

Whitehead and Sheffer's Incompatibility: An Investigation of the Relationship between Metaphysics and Logic

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Abstract

In this article, I investigate the relationship between Whitehead's metaphysics and logic with relevance to Henry Sheffer's idea of "incompatibility." In chapter 1, I clarify my interpretation of Whitehead's position on this problem via Ernst Tugendhat's classification and the historical context of the relationship between logic and metaphysics. In chapter 2, I detail one element of this interpretation by considering Whitehead's usage of the concept of "incompatibility." I also show that Russell, Wittgenstein, and Whitehead adopt Sheffer's idea in different ways, and suggest that Whitehead's acceptance of this idea is unique.

Key words: Whitehead, Metaphysics, Logic, Sheffer, Incompatibility

Introduction

Alfred North Whitehead was a mathematical logician who is best known for writing *Principia Mathematica* with Bertrand Russell. However, in his later years, Whitehead constructed a metaphysical system which he called the philosophy of organism. This fact raises some questions: how did the later Whitehead think of logic? How is logic related to his metaphysics? ¹Some might claim that Whitehead had little interest in logic later in life. However, I claim that Whitehead continued to have an interest in logic late in life and, in fact, a logical concept has a fundamental role in his metaphysics. This concept is Henry Sheffer's notion of "incompatibility," which he

¹ Some scholars discussed about this relationship. For example, Wolfe Mays attempts to interpret Whitehead's metaphysics as "applied logic." (Mays 1959, p. 20.) However, He doesn't pay enough attention to the fact that Whitehead says "logic presupposes metaphysics." (MT54) In contrast with Mays' study, my purpose of this paper is to examine how Whitehead gives metaphysical grounds to logic.

devised in 1913. I believe that this concept is the key to understanding the relationship between Whitehead's logic and his metaphysical system, and I have studied his philosophy based on this hypothesis. In this paper, I examine incompatibility's role in Whitehead's metaphysics and discuss the originality of Whitehead's ideas by presenting them in their wider context.

This paper deals with two topics. Each topic has two sub-topics.

The first topic is "logic and philosophy (or metaphysics)," or a discussion of the general relationship between logic and philosophy. In 1-1, I situate Whitehead within Ernst Tugendhat's classification of philosophical positions on logic. In 1-2, I survey the historical context of the relationship between logic and philosophy (or metaphysics). The second topic is "Sheffer's concept and its influence." Sheffer's ideas inspired several philosophers, and each used his ideas in different ways. In 2-1, I will trace Sheffer's influence on Russell and Wittgenstein. In 2-2, I consider the relationship between Whitehead's metaphysics and Sheffer's concept of incompatibility.

1. Logic and Philosophy (or Metaphysics)

1-1. Tugendhat's classification

First, let us consider the general relationship between logic and philosophy.

Logic is the science that deals with laws and rules. But this begs the question: the laws and rules of what, exactly?

Logician and phenomenologist Ernst Tugendhat discerns three distinct philosophical positions regarding this problem; he calls them ontological, psychological, and linguistic positions.² In the first position, philosophers regard the

² Tugendhat and Wolf (1983), pp. 7-8.

laws of logic as the laws of reality. In the second position, they regard laws as psychological laws, or the laws of our cognitive capacities. In the third position, logical laws are laws of linguistic usage. According to Tugendhat, Husserl and the earlier Wittgenstein are typical of the first position and Kant is the typical philosopher of the second position. He classifies Aristotle as a philosopher of the third position because Aristotle explicated theories of judgment based on discourse. Perhaps most linguistic philosophers belong to this third group.

Of course, this classification is merely a convenient tool; each of these positions is intertwined and cannot be completely separated from the other. For instance, Aristotle might belong to the first position because his logic depends on his ontology.

I refer to Tugendhat's classification here because it helps me clarify my interpretation of Whitehead's view of logic. I believe that Whitehead's view belongs to the ontological position. In Whitehead's metaphysics, the fundamental reality is an "actual entity." I will later show that this actual entity is relevant to the basis of logic. Note that we might also classify Whitehead under the second psychological position because all actual entities are subjects of experiences and feelings. I will examine this point in more detail in chapter 2, but first, I will examine the historical peculiarity of Whitehead's position.

1-2. The development of logic and its exclusion of metaphysics

A revolution in logic occurred between the mid-nineteenth and early twentieth centuries: logicians began denying some presuppositions of traditional logic and invented modern logic (or symbolic logic). Traditional logic had presupposed Aristotelian metaphysics, and this bias had inhibited the development of logic; therefore, logicians attempted to disconnect logic from Aristotelian metaphysics in

order to improve it. Since the 1930s, some philosophers and logicians became more radical, and excluded all types of metaphysics from logic. The logical positivist Ernest Nagel's slogan, "logic without ontology," is symbolic of this moment; so is Rudolf Carnap's conventionalism. However, logicians who were transitioning from traditional logic to modern logic did not abandon metaphysics entirely – rather, they felt the need to make a new metaphysics which harmonized with modern logic (e.g. Bertrand Russell's metaphysics, which he called "logical atomism").

I believe Whitehead also felt the need to connect modern logic with his metaphysics by developing a metaphysics which could give grounds to logic. We will examine this argument in detail in 2-2, but first, consider Whitehead in *Modes of Thought*:

We thus dismiss deductive logic as a major instrument for metaphysical discussion, Such discussion is concerned with the eliciting of self-evidence. Apart from such self-evidence, deduction fails. Thus logic presupposes metaphysics.(MT106)

Whitehead states that, "logic presupposes metaphysics" i.e. that logic depends on some form of self-evidence. According to him, philosophers' and metaphysicians' task is to manifest this self-evidence: "The attempt of any philosophic discourse should be to produce self-evidence."(MT49) This is his general view of the relationship between logic and metaphysics.

This raises some questions. What about Whitehead's metaphysics? Does his own metaphysical system actually give grounds to logic? In my view, Sheffer's concept of incompatibility is the key to answering these questions.

2. Sheffer's Concept and its Influences

First, let me introduce Sheffer's life and career.³ Sheffer was born in the

³ Information on Sheffer's life and career comes from Scanlan (2000).

Ukraine in 1883 and his family moved to America ten years after his birth. Sheffer studied philosophy and logic at Harvard, where he was a student of Josiah Royce. He wrote his doctoral thesis in 1908, and became a temporary instructor at Harvard in 1917. Ten years after that, he became an assistant professor. Some researchers point out that one factor in his promotion was Whitehead's recommendation. He wrote very few articles. According to his manuscript, he had an idea to construct a theory of a super-postulate system, but didn't complete it. His name is known only as an inventor of the "Sheffer stroke" today. He proposed the idea and its sign in 1913.⁴

So, what did Sheffer discover in 1913? I'd like to cite a passage from the second edition of *Principia Mathematica* (PM) where Russell clearly summarizes Sheffer's discovery and its importance:

The most definite improvement resulting from work in mathematical logic during the past fourteen years is the substitution, in Part I, Section A, of the one indefinable "p and q are incompatible" (or, alternatively, "p and q are both false") for the two indefinables "not-p" and "p or q." This is due to Dr. H. M. Sheffer. (...) From this there follows a great simplification in the building up of molecular propositions and matrices (...).⁵

Logicians require some primitive notions to combine propositions. In the first edition of PM, Whitehead and Russell introduced negation and disjunction as two indefinable notions. Sheffer showed that we can define these two notions via one primitive notion—that of incompatibility. For example, not-p can be defined as "p is incompatible with p." He used the sign " $|$ " to refer to this concept. Thus, we can express "p is incompatible with q" in the form of " $p|q$." In the second edition of PM, Russell adopted this notion and the sign (which is now called the Sheffer stroke).

This is the general understanding of the Sheffer stroke. However, I propose

⁴ Sheffer (1913)

⁵ Russell and Whitehead (1925) p. xiii.

a different interpretation. Please direct your attention to the passage from PM again. Russell says, "the one indefinable 'p and q are incompatible' (or, alternatively, 'p and q are both false') for the two indefinables 'not-p' and 'p or q.'" This passage shows that Sheffer suggested *two* options. "Not-p" and "p or q" can be derived from "p and q are incompatible," but they can also be derived from "p and q are both false" or "neither p nor q." Sheffer called the latter notion "rejection."

Sheffer himself was the first to suggest the concept of rejection. He then wrote that another interpretation is also possible. Furthermore, Sheffer did not use the word "incompatibility" in 1913—this is the word Russell used to express Sheffer's concept in the 1925 edition of PM, which then became influential. Whitehead, like Russell, interprets Sheffer's idea using the notion of incompatibility, but some of his contemporaries adopted the other interpretation. For example, the early Wittgenstein uses the concept of rejection in his *Tractatus Logico-Philosophicus*.

We have seen the logical importance of Sheffer's idea. However, his idea also has broader philosophical importance. In the next section, we will examine Sheffer's influence on Russell, the early Wittgenstein, and Whitehead. This comparison will help us to understand Whitehead's uniqueness.

2-1. Sheffer's Influences on Some Philosophers

2-1-1. Russell's Logical Atomism and Sheffer

First, let me examine Russell's logical atomism. As mentioned above, he regarded logical atomism as a new metaphysics, one in harmony with modern logic. Russell leverages Sheffer's concept to build his philosophy. First, he asserts that this world consists of atomic facts. Atomic facts consist in particulars and in universals. Then, he sets atomic propositions that express these facts. These atomic propositions are logically independent of each other, but we can combine them via logical constants

and they can thus constitute more complex, molecular propositions. Here, Russell introduces Sheffer's notion of incompatibility as the only logical constant to combine atomic propositions.⁶ Thus, in Russell's logical atomism, the field of our thinking consists of complexes of propositions combined by incompatibility. Thus, Sheffer's notion of incompatibility is a basic, foundational tool for Russell's metaphysics.

2-1-2. Wittgenstein and Sheffer

Next, let us examine Sheffer's influence on the early Wittgenstein. It may be difficult to express the content of Wittgenstein's *Tractatus* as metaphysical—however, via Tugendhat, we can describe Wittgenstein's position as ontologically driven because his logic indicates forms which both the world and language have.

In the *Tractatus*, the early Wittgenstein describes combinations of simple propositions. For him, the simplest propositions are elemental propositions; complex propositions consist of these propositions and logical constants. For Wittgenstein, logical constants are not names of objects but operations. There are some operations such as "negation (not)," "disjunction (or)," and so on—however, we can reduce them to a single operation. This operation is "nor," or "neither nor." Wittgenstein expresses this operation with a Sheffer stroke. Therefore, in the *Tractatus*, all propositions can be created by the repetitive applications of the same operations which govern elemental propositions.

Wittgenstein writes about the uniqueness of this operation in an impressive way.

5.511 How can logic—all-embracing logic, which mirrors the world—use such peculiar crochets and contrivances? Only because they are all connected with

⁶ Russell (2009), p. 41.

one another in an infinitely fine network, the great mirror.⁷

In this passage, Wittgenstein talks about "peculiar crochets." Crochets are needles for knitting. The shape of the needle is like the "stroke."⁸ For him, logic constitutes the texture of propositions by using these crochets (Sheffer stroke).

2-2. Whitehead and Sheffer

2-2-1. "Incompatibility (Inconsistency)" in Modes of Thought

Now, let us examine Sheffer's influence on Whitehead's philosophy. To do this, I will examine some passages from Whitehead's *Modes of Thought* where he mentions Sheffer. Here, I must alert the reader that while Whitehead calls Sheffer's concept "incompatibility" in other words, he calls the same notion "inconsistency" in *Modes of Thought*. It is not clear why he does this, but the reader should read "inconsistency" in the following passages as "incompatibility."

In the chapter "Understanding," Whitehead introduces Sheffer's discovery:

The foundation of logic upon the notion of inconsistency was first discovered and developed by Professor Henry Sheffer of Harvard, about twenty years ago(...).(MT52)

Then, he explains the philosophical meaning of inconsistency in the following way:

Inconsistency is the fact that the two states of things which constitute the respective meanings of a pair of propositions cannot exist together. (Ibid.)

This passage suggests that Whitehead's acceptance of Sheffer's idea is unique. Both Russell and the early Wittgenstein used Sheffer's notions of incompatibility or

⁷ Wittgenstein (1971/1921) p. 99.

⁸ Fujimoto and Sakai, Japanese translators of *Tractatus*, interpret "peculiar crochets" as Sheffer stroke in translator's notes (Wittgenstein, 1968/1921, p. 152)

rejection with reference to combining propositions; however, Whitehead uses the concept of incompatibility with reference to the states of things. In short, Whitehead asserts that incompatibility means that two states of things cannot exist together. For Whitehead, Sheffer's concept describes not only propositions, but reality. Moreover, according to him, incompatibility / inconsistency defines the finitude of things:

...by basing logic upon the concept of inconsistency, the notion of the finite is definitely introduced. For as Spinoza pointed out, the finite is that which excludes other things comparable to itself. (MT52)

In addition, Whitehead points out the concept is useful to understand process:

[T]his basis for logic enlightens our understanding of process, which is a fundamental fact in our experience. (52)

Why is incompatibility / inconsistency important for understanding process? In order to explain this, Whitehead asks, "if all things can be together, why should there be process?"(MT53) One of his possible answers is to deny the existence of process – i.e., to assert that process is only an appearance. Some monists, including Neo-Hegelians, might choose this answer. However, this is not Whitehead's choice – for him, the existence of process is an "inexorable fact." (Ibid.) Whitehead is a pluralist, so he suggests another answer; one which admits the existence of process and denies the possibility that all things can be together at the same time because of the finitude of things. In other words, finite things have incompatible states—they must exclude something in order to exist.

In this universe, everything except God is finite. Therefore, there are incompatible states in all things; one finite thing's existence necessarily excludes something. But if we see the states of things from the view of the process of the

universe, this fundamental incompatibility dissolves. For example, even if thing A and thing B cannot exist together at the same time, they may exist at different times. Moreover, even if the A-state of things cannot be realized simultaneously with the B-state, it is possible that the A-state becomes the B-state through the process. For Whitehead, incompatibility can be escaped through process. On this point, he says:

Now process is the way by which the universe escapes from the exclusions of inconsistency. (MT54)

By means of process, the universe escapes from the limitations of the finite. Process is the immanence of the infinite in the finite; whereby all bounds are burst, and all inconsistencies dissolved.(Ibid.)

In this way, Whitehead uses the concept of incompatibility to explain the finitude of things, and details how finitude and incompatibility can be overcome through the process. This discussion implies the relevance of Whitehead's "process philosophy" to incompatibility, but this relevance is not sufficiently clear. It is unclear in part because Whitehead wrote this book not as a systematic discussion, but as a free investigation of some fundamental notions (MT1-2). Thus, he does not clearly show us the connection of the discussion on incompatibility to his metaphysical system.

2-2-2. "Incompatibility" in *Process and Reality*

Whitehead also mentions Sheffer's incompatibility in *Process and Reality*, which is widely regarded as his most systematic work. Incompatibility has an important role in this text's philosophical system.⁹ Thus, we will examine the role which incompatibility has in his system, which will thereby clarify the relationship between

⁹ I have written about the role of incompatibility in this text before. See Arimura (2016). In the paper, I pointed out that Whitehead uses two meanings of "incompatibility" (a logical meaning and an aesthetic meaning) and argued that both are relevant to categorical obligations. I will now limit my discussion only to logical meaning, and address some points which clarify the similarities and differences between *Modes of Thought* and *Process and Reality*.

Whitehead's metaphysics and logic.

In *Process and Reality*, incompatibility is intertwined with process—especially the process of becoming an actual entity. As we all know, actual entities realize themselves through the process of integrating feelings. These entities, in becoming, are subjects of experiences. Each subject has its own perspectives, and it feels other entities from that perspective. The concept of incompatibility has its role in this basic scheme of Whitehead's philosophy. Let us examine some quotations.

...'feelings' are the entities which are primarily 'compatible' or 'incompatible.'(PR148)

The compatibilities and incompatibilities which impose the perspective, transforming the actual world into the datum, are inherent in the nature of things. (PR154)

The perspective is provided by the elimination of incompatibilities. (PR150)

All actual entities except God are finite, so all actual entities feel some things and exclude other things. What an actual entity feels and what it excludes determines its perspective; we can even assert that what an entity feels and what it excludes determine every one of its peculiarities. Whitehead explains this fact in terms of compatibility and of incompatibility.

The following quotation is important:

The actual entities enter into each others' constitutions under limitations imposed by incompatibilities 4 of feeling. (PR149)

In the footnote of this same passage, Whitehead says:

4 Dr. H. M. Sheffer has pointed out the fundamental logical importance

of the notion of 'incompatibility'; cf. Trans. Amer. Math. Soc., Vol. XIV, pp. 481- 488; and Introduction to Vol. 1 of Principia Mathematica (2nd edition). (Ibid.)

As this quotation shows, Whitehead deliberately uses the word "incompatibility" with reference to Sheffer in his metaphysics, especially his theory of feeling.

The above quotations are all from part 2 of *Process and Reality*. However, in the above passages Whitehead does not explain the meaning of this incompatibility in context. He also mentions incompatibility in part 3 of this text, when discussing genetic analysis. There, he explains the meaning of categoreal obligations which are, for him, the conditions of becoming. He explains how incompatibility functions in concrescence (actual entities' process of becoming).

In part 2 of *Process and Reality*, Whitehead states that "compatibilities and incompatibilities are inherent in the nature of things." In part 3, he lists three categoreal obligations as derived from "the final nature of things". These categories are Subjective Unity, Objective Identity, and Objective Diversity. He regards them as ultimate metaphysical principles and isolates them from other categoreal obligations (PR222).

This distinction is important for this paper because these three categories are mentioned in reference to logic. Let us see one example. Here, Whitehead explains the implication of the category of Subjective Unity with respect to logical conditions of becoming:

In [the Category of Subjective Unity] the intrinsic inconsistencies, termed 'logical,' are the formative conditions in the pre-established harmony. (PR255)

In other words, the Category of Subjective Unity orders actual entities to make their

feelings compatible in their concrescence.¹⁰ This means that actual entities need to exclude incompatible feelings. Whitehead describes this condition with relevance to logical inconsistencies.

The other two categories, Objective Identity and Objective Diversity, are more significant for our discussion because Whitehead explains these categories as the "ground of incompatibility." (PR225). I do not have enough space here to interpret these categories in detail, but it is worth examining the following passage, where Whitehead summarizes the meanings of these categories:

There are thus two laws respecting the feelings constituting the complex satisfactions of an actual entity: (i) An entity can only be felt once, and (ii) the diverse feelings, in the same subject, of the same entity as datum which are to be unified into one feeling, must be compatible in their treatment of the entity felt. In conformity with this pre-established harmony, 'incompatibility' would have dictated from the beginning that some 'feeling' be replaced by a negative prehension. (PR231)

When an actual entity reaches satisfaction, its component feelings must be compatible: this is the meaning of the pre-established harmony of an actual entity's becoming. Each felt object must have one role in the entity, and it cannot have a contradictory role: this is the meaning of the Category of Objective Identity. Moreover, each felt object must have its status in the actual entity: this is the meaning of Category of Objective Diversity. In order to fulfill these conditions, an actual entity, in becoming, needs to exclude incompatible elements by negative prehensions. For Whitehead, these obligations are the ground of incompatibility.

Sheffer's incompatibility was a purely logical notion, yet Whitehead

¹⁰ "(i) The Category of Subjective Unity. The many feelings which belong to an incomplete phrase in the process of an actual entity, though unintegrated by reason of the incompleteness of the phase, are compatible for integration by reason of the unity of their subject."(PR26)

incorporates it into his metaphysical system and thereby makes it a fundamental condition of becoming. How should we understand Whitehead's intention here? As mentioned earlier, Whitehead says in *Modes of Thought* that "logic presupposes metaphysics." He makes a similar claim in *Process and Reality* when he states that "logical notions must themselves find their places in the scheme of philosophic notions."(PR3) Based on this statement, I claim that incompatibility is an instance which finds its place in the scheme of philosophical notions. This incompatibility is an indefinable, logical concept in PM. However, Whitehead attempts to give it metaphysical grounds in *Process and Reality*. Therefore, it behooves us to understand that the existence of the conditions of process which these categories describe are themselves the basis of logic, not that these categories depend on logic. Indeed, the fact that reality has such conditions is the basis of logic. In other words, we might say that incompatibility as logical concept is rooted in the ontological incompatibility demonstrated in the process of becoming.

Before finishing this section, let us further compare *Modes of Thought* and *Process and Reality*. In the former, Whitehead discusses the inconsistency of states of things; in the latter, he discusses the incompatibility in the feelings of actual entities. In both works, Whitehead points out that finite things have to exclude something in order to exist (either inconsistent states or incompatible feelings). For him, the basis of logic is interconnected with the finitude of reality. However, process has a different implication in each text. We should interpret this difference as a difference of emphasis rather than a contradiction in thought. In *Modes of Thought*, Whitehead invokes process from a broader point of view¹¹. According to him, even if finite things

¹¹ So, he adds, "If we extend the stretch of our attention throughout the passage of time (...)" in this discussion (MT54).

are limited by inconsistency, this limitation can be escaped through the process of universe. On the other hand, in *Process and Reality*, Whitehead emphasizes that these finite entities themselves arise from microscopic processes, namely concrescence. The categoreal obligations explicate how incompatibility limits this concrescence in detail, and this also explains the ontological ground of logical incompatibility.

Concluding Remarks

We have seen how the later Whitehead describes the relation between metaphysics and logic and how he uses Sheffer's concept of incompatibility. The later Whitehead continues to have an interest in logic. According to his position, logic presupposes metaphysics and philosophy gives logical notions their positions in philosophical systems. I interpret Whitehead as attempting to give the concept of incompatibility its place via his metaphysics. The concept of incompatibility is incorporated into the very heart of his theory of becoming, because incompatibility is intertwined with the finitude of entities and with the process of becoming.

We also saw that Russell, the early Wittgenstein, and Whitehead adopted Sheffer's idea in different ways. Such difference may be explained via the differences between their philosophical positions. This paper seems to highlight the uniqueness of Whitehead's acceptance of Sheffer's ideas; however, philosophers need more and more detailed investigations into these three thinkers' use of the concept of incompatibility in order to better understand the differences between them. For now, we have to be satisfied with catching glimpses of future discussions.

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