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Evaluation of university libraries using a multi-stage non-parametric model

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Abstract

The present work aims to evaluate the efficiency of a set of university libraries belonging to a Brazilian federal higher education institution using Data Envelopment Analysis (DEA). To prevent the internal stages of resource processing in services from being neglected, we chose to use a multi-stage method known as Network DEA (NDEA). Thus, to obtain efficiency indices in the two internal stages and consequently the verification of the overall efficiency index, seven variables were used: three exogenous inputs, one intermediate variable, and three exogenous outputs. The results showed that the libraries located in inland cities obtained greater efficiencies in the first stage because they are smaller and offer their services in the number of hours near the larger libraries. It was also possible to identify the best

performing libraries in the second stage and those that performed well in both evaluation stages. For better visualization of the results, we developed a bi-dimensional representation of the results. It was noticed that in future works the evaluation can be expanded, being analyzed libraries belonging to different institutions and different countries, in addition to using a model that allows the identification of benchmarks for inefficient units.
Keywords: Academic libraries. Library management. Evaluation. Mathematical models.

Avaliação de bibliotecas universitárias utilizando um modelo não paramétrico multiestágio
O presente trabalho tem como objetivo avaliar a eficiência de um conjunto de bibliotecas universitárias pertencentes a uma instituição de ensino superior federal brasileira utilizando Análise Envoltória de Dados (DEA). Para evitar que as etapas internas de processamento de recursos em serviços sejam negligenciadas, optou-se por utilizar um método multiestágio conhecido como Network DEA (NDEA). Assim, para obtenção dos índices de eficiência nas duas etapas internas e consequentemente a verificação do índice de eficiência global, foram utilizadas sete variáveis: três insumos exógenos, uma variável intermediária e três produtos exógenos. Os resultados mostraram que as bibliotecas localizadas em cidades do interior obtiveram maiores eficiências na primeira etapa por serem menores e geralmente oferecerem seus serviços no número de horas próximas às bibliotecas maiores. Também foi possível identificar as bibliotecas com melhor desempenho na segunda etapa e aquelas que tiveram bom desempenho nas duas etapas de avaliação. Para melhor visualização dos resultados, desenvolveu-se uma representação bidimensional dos resultados. Percebeu-se que em trabalhos futuros a avaliação poderá ser ampliada, analisando bibliotecas pertencentes a diferentes instituições e países, além do uso de um modelo que permita a identificação de *benchmarks* para as unidades ineficientes.

Palavras-chave: Bibliotecas universitárias. Gestão de bibliotecas. Avaliação. Modelos matemáticos.

Evaluación de bibliotecas universitarias mediante un modelo no paramétrico de múltiples etapas
El presente trabajo tiene como objetivo evaluar la eficiencia de un conjunto de bibliotecas universitarias pertenecientes a una institución de educación superior federal brasileña utilizando Análisis Envolvente de Datos (DEA). Para evitar que se descuiden las etapas internas del procesamiento de recursos en los servicios, optamos por utilizar un método de varias etapas conocido como Network DEA (NDEA). Así, para obtener índices de eficiencia en las dos etapas internas y consecuentemente la verificación del índice de eficiencia global, se utilizaron siete variables: tres insumos exógenos, una variable intermedia y tres productos exógenos. Los resultados mostraron que las bibliotecas ubicadas en ciudades del interior obtuvieron mayores eficiencias en la primera etapa porque son más pequeñas y generalmente ofrecen sus servicios en el número de horas cercanas a las bibliotecas más grandes. También fue posible identificar las bibliotecas con mejor desempeño en la segunda etapa y aquellas que tuvieron un buen desempeño en ambas etapas de evaluación. Para una mejor visualización de los resultados, desarrollamos una representación bidimensional de los resultados. Se observó que en futuros trabajos se puede ampliar la evaluación, analizándose bibliotecas pertenecientes a diferentes instituciones y diferentes países, además de utilizar un modelo que permita identificar puntos de referencia para unidades ineficientes.
Palabras-claves: Bibliotecas universitarias. Gestión de bibliotecas. Evaluación. Modelos matemáticos.

1 Introduction

Higher Education Institutions (HEIs) have an important social function and a fundamental value for the economy. They are challenged to stay ahead of the rapid and constant changes determined by the modus operandi of modern society and the demands imposed by their organizations, especially considering their growing budget constraints. Thus, there are many challenges inherent to university management, which implies the need to continuously monitor them (Tavares; Angulo-Meza; Sant´Anna, 2021).

As part of these institutions, university libraries play a key role in supporting the achievement of the university's overall objective, which encompasses diverse activities such as learning, teaching, research and other community services (Enweani, 2018), recognizing that the generation, acquisition and transfer of knowledge go beyond the confines of classrooms and laboratories.

In this way, the institutional results pursued by the HEIS impose the perception of the importance of the elements that compose it and the evaluation of the performance of each of these parts in relation to its main activities, considering the demands for financial, human, material and technological resources, in addition of the efficiency of the use of these resources when related to the contributions and services offered.

Maia and Santos (2015) state that, in the case of libraries, it is necessary to measure the quantified data to improve them, so that they can meet the needs of the academic community and establish themselves as a necessary body within the university. In Brazil, libraries are included in the assessment instruments proposed by INEP (National Institute for Educational Studies and Research Anísio Teixeira), an agency linked to the Ministry of Education (MEC), mainly related to indicators of the infrastructure assessment axis.

In this sense, Bernardo *et al.* (2020) points out the need for constant evaluation of the performance of university libraries, since their efficiency directly influences the quality of the courses offered by the HEIS of which they are a part, since it mainly directs its collection to the programmatic contents or the academic projects of the courses offered by the linked university.

Thus, given the importance of university libraries and their contribution to the achievement of HEIs goals, the present study aims to propose a multistage evaluation methodology, with the subsequent application in a set of university libraries belonging to a Higher Education Institution (HEI), using a Network Data Envelopment Analysis (DEA) model, specifically the multiplicative relational NDEA model.

The methodology applied proposes a specific evaluation of university libraries, as isolated units of the university, highlighting how these units use their resources to offer quality services to the community served, ensuring a balance between the use of resources and the provision of services available to society.

Regarding the evaluation of efficiency, several studies have used DEA models applied to university libraries, as stated by Najafi et al. (2020). The authors in question also attest that the use of DEA to evaluate the performance of libraries can provide useful inputs for the continuous improvement of libraries. In Shahwan and Kaba (2013), Stroobants and Bouckaert (2014) and Tavares *et al*. (2018), several studies that use DEA to evaluate libraries are listed, many of which focus on university libraries. These studies also mention the main inputs and outputs used in this scenario. In the Brazilian context, Carvalho et al. (2012) and Tavares et al. (2018) verify that studies in evaluation of efficiency in public libraries are scarce, one of the motivators for the realization of this article.

It should also be noted that the option for the NDEA model is justified, since in this model the internal stages of each evaluated library are taken into account, interconnected through input and output variables, going beyond the simplistic way in which these units are considered in classical models (Kao, 2014). In addition, the referred model supports the decision making of the managers of the evaluated units, as it facilitates the identification of the stages in which libraries are most deficient in terms of efficiency and, thus, subsidizes the proposal of specific improvements. Mahmoudi, Emrouznejad and Rasti-Barzoki (2019) highlight that the analysis of the efficiency of complex organizations with network structures is more realistic than the analysis with these classic structures.

Even considering all these advantages of the NDEA model, there are few studies that use this model in the evaluation of university libraries. To the best of our knowledge, one of the few studies found, however with a focus on urban public libraries, is the one proposed by Del Barrio-Tellado et al. (2021). For this, the authors use a Dynamic-Network DEA model to evaluate the efficiency of a library system in the city of Medellín, Colombia.

This study is organized into six sections, this introduction being the first of them. Sections 2 and 3 deal with conceptual aspects inherent to the methodology used to achieve and display the results. Section 4 aims to present the case study and the methodological aspects involved. Section 5 concentrates on the results and discussions of the findings of this research, and finally, section 6 refers to the main conclusions of the study.

2 Data envelopment analysis and DEA network

Popularly known as DEA, due to the abbreviation - Data Envelopment Analysis, this method is based on mathematical programming models, and its function is to compare the performance of Decision-Making Units (DMUs).

To calculate the efficiencies, initially, the return to scale and the model's orientation must be defined. As for the returns to scale, we have the classic DEA models, that is, the CCR model (Charnes; Cooper; Rhodes, 1978), which is used when there is evidence of proportionality between the resources used by the evaluated units (inputs) and the results achieved (outputs), and the BCC model (Banker; Charnes; Cooper, 1984), which is characterized by variable losses or gains in the input/output ratio, which characterizes a convex efficiency frontier, taking into account differences in size and/or scale.

Regarding the model's orientation, when it is input-oriented, the objective is to reduce the resources consumed by the unit to reach the efficiency frontier. When it is output-oriented, an increase in output is chosen to reach the efficiency frontier.

Classic DEA models (CCR and BCC) consider each DMU as a “black box,” with only inputs and outputs, which can be called exogenous (Färe; Grosskopf; Whittaker, 2007). The term exogenous here refers to inputs and outputs that come directly from the outside and not from any of the DMU's internal variables. However, over time, other advanced models have been developed to consider the internal processes of this “black box.” An example is the Network DEA (NDEA), which considers the internal stages of each DMU interconnected through input and output variables (Färe; Grosskopf, 2000). And so, this set of stages forms a network that determines the overall efficiency of the DMU.

There is several NDEA models pointed out in the literature (Kao, 2014). The most used, however, are the multiplicative model (Kao, 2009) and the additive model (Chen; Cook; Zhu, 2010). The main difference between these models is how the efficiencies of the internal stages of the DMUs are aggregated to obtain overall efficiency. The model proposed in Kao (2009) obtains global efficiency through the multiplication of its internal stages, while the Chen, Cook and Zhu (2010) model obtains global efficiency through a weighted sum of the internal stages.

Unlike classic DEA models, in NDEA, in addition to exogenous inputs and outputs, there are also intermediate variables, which play both the input function and the output function, depending on the stage to be analyzed. These are the intermediate variables that keep the system in a network, as shown in Figure 1.

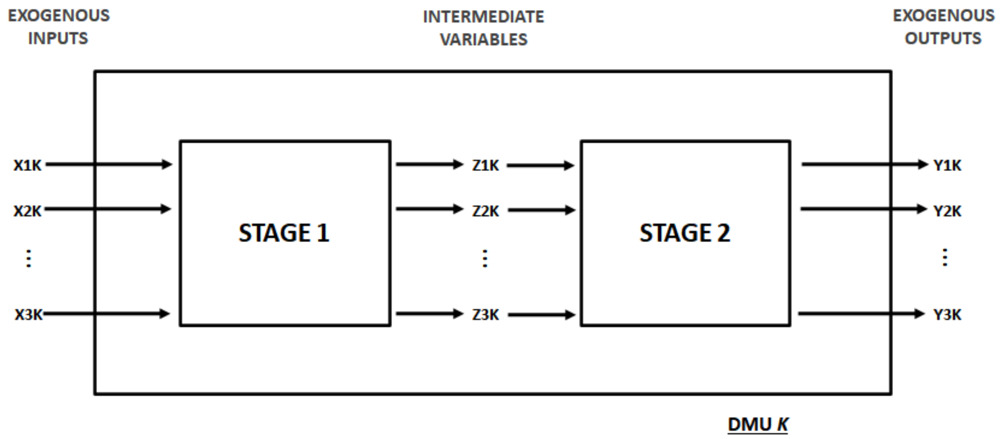


Figure 1 -Representation of the NDEA multi-stage model.

In Figure 1, the intermediate variables are simultaneously output for the first stage and input for the second stage, thus interconnecting the presented network.

The NDEA model used in this study will be the multiplicative relational Kao (2009), in which a DMU is only globally efficient if all its internal stages are efficient since the overall efficiency is obtained by multiplying the efficiency values of the internal stages, making it more rigorous than the classic DEA model.

The information regarding the efficiency of internal stages, presented in the NDEA, supports decision making by the managers of the evaluated units since it facilitates the identification of which is the most deficient stages in terms of efficiency and thus supports the proposal for specific improvements.

The linear programming problem (LPP) of the NDEA model that calculates the overall efficiency is based on the classic CCR DEA model. Model 1 presents the output-oriented linearized LPP for two stages in series. The LPP objective function will provide the inverse of overall efficiency. In addition, to obtain the efficiencies of each internal stage, the sum of the multiplication of inputs and their respective multipliers (called virtual input) must be divided by the multiplication of outputs and their respective multipliers (called virtual output).

It is necessary to clarify that, in model 1, h_0 is the inverse of the global efficiency and u_r, v_i and w_d are the multipliers of the variables, and yet, x, y, and z are the values of the inputs, outputs, and intermediate variables, respectively.

$$
\begin{aligned}
& \boldsymbol{h_o} = \min \sum_{i=1}^{m} v_i x_{i0} \\
& \quad s.t. \sum_{r=1}^{s} u_r y_{r0} = 1 \\
& \quad \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, j = 1,2,\ldots,n \\
& \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{d=1}^{D} w_d z_{dj} \leq 0, j = 1,2,\ldots,n \\
& u_r, v_i, w_d \geq 0, i = 1,2,\ldots,m, r = 1,2,\ldots,s, d = 1,2,\ldots,D
\end{aligned}
\tag{1}
$$

There is a discussion about the use of the multiplicative relational BCC NDEA model since the model does not follow the properties of the classic DEA. As an example, the efficiencies of some DMUs are lower in the BCC NDEA model than in the CCR DEA, which does not occur between the classic BCC and CCR models (Chen *et al.,* 2013). Thus, the present study will use only the output-oriented CCR DEA model. More details on the NDEA model can be found in Appendix A.

3 Graphical representation models for DEA efficient frontier

Because they are mathematical, the results obtained by DEA models can be difficult to interpret for those unfamiliar with linear programming or DEA. In addition, graphically represented results aid in understanding and interpreting results. For this reason, graphical representations of the efficiency frontier are important tools for disseminating results. They show the unit manager or administrator, visually, how close their DMU is to the efficiency frontier and how much their inputs should be reduced, or their outputs added to reach a 100% efficiency level. Additionally, for the manager, it is easier to make comparisons between his DMU and the others.

However, the graphical representation of the efficient frontier is very restricted and known only for the classic, constant return (CCR) models, for the case of at most two inputs and one output or one input and two outputs, and variable return scale (BCC), just for the case of an input and an output. Additionally, for NDEA (Färe; Grosskopf, 2000) and other advanced DEA models, the representation is even more restricted or unknown.

Bana and Costa, Soares de Mello and Angulo Meza (2016) proposed a bi-dimensional representation of the efficient frontier that includes the constant and variable returns to scale, for an unlimited number of DMUs and unlimited variables. Based on this model, Torres, Soares de Mello and Almeida (2017) presented a method to obtain a bi-dimensional representation of the efficient frontier for the NDEA, which will be used in the case study presented. An example of the bi-dimensional representation of the efficient frontier for the overall stage of the NDEA can be seen in Figure 2.

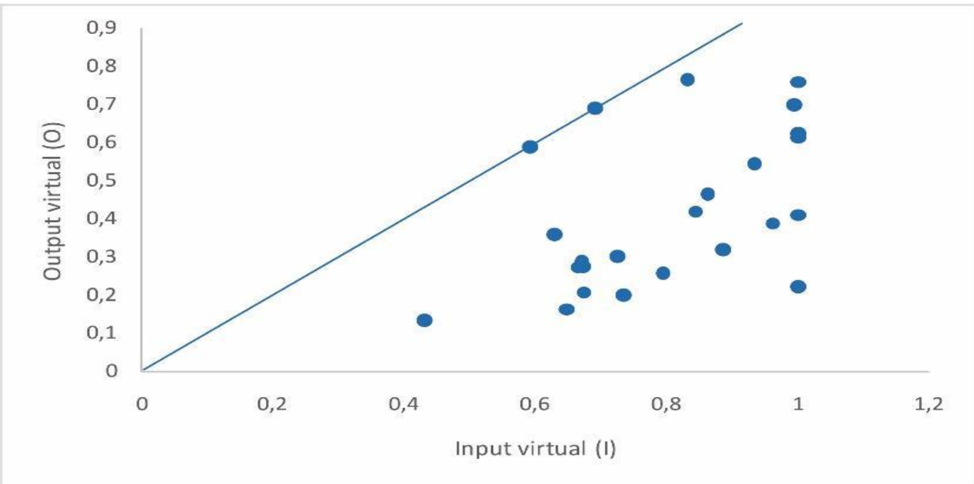


Figure 2 – Example of bi-dimensional representation of the efficiency frontier.

Source: Torres, Soares de Mello and Almeida, 2017

In Figure 2, efficient DMUs are represented on the frontier, while inefficient DMUs are below it. The targets of inefficient DMUs are obtained, for output orientation, through the tracing of a vertical line, from the evaluated DMU to the represented frontier. Where the vertical line meets the frontier is the target of that DMU. It is also noteworthy that, unlike the classic DEA models, this representation of the efficiency frontier does not provide the benchmarks for each inefficient DMU. Mariano et al. (2021) and Ferraz et al. (2021) developed studies using the bi-dimensional representation of the efficiency frontier proposed for NDEA models.

4 Methodology

This study proposes an efficient evaluation of 26 university libraries belonging to a given HEI, according to the capacity that each of these units must use its resources, to meet the demand of the university community, in an optimized way, through the services provided by these institutions.

The use of network data envelopment analysis (NDEA) thus seeks to detect possible inefficiencies in this process and align the library's impact on that academic community, making it possible to identify potential improvements for each library belonging to the system.

It is important to emphasize that the NDEA method will make a comparison between the analyzed libraries, and the efficiency indices obtained refer exclusively to this set.

In this section, the DMU's that make up the assessment set will be defined, the variables that will determine how the efficiency will be determined, and the choice of the orientation of the NDEA model, following the three steps proposed by Golany and Roll (1989), referring to the implementation of the problem during the modeling of Data Envelopment Analysis.

4.1 Definition of evaluated units

The Higher Education Institution (HEI) in question has a ULs system comprising a total of 30 libraries. It is necessary to emphasize that, in the analysis proposed by this study, four libraries will not be considered, as they escape the scope of this work. Two of these libraries are characterized as a Memory Center and a Rare and Special Works Center and were excluded from the analysis because they have an exclusively cultural character, not directly serving teaching and research activities, and in some cases, imposing restrictions on user access to shelves given the rarity and state of conservation of part of the collection.

The other two libraries mentioned were removed from the analysis because they are not intended to serve the university community and are linked to a school and a childcare center maintained by the university in question. Thus, 26 ULs will compose the efficiency evaluation process through the NDEA method.

It should be noted that among the selected DMUs there are two possibilities concerning the locations where they are situated. Due to the expansion process of the analyzed university, there are campuses scattered in smaller cities in the state to which the university belongs. Thus, there are libraries linked to campuses in the interior of the state, generally of a smaller size, which in this study will be identified as libraries in INLAND CITIES. Others are located on campuses that are part of the host city in which the university is established. These ULs are identified in this study as MAIN CITY libraries. Table 1 identifies the 26 libraries included in the analysis set (DMU's), their location, and the subject area in which they are inserted.

Table 1 - University libraries evaluated.

LIBRARIES	KNOWLEDGE AREA	LOCALIZATION
DMU 1	Applied Social Sciences	INLAND CITIES
DMU 2	Applied Social Sciences	MAIN CITY
DMU 3	Applied Social Sciences	MAIN CITY
DMU 4	Applied Social Sciences	MAIN CITY
DMU 5	Applied Social Sciences	MAIN CITY
DMU 6	Health Sciences	INLAND CITIES
DMU 7	Health Sciences	MAIN CITY
DMU 8	Health Sciences	MAIN CITY
DMU 9	Health Sciences	MAIN CITY
DMU 10	Health Sciences	MAIN CITY
DMU 11	Health Sciences	MAIN CITY
DMU 12	Multidisciplinary	INLAND CITIES
DMU 13	Exact and Earth Sciences	MAIN CITY
DMU 14	Exact and Earth Sciences	MAIN CITY
DMU 15	Exact and Earth Sciences	MAIN CITY
DMU 16	Biological Sciences	MAIN CITY
DMU 17	Engineering	INLAND CITIES
DMU 18	Engineering	MAIN CITY
DMU 19	Multidisciplinary	INLAND CITIES
DMU 20	Multidisciplinary	INLAND CITIES
DMU 21	Multidisciplinary	MAIN CITY
DMU 22	Multidisciplinary	MAIN CITY
DMU 23	Engineering	INLAND CITIES
DMU 24	Applied Social Sciences	INLAND CITIES
DMU 25	Exact and Earth Sciences	MAIN CITY
DMU 26	Applied Social Sciences	INLAND CITIES

4.2 Variables used

The choice of variables, illustrated in Figure 3, was based on meeting the concept of efficiency that we seek to achieve in this article, seeking to understand how each assessed unit uses its resources to produce knowledge and support university activities, justifying the maintenance of the number of inputs necessary for its operation. For a more in-depth analysis of variables used in the context of efficiency assessment in ULs, it is suggested to read the work proposed by Tavares et al. (2018).

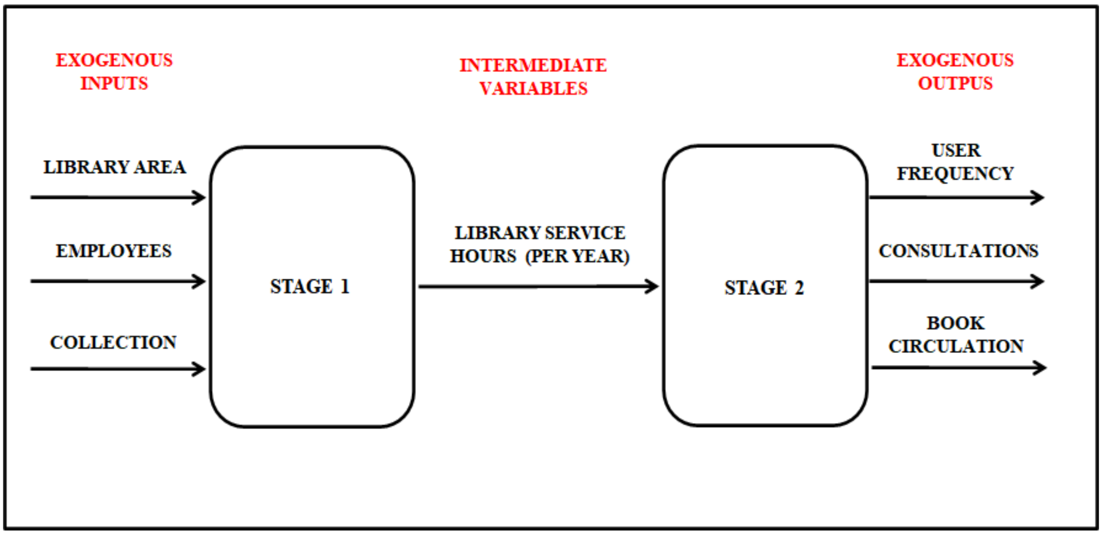


Figure 3 - NDEA model proposed for the evaluation of a set of Uls linked to a Brazilian HEI.

The model shown in Figure 3 was adapted from the model proposed by Tavares et al. (2017), which bases the analysis on variables widely used in efficiency assessments in the context of university libraries. However, in the present study, a two-stage model is chosen, to enable the understanding of the sub-processes inserted between the input of resources and the results obtained by each UL. In the first stage of the established model, the capacity of each library to offer its services in a period is evaluated according to the available inputs, thus avoiding the lack of use of the collection, labor, and physical space, while in the second stage of the NDEA model, it is verified how the library offers its services, considering its opening hours.

At this stage, to be efficient, the library should serve the greatest number of interested parties, concerning the number of hours it was open in the year evaluated. The variable "Library service hours" (or Operating Hours) is defined as an intermediate variable, as it presents input and output characteristics at different stages of the process. In addition, other adaptations in the model, in relation to that proposed by Tavares et al. (2017), include the insertion of the variable library area, in square meters, and the subdivision of the variables "Consultations" and "Loans", since they have different purposes, as detailed in Table 2.

The data used in the evaluation of efficiency were collected from the statistical report of Pergamum, an integrated management system of the bibliographic collection used by the university libraries, from the annual report of the Documentation Superintendence (SDC) for the year 2016, and contacts made with the university libraries.

Table 2 - Description of the variables used.

INPUTS (RESOURCES)	
LIBRARY AREA (AREA)	Area, in m^2, available for the functioning of the activities of each UL, including storage of the collection, accommodation of staff, space for books consultation, space for study, administrative support, and reception and control of users.
EMPLOYEES (EMPL)	Permanent operational team, allocated in a specific UL. This variable includes librarians and administrative support. It is understood that human resources represent a large portion of library costs.
COLLECTION (COLL)	It refers to the number of copies (books) available in each library. It is also one of the main resources of the units since its maintenancerepresentssignificantcostsarisingfromtheallocationand updating (acquisition), and this directly reflects the following services provided by the ULs: consultations and loans.
VARIÁVEL INTERMEDIÁRIA	
LIBRARY SERVICE HOURS (LSHO)	A key variable for the two-stage model since it represents the outputs of the first stage and the resources of the second stage. To be efficient, the Uls need to make their services available for a period that fully meets the needs of users and, on the other hand, fits the number of available resources, such as collections and number of employees. This variable indicates the number of hours that the library was open in 2016.
OUTPUTS (SAÍDAS)	
USER FREQUENCY (FREQ)	The number of users circulating annually in each library. This variable comprises the entire public that uses the Uls spaces, both to access the collection and to use the spaces for various purposes, such as reading and studying.
CONSULTATIONS (CONS)	It expresses the number of volumes consulted throughout 2016 in each UL. It is understood that when analyzing this variable separately from the loan variable, it is possible to infer about the use of the physical space of the library since the consultation is conducted in the unit itself.
BOOK CIRCULATION (CIRC)	Expresses the number of volumes lent (home loan) to registered users throughout 2016 in each UL.

Finally, it is essential to clarify that the variables selected for the proposed model were chosen to align with the objective of the efficiency assessment. This assessment will enable the measurement of ULs performances, first in terms of the efficient use of infrastructure and later in terms of DMUs operations, particularly when compared to the services provided to the academic community.

It is important to highlight that the selection of variables was based on a review of the most used variables in similar contexts (see the systematic literature review propose by Tavares et al. 2018), as well as on the availability of data from both the published Management Report and additional data collected from the analyzed university.

Furthermore, university library managers seeking to replicate the proposed evaluation model may adapt it according to data availability and the specific objectives of their assessment.

This may involve including, excluding, or replacing one or more of the variables considered in this study.

4.3 Model, orientation, and efficiency indices analysis

To obtain efficiency scores, we will use the output-oriented NDEA multiplicative relational model, as represented by the linear programming problem shown in model 1. Librarians might be interested in providing a maximum of services given their resources (Reichmann; Sommersguter-Reichmann (2006), which justifies an output-oriented model.

It is also noteworthy that the "library service hours" variable is treated as an intermediate variable in the model due to its input characteristics, when you want to serve the largest number of users, in a shorter period, and at the same time, its output characteristics, when there is an interest in making its services available for as long as possible to the interested public.

The orientation to outputs is desirable in this scenario since it was decided to maintain resources to maximize the opening hours in the first stage and the volume of services provided (users, consultations, and loans).

Once defined the variables used, and resolved the LPPs for each library, the efficiency indices for each stage will be obtained. It will also be possible to propose an ordering according to the overall efficiency score of each unit, and, from the results obtained, suggest points of improvement for the libraries with the worst performance.

5 Results and discussions

Through reports and contacts made directly with the evaluated ULs, data referring to the seven variables used in the efficiency evaluation model, such as those used in Tavares et al. (2017), are summarized in Table 3.

Table 3 -Descriptive statistics of all the variables employed in this study.

	INPUT			INT. V.	OUTPUT		
	AREA	**EMPL**	**COLL**	**LSHO**	**FREQ**	**CONS**	**CIRC**
Maximum	7251	42	164323	3159	105762	60670	29294
Minimum	84	4	459	1896	853	78	455
Average	558	8,54	15209	2569	21785	5608	8677
St. Deviation	1386	7,18	31026	347,6	28228	12301	8984

With this data, we calculated the efficiency of the DMUs using the model described in (1). It was possible to obtain efficiency of the overall stage (e0), as well as the first (e1) and second (e2) stage. The data collected and the efficiencies values are described in Table 4.

Table 4 - Efficiencyindicesofuniversitylibraries.

DMU	**e1**	**e2**	**e0**	**DMU**	**e1**	**e2**	**e0**
1	0.442	0.253	0.112	**14**	0.515	0.930	0.479
2	0.548	0.118	0.065	**15**	0.787	0.087	0.068
3	0.486	0.285	0.139	**16**	0.301	0.989	0.297
4	0.394	0.442	0.174	**17**	0.492	0.843	0.415
5	0.585	0.080	0.047	**18**	0.401	0.454	0.182
6	0.826	0.189	0.156	**19**	0.509	1.000	0.509
7	0.355	0.256	0.091	**20**	0.449	0.268	0.120
8	0.442	0.357	0.158	**21**	0.374	0.479	0.179
9	0.344	0.276	0.095	**22**	0.098	1.000	0.098
10	0.647	0.182	0.118	**23**	1.000	0.015	0.015
11	0.540	0.203	0.110	**24**	1.000	0.023	0.023
12	0.796	0.122	0.097	**25**	0.558	0.163	0.091
13	0.490	0.147	0.072	**26**	0.983	0.047	0.046

From the observation of the efficiency scores shown in Table 4, it can be seen that referring to the first stage, only DMUs 23 and 24 are efficient. These two university libraries are located outside the host city (inland city), as can be seen in Table 1, as well as the third most efficient, the DMU 26, which has an efficiency value of 0.983.

These DMUs were able to offer their services throughout the year analyzed, using a reduced number of inputs concerning the number of hours they were working, thus avoiding waste and idleness of the collection, labor, and physical space.

As we can see in Table 5, which proposes a comparison between the averages of the DMUs efficiencies, it is possible to verify that the inland libraries have overall efficiency values, slightly higher than the DMUs located on campus in the main city, mainly influenced by the performance in the first stage, since, in the evaluation of this stage, inland libraries, as they are smaller, consequently use fewer resources concerning libraries in the main city, operating, even so, in a number of hours close to the group's average. Furthermore, in general, the ULs in the main city are older and deal with problems of infrastructure and lack of resources, given the recent interiorization process of the university considered.

Table 5 **–** Comparison of average efficiency between ULs: Main city x Inland.

LOCALIZATION	LIBRARIES	E1	E2	E0
Inland city libraries	9	0.722	0.307	0.166
Main city libraries	17	0.463	0.379	0.145

Analyzing the second stage, it is possible to observe that two libraries are efficient, the DMUs 19 and 22, both multidisciplinary that, given their characteristics, meet the demand of several undergraduate courses, and these units should theoretically be well prepared to receive a quantity larger and differentiated number of students, aiming to fulfill its function efficiently. It is noteworthy that these units stand out for offering their services to users fully, without needing, to extend their opening hours, causing unnecessary costs to the units. DMU 19, located in an inland city, received more than 52,000 users in 2016, more than double the average for libraries, recording consultations four times higher than the average, and it was the DMU that made more loans among all analyzed. This certainly contributed to an impressive performance in the second stage. DMU 22 is the university's central library, having received more than 100,000 users in the year considered, the largest number of visitors among the units evaluated, as well as the largest number of works consulted. It is noteworthy that these two DMUs, even though they present a large circulation of users and offer their consultation and loan services with greater intensity than the others, worked a few hours close to the average of the DMUs, which favored their performance at this stage.

Likewise, it is necessary to highlight in this second stage, the excellent performance of DMUs 14 and 16, both located in the main city of the HEI. These ULs, even though they are not located on the efficient frontier of stage two, achieved scores very close to 1. In addition, the first UL attends to the Physics course and the second to courses in the health area, and their good performances can be justified by the areas of knowledge to which they are inserted, characterized by the expressive demand for library services, in addition to being areas in which the evaluated university is recognized for the quality of its undergraduate courses.

It is important to emphasize that this study does not aim to propose specific improvements for DMUs classified as inefficient, nor to establish benchmarks for these units. This approach could be explored in future research focused on investigating how DMUs outside the efficiency frontier might enhance their performance and identify which aspects require adjustments. Nevertheless, library managers may analyze the exogenous input and output variables at each stage and compare them with those of efficient DMUs, thereby identifying potential strategies to improve the performance of inefficient libraries, as can be seen in the example proposed below.

DMU	UNIVERSITY LIBRARIES	**AREA**	**EMPL**	**COLL**	**LSHO**	**FREQ**	**CONS**	**CIRC**
5	BAU	120	5	5800	2090	3537	1112	1697
14	BIF	285,67	8	9303	2938	99440	4532	20818
22	BCG	7251,07	42	164323	2925	105762	60670	23683

In the table above, we compare the performance of three randomly selected DMUs, each exhibiting different levels of efficiency, based on the resources they consume and the results they produce. The aim is to help managers better understand how efficient indices can be analyzed according to the variables of each unit and, from there, how projections can be developed to enable inefficient DMUs to optimize their performance.

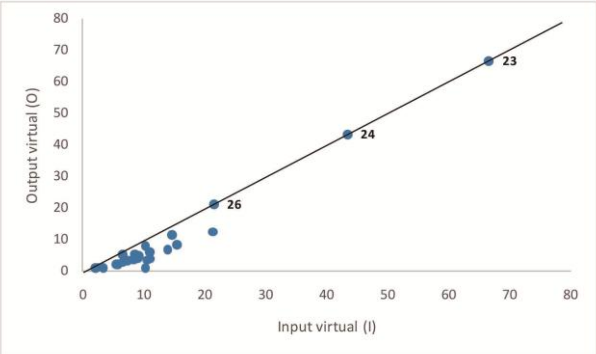
In this example, we compare the BCG library, which was efficient in stage 2, with the BIF library, which in the same stage performed very close to the efficiency frontier (0.930), and the BAU library (0.189), which had one of the lowest performances in this stage. As observed, the BIF unit operates with significantly fewer inputs than the BCG unit; however, its results are quite competitive when analyzed in terms of USER FREQUENCY and BOOK CIRCULATION.

This explains why the BIF library is positioned so close to the efficient frontier. A possible reason for this library not reaching an efficiency index of 1 is its performance in the CONSULTATIONS output, which is significantly lower than that of the BCG library.

On the other hand, when comparing the BIF and BAU DMUs, it is evident that, despite having half the infrastructure of BIF, the BAU library delivers drastically inferior results. Given the resources available, it would be expected that this library could provide a greater number of services to its community, highlighting either an oversized range of resources or an inefficient provision of services.

Finally, this example demonstrates that, although infrastructure is a key factor, some smaller libraries can achieve superior results with limited resources, whereas others must adjust their resource allocation in line with the volume of services provided. Managers should not only monitor efficiency indices but also develop a deep understanding of each DMU's characteristics, assessing how they perform given the resources at their disposal and the outcomes they generate.

Finally, to help visualize the performance of the DMUs in the internal stages, Figures 4 and 5 represent the bi-dimensionally efficient frontier for these stages. This graphical representation follows the procedure suggested by Torres, Almeida and Soares de Mello (2017), where the axes used for the representation are the modified virtual inputs and outputs, which represent the weighted sum of the inputs and outputs of each stage, respectively.



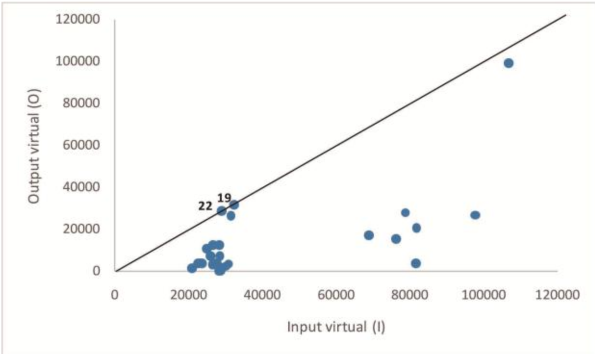


Figure 4 - Bi-dimensional representation of stage 1 efficiency.

Figure 5 -Bi-dimensional representation of stage 2 efficiency.

From the bi-dimensional representation of the first stage (Figure 4), it is possible to observe that the two efficient DMUs (Golany; Roll, 1989; Tavares et al., 2018) are located on the efficient frontier, while the DMU 26, which has an efficiency very close to 1, is represented very close to the frontier. The other DMUs are well distributed in the graph, which demonstrates that there is a large variation in the value of efficiencies for this stage.

In the bi-dimensional representation of the second stage (Figure 5), as well as in the first stage, the efficient DMUs (Bana e Costa; Soares de Mello; Angulo Meza, 2016; Ferraz et al., 2021) are on the efficiency frontier and are represented superimposed in the presented graph, while the DMUs with high efficiencies, higher than 0.8 are represented near the border. It is also verified that there is a clear separation between the more efficient DMUs and the less efficient DMUs, which is also observed in the results and does not happen in the representation for the first stage. This occurs since there are no DMUs, in the second stage, with efficiency indices between 0.5 and 0.8.

Finally, analyzing the overall stage, we can observe that no DMU is efficient, with DMU 19 being the closest to the overall efficient frontier, with an index of 0.509. Low global efficiency scores are common in NDEA, as to be globally efficient the DMU must have an efficiency value equal to 1 for both internal stages. DMU 19, for example, is efficient in the second stage and has a value equal to 0.509 in the first stage.

In addition to DMU 19, only two other ULs achieved an overall efficiency index higher than 0.4, the aforementioned DMU 14, due to its good performance in stage 2, and the DMU 17, which operates in the "engineering" knowledge area and, despite being only the fourteenth best library in stage 1, it is the fifth-best library in stage 2, and the second-best inland UL, at that stage. Among its characteristics, it is the library with the most service hours in the evaluated period and has the largest loan volume among the DMUs, even ahead of the university's central library. It is also noteworthy that DMU 14, despite having a high number of users and loans, in addition to extended opening hours, had only 9 employees in that period.

It is also possible to highlight the libraries with the greatest potential for improvement. DMUs 23, 24, and 26 perform very well in the first stage, however, they have a frustrating performance in the second. This brings a huge loss to the overall efficiency of these ULs and alerts us to the need for changes in these units. All three units are inland ULs and two of them were recently opened. Another common characteristic among these units is the small number of employees, collection, and area, which may have favored their performance in the first stage. However, when analyzing the relationship between the outputs and the opening hours of these ULs, it is clear that they are underutilized and thus, a more in-depth study is suggested to be carried out by managers to verify whether it is possible to increase the volume of services provided in these ULs or reduce their opening hours, avoiding waste of resources. Figure 6 presents the bi-dimensional representation of the efficient frontier for the overall stage.

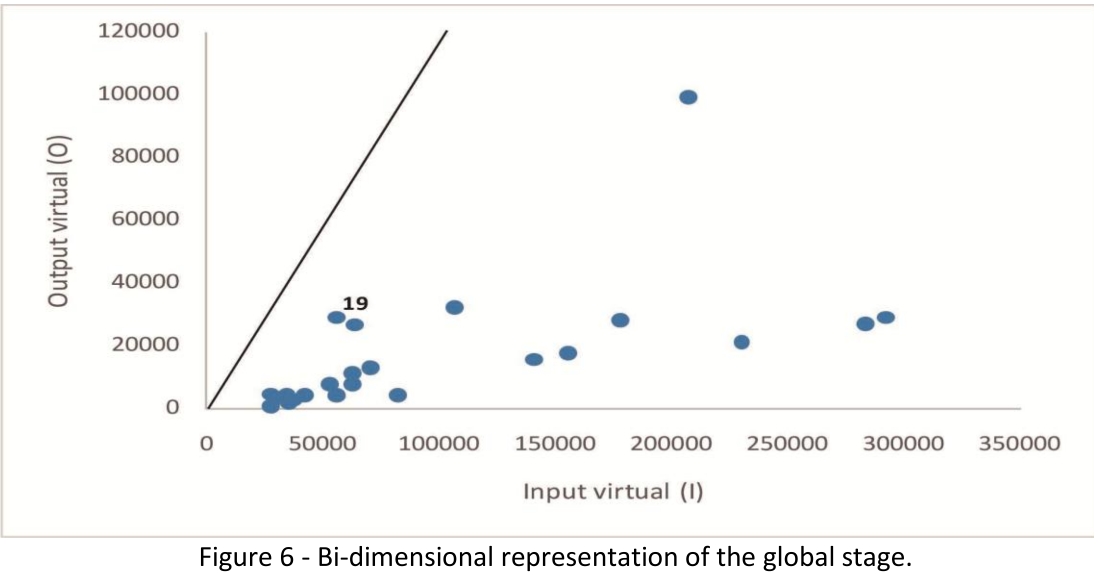


Figure 6 - Bi-dimensional representation of the global stage.

Figure 6 confirms the results obtained for overall efficiencies. It is possible to observe that there is no DMU represented in the efficient frontier, and we can note that the DMUs were extremely far from the efficient frontier. The low efficiency of DMUs indicates that libraries can improve their performance, both in the first stage and in the second stage.

6 Conclusions

This manuscript presented the efficiency evaluation of 26 university libraries of a Higher Education Institution. This efficiency may be evaluated in two distinct stages, with the help of an output-oriented NDEA model, which allowed for a more in-depth analysis by proposing the opening of the "black box" relative to classic DEA models and evaluating the first stage according to the infrastructure provided and the service hours of these DMUs and the second stage according to the services provided concerning the opening hours.

Two inland libraries were efficient in the first stage, since, as they are small ULs, which offer above-average opening hours, they managed to optimize the use of their resources, while the other two were efficient in the second stage, due to their large volume service provided to users during optimized service hours.

It is noteworthy that no DMU was efficient on the global stage, which indicates that there is space for improvements in the performance of university libraries.

Bi-dimensional representations of the efficiency frontier were elaborated for each stage, and, thus, it was possible to graphically observe the performance of the DMUs. The bi-dimensional representations helped in the analysis of the results of the DMUs.

As future research, we suggest the analysis of several university libraries from other HEIs in Brazil and other countries, so that they can be compared with the results obtained in this study. Moreover, it is also possible to analyze other models to calculate library efficiencies, such as a model with exogenous inputs in the second stage, or the use of a model that allows identifying targets for inefficient ULs.

References

Bana e Costa, Carlos A.; Soares de Mello, João Carlos C.B.; Angulo Meza, Lídia. A new approach to the bi-dimensional representation of the DEA efficient frontier with multiple inputs and outputs. **European Journal of Operational Research**, v. 255, p. 175-186, 2016.

Banker, R. D.; Charnes, A.; Cooper, W. W. Some Models for Estimating Technical and Scale Inefficiencies in Data Envelopment Analysis. **Management Science**, v. 30, p. 1078-1092, 1984. https://doi.org/10.1287/mnsc.30.9.1078

Bernardo, Marcella; Souza, Matheus Alves Madeira de, Lopes, Ramon Sávio Moreira, RODRIGUES, Lásara Fabrícia. University library performance management: Applying zero-sum gains DEA models to resource allocation. **Socio-Economic Planning Sciences**, 100808, 2020. DOI:10.1016/j.seps.2020.100808

Carvalho, Frederico A.; Jorge, Marcelino José; Jorge, Marina Filgueiras; Russo, Mariza; SÁ, Nysia Oliveira de. Library performance management in Rio de Janeiro, Brazil: applying DEA to a sample of university libraries in 2006-2007. **Library Management**, v. 33, n. 4/5, p. 297-306, 2012. DOI: 10.1108/01435121211242335

Charnes, A.; Cooper, W. W.; Rhodes, E. Measuring the efficiency of decision-making units. **European Journal of Operational Research**, v. 2, p. 429-444, 1978. DOI: 10.1016/0377-2217(78)90138-8

Chen, Yao; Cook, Wade D; Kao, Chiang; Zhu, Joe. Network DEA pitfalls: Divisional efficiency and frontier projection under general network structures. **European Journal of Operational Research**, v. 226, n. 3, p. 507-515, 2013. https://doi.org/10.1016/j.ejor.2012.11.021

Chen, Yao; Cook, Wade D.; Zhu, Joe. Deriving the DEA frontier for two-stage processes. **European Journal of Operational Research**, v. 202, p. 138-142, 2010.

Cooper, William W.; Seiford, Lawrence M.; Tone, Kaoru. **Data Envelopment Analysis**: A Comprehensive Text with Models. Applications, References and DEA-Solver Software. Second Edition. New York, NY: Springer, 2007. https://doi.org/10.1007/978-0-387-45283-8.

Del Barrio-Tellado, María José; Gómez-Vega, Mafalda; Gómez-Zapata, Jonathan Daniel; Herrero-Prieto, Luis César. Urban public libraries: Performance analysis using dynamic-network-DEA. **Socio-Economic Planning Sciences**, v. 74(C), 100928, 2021. DOI: 10.1016/j.seps.2020.100928

Enweani, Uchechukw Victoria. Challenges of Managing University Libraries in Contemporary Digital Environment. **Library Philosophy and Practice (e-journal**), 2073, 2018.

Färe, Rolf; Grosskopf, Shawna. Network DEA. **Socio-Economic Planning Sciences**, v. 34, p. 35-49, 2000. https://doi.org/10.1016/S0038-0121(99)00012-9

Färe, Rolf; Grosskopf, Shawna; Whittaker, Gerald. Network DEA. *In*: Zhu, Joe; Cook, Wade D. (Ed.). **Modeling DATA Irregularities and Structural Complexities in Data Envelopment Analysis***.* USA: Springer, 2007. p. 209–240.

Ferraz, Diogo; Mariano, Enzio Barberio; Manzine, Patrícia Regina; Moralles, Herick Fernando; Morceiro, Paulo César; Torres, Bruno Guimarães; Almeida, Mariana Rodrigues de; Soares de Mello, João Carlos Correia Baptista; Rebelatto, Daisy Aparecida do Nascimento. COVID Health Structure Index: The Vulnerability of Brazilian Microregions. **Social Indicators Research**, v. 158, n. 1, p. 197-215, 2021.

Golany, B; Roll, Y. An application procedure for DEA. **Omega**, v. 17, n. 3, p. 237-250, 1989.

Kao, Chiang. Efficiency decomposition in network data envelopment analysis: A relational model. **European Journal of Operational Research**, v. 192, p. 949-962, 2009.

Kao, Chiang. Network data envelopment analysis: A review. **European Journal of Operational Research**, v. 239, p. 1-16, 2014. DOI: 10.1016/j.ejor.2014.02.039

Kao, Chiang. **Network data envelopment analysis**: foundations and extensions. Berlin: Springer, Berlin, 2017.

Kao, Chiang; Hwang, Shiuh-Nan. Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan. **European Journal of Operational Research,** v.185, n.1, p.418-429, 2008. https://doi.org/10.1016/j.ejor.2006.11.041

Mahmoudi, Reza; Emrouznejad, Ali; Rasti-Barzoki, Morteza. A bargaining game model for performance assessment in network DEA considering sub-networks: a real case study in banking. **Neural Computing and Applications**, v. 31, p. 6429-6447, 2019. DOI: 10.1007/s00521-018-3428-y

Maia, Luiz Cláudio; Santos, Maria de Souza Lima. Gestão da biblioteca universitária: análise com base nos indicadores de avaliação do MEC. **Perspectivas em Ciência da Informação**, v. 20, n. 2, p. 100-119, 2015. DOI: 10.1590/1981-5344/2079.

Mariano, Enzo; Torres, Bruno; Almeida, Mariana; Ferraz, Diogo; Rebelatto, Daisy; Soares de Mello, João Carlos. Brazilian states in the context of COVID-19 pandemic: an index proposition using Network Data Envelopment Analysis. **IEEE Latin America Transactions**, v. 19, p. 917-924, 2021.

Najafi, Ali; Sefiddashti, Sara Emamgholipour; Sheikhshoaei, Fatemeh; Hajiagha, Seyed Hossein Razavi; MasoumI, Davoud. DEA-based Performance Evaluation of Libraries: A Systematic Mapping study. **Investigación Bibliotecológica**: archivonomía bibliotecología e información, v. 34, n. 85, p. 227-251, 2020. DOI: 10.22201/iibi.24488321xe.2020.85.58159

Reichmann, Gerhard; Sommersguter-Reichmann, Margit. University library benchmarking: An international comparison using DEA. **International Journal of Production Economics**, v. 100, n. 1, p. 131-147, Mar. 2006.

Shahwan, Tamer Mohamed; Kaba, Abdoulaye. Efficiency analysis of GCC academic libraries: application of data envelopment analysis. **Performance Measurement and Metrics**, v. 14, n. 3, p. 197-210, 2013. DOI: 10.1108/pmm-07-2013-0023

Soares de Mello, João Carlos Correia Baptista; Angulo Meza, Lidia; Gomes, Eliane Gonçalves; Biondi Neto, Luiz. Curso de Análise Envoltória de Dados. In: SIMPÓSIO BRASILEIRO DE PESQUISA OPERACIONAL, 37., 2005, Gramado. **Anais [**...]. Gramado: UFSM, 2005. p.2520-2545.

Stroobants, Jesse; Bouckaert, Geert. Benchmarking local public libraries using non-parametric frontier methods: a case study of Flanders. **Library & Information Science Research**, v. 36, p. 211-224, 2014. DOI: 10.1016/j.lisr.2014.06.002

Tavares, Rafael Santos; Angulo-Meza, Lídia; Sant'Anna, Annibal Parracho. A proposed multistage evaluation approach for Higher Education Institutions based on network Data envelopment analysis: A Brazilian experience. **Evaluation and Program Planning**, v. 89, p. 101984, 2021. DOI: 10.1016/j.evalprogplan.2021.101984

Tavares, Rafael Santos; Drumond, Geisa Meirelles; Angulo Meza, Lidia; Méxas, Mirian Picinini. Aplicação da metodologia DEA na avaliação de eficiência de bibliotecas universitárias de uma

Instituição Federal de Ensino Superior. *In*: SIMPÓSIO BRASILEIRO DE PESQUISA OPERACIONAL, 49., 2017, Blumenau. **Anais** [...]. Blumenau, SC: SOBRAPO, 2017.

Tavares, Rafael Santos; Drumond, Geisa Meirelles; Angulo Meza, Lídia; Méxas, Mirian Picinini. Efficiency assessment in university libraries. **Transinformação**, v. 30, n. 1, p. 65-79, 2018. DOI: 10.1590/2318-08892018000100006

Torres, Bruno Guimarães; Soares de Mello, João Carlos Correia Baptista; Almeida, Mariana Rodrigues de. Representação Bidimensional do Network DEA: Um estudo de caso com avaliação de docentes. *In*: SIMPÓSIO BRASILEIRO DE PESQUISA OPERACIONAL, 49., 2017, Blumenau. **Anais** [...]. Blumenau, SC: SOBRAPO, 2017.

APPENDIX A

As explained previously, Data Envelopment Analysis (DEA) is a non-parametric methodology that evaluates the efficiency of decision-making units (DMUs) that use linear programming models to determine the efficiency of the DMUs. The DMUs, the units under evaluation, use the same inputs and the same outputs. The proposed programming model is in (2).

$$Max \ \frac{\sum_{\mathrm{r=1}}^{s} u_r Y_{\mathrm{r0}}}{\sum_{\mathrm{i=1}}^{m} v_i \mathrm{X}_{\mathrm{i0}}}$$

$$\textit{Subject to:} \qquad (2)$$

$$\frac{\sum_{\mathrm{r=1}}^{s} u_r Y_{\mathrm{rj}}}{\sum_{\mathrm{i=1}}^{m} v_i \mathrm{X}_{\mathrm{ij}}} \leq 1, j = 1, \dots, n$$

$$v_i, u_r \geq 0, \forall i, j$$

where:
v_i = weight of input *j*
u_r = weight of the output *r*
x_{jj} = amount of input *i* of the DMU $_j$
y_{rj} = amount of output *r* of the DMU $_j$
x_{i0} =amount of input *i* of the observed/analyzed DMU_0
y_{r0}= amount of output *j* of the observed/analyzed DMU_0
n = number of DMUs

This model is used to determine the efficiency index of the analyzed DMU, which is placed in the first line, as well as the weights to achieve the DMUs index. These weights are determined by comparison which is guaranteed by the second line of this model.

This is an input-oriented model, which focuses on reducing the input while maintaining the outputs, this means that the focus is on reducing the resources in the process to achieve the same production or results). As the evaluation is made by comparison, there is a real DMU on the set that can produce consuming less resources.

$$Min \frac{\sum_{i=1}^{m} v_i X_{i0}}{\sum_{r=1}^{s} u_r Y_{r0}}$$

Subject to:

$$\frac{\sum_{r=1}^{s} u_r Y_{rj}}{\sum_{i=1}^{m} v_i X_{ij}} \leq 1, j = 1, \dots, n \qquad (3)$$

$$v_i, u_r \geq 0, \forall i, j$$

As expected, there is an output-oriented programming model, which focuses on increasing the output while maintaining the inputs. This means that the focus is on increasing the production or results of the process while maintaining the actual consumption of resources. As in the previous case, the comparison ensures that there is a real DMU on the set that is able to produce more with the same number of resources. The programming model is in (3), where the variables are the same as explained in model (2).

Note that, compared to the previous model, this model has an inversion of the ratio in the first line of the model. In this way, the solution found by this model is the inverse of the efficiency index for the analyzed DMU 0.

Due to mathematical characteristics, the previous models (2 and 3) need to be linearized. This is done by taking the denominator of the ratio in the objective function to the restrictions and equaling it to 1. The linear programming problem (LPP) of the output-oriented model in (3) is found in (4).

$$Min \sum_{i=1}^{m} v_i X_{i0}$$

Subject to:

$$\sum_{r=1}^{s} u_r Y_{rj} = 1 \qquad (4)$$

$$\sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \leq 1, j = 1, \dots, n$$

$$v_i, u_r \geq 0, \forall i, j$$

In the case of the Network DEA approach, the efficiency is measured in each stage, that is, the ratio seen in the previous models is replicated for each stage. Figure 7 depicts the network structure of this study with the efficiency (ratio) for each stage.

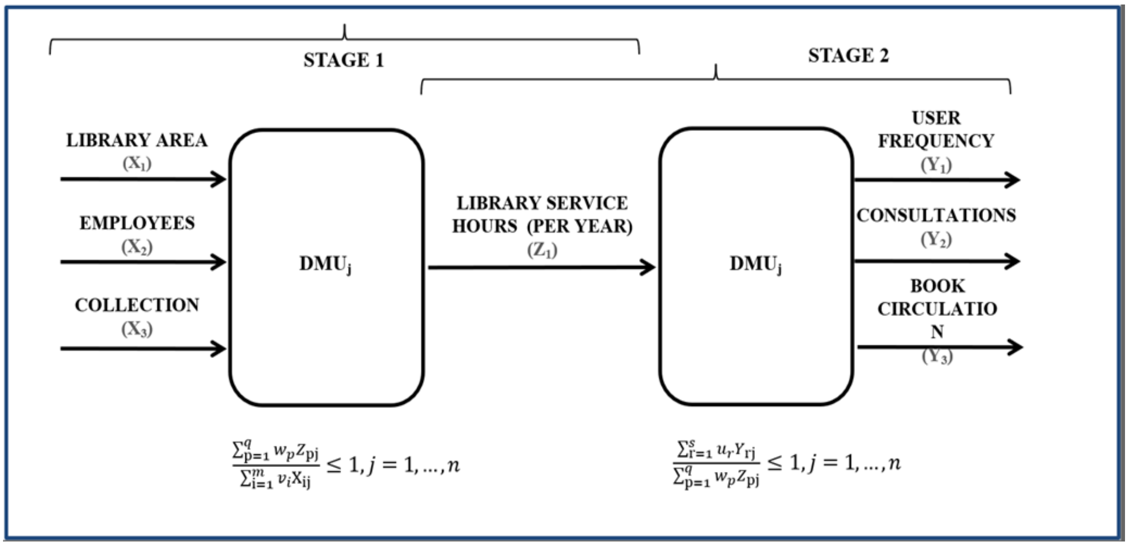


Figure 7 – Efficiencies for each stage.

For stage 1, the inputs (exogenous inputs) are identified by X_{ij} and the outputs (intermediate variables) by z_{pj}. The ratio has the weighted sum of inputs, X_{ij}, in the denominator and the weighted sum of the outputs, Z_{pj}, in the numerator. For the second stage, the inputs (intermediate variables) are identified by Z_{pj} and the outputs (exogenous outputs) by Y_{rj}. Its ratio has the weighted sum of the inputs, Z_{pj}, in the denominator and the weighted sum of the outputs, Y_{rj}, in the numerator.

For a DMU 0, under evaluation, both equations must be considered to determine their global efficiency. Therefore, the programing model considering an output orientation as in 3 is in 5:

$$Min \frac{\sum_{i=1}^{m} v_i X_{i0}}{\sum_{r=1}^{s} u_r Y_{r0}}$$

Subject to:

$$\frac{\sum_{p=1}^{q} w_p Z_{pj}}{\sum_{i=1}^{m} v_i X_{ij}} \leq 1, j = 1, \ldots, n \qquad (5)$$

$$\frac{\sum_{r=1}^{s} u_r Y_{rj}}{\sum_{p=1}^{q} w_p Z_{pj}} \leq 1, j = 1, \ldots, n$$

$$v_i, u_r, w_p \geq 0, \forall i, j, p$$

The linearized version of this model is in [6] (used in this manuscript and similar to model [1]):

$$Min \sum_{i=1}^{m} v_i X_{i0}$$

Subject to:

$$\sum_{r=1}^{s} u_r Y_{rj} = 1$$

$$\sum_{p=1}^{q} w_p Z_{pj} - \sum_{i=1}^{m} v_i X_{ij} \leq 1, j = 1, \ldots, n \qquad (6)$$

$$\sum_{r=1}^{s} u_r Y_{rj} - \sum_{p=1}^{q} w_p Z_{pj} \leq 1, j = 1, \ldots, n$$

$$v_i, u_r, w_p \geq 0, \forall i, j, p$$

As in the previous cases, this model needs to be solved for each DMU and it will provide the global efficiency index for the analyzed DMU, as well as the efficiency indexes for each stage. For more information about the classic DEA model and the NDEA model, we suggest reading: Soares de Mello et al. (2005), Cooper, Seiford and Tone (2007), Kao and Hwang (2008); Chen et al. (2013) and Kao (2017).