi_2(\beta)\Big]\times\tanh^4\left(\frac{\beta E_{CM}}{2}\right), \quad (47)$$

where

$$\Pi_1(\beta) = \frac{64\pi^2 e^4 E^4}{(e^{\beta E_{CM}} - 1)^2}\Big[(11 + 4\cos\theta + \cos 2\theta)\delta^2\left(-2E^2(1-\cos\theta)\right) + (6 + 2\cos 2\theta)\delta^2\left(4E^2\right) + 16\cos^4(\theta/2)\,\delta\left(-2E^2(1-\cos\theta)\right)\delta\left(4E^2\right)\Big],$$

$$\Pi_2(\beta) = \frac{256\pi^2 e^2 g^2 E^6 \sin^2(\theta/2)}{(e^{\beta E_{CM}} - 1)^2}\Big[(\cos 2\theta + 32\sin^2(\theta/2) + 15)\delta^2\left(-2E^2(1-\cos\theta)\right) + 8\cos^4(\theta/2)\,\delta\left(-2E^2(1-\cos\theta)\right)\delta\left(4E^2\right). \quad (48)$$

Note that, the propagator at finite temperature introduces product of delta functions with identical arguments [19–22]. This problem may be avoided by working with the regularized form of delta-functions and their derivatives [23].

Therefore corrections for Bhabha scattering due to Lorentz violation are altered at finite temperature. Furthermore the temperature effect is very large at very high temperature, then these corrections are relevant. For more details about these calculations, see [24].

VI. CONCLUSION

The Standard Model Extension is an effective theory that allows to study violations of Lorentz and CPT symmetries. Besides this theory, there is an alternative way that modifies the interaction part using a non-minimal coupling. Here a CPT-odd non-minimal coupling term is considered. Then the effect of Lorentz violation at zero and finite temperature on Bhabha scattering are investigated. The TFD formalism is used to introduce the temperature effects. It is important to observe that the results give us a reasonable estimate of the Lorentz violating operators in the SME at high temperatures on the Bhabha Scattering.

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