A STUDY ON DYNAMIC LOAD HISTORY RECONSTRUCTION USING PSEUDO-INVERSE METHODS

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Abstract. Considering that the vibratory forces generally cannot be measured directly at the interface of two bodies, an inverse method is studied in the present work to recover the load history in such cases. The proposed technique attempts to reconstruct the dynamic loads history by using a frequency domain analysis and Moore-Penrose pseudo-inverses of the frequency response function (FRF) of the system. The methodology consists in applying discrete dynamic loads on a finite element model in the time domain and retrieve estimates of the loads at points that cannot be measured, and complementing these data with the experimental results. Since the number of points of interest in general differs from the number of points experimentally tested, the FRF matrix results rectangular, then the need of pseudo-inverses. Additionally, a regularization procedure is used to smooth the reconstructed load history, aiming the removal of spurious peaks. A case is illustrated to describe the method.

Keywords: Load Reconstruction, Pseudo-Inverse, Moore-Penrose Pseudo-Inverse, Structural Dynamics, Load History.
1 INTRODUCTION

Load reconstruction or load identification constitutes an important type of engineering problem. There are many cases in which it is not possible to obtain a correct description of the excitation conditions directly. For example, vibratory forces generally cannot be measured directly at the interface of two bodies, in such cases an indirect or inverse method is required to recover the load history.

A problem is considered direct if when subjected to a particular input is able to determine the system response. As for the problem to be considered inverse or indirect, system responses are measured in order to establish your inputs (Bekey, 1970).

Structural systems can be modelled as continuous or discrete. Exact solutions in continuous form can only be obtained for relatively simple structures. Therefore, discrete formulations are commonly used. There are three forms of discrete modelling that are widely used in structural dynamics. These are based upon the three domains of spatial, modal and frequency (Dobson and Rider, 1990). This paper will concentrate on the use of frequency domain models.

In the literature, one of the earliest investigations into load reconstruction using Moore-Penrose pseudo-inverse was undertaken by Barlett and Flannelly (1979) and one of the most recently study was conducted by Vishwakarma et al. (2010).

The paper presents an ongoing work which proposes a technique that attempts to reconstruct the dynamic loads history by using a frequency domain analysis and Moore-Penrose pseudo-inverses of the frequency response function (FRF) of the system.

There are five sections in this paper. Section 2 describes the frequency response function (FRF) model. Section 3 presents the Moore-Penrose pseudo-inverse. In Section 4 contain a case of study which utilizes the theory of load reconstruction technique outlined in Sections 2 and 3. Finally, the concluding remarks are presented in Section 5.

2 FREQUENCY RESPONSE FUNCTION MODEL

A frequency response function (FRF) is a transfer function which response may be given in terms of displacement, velocity, or acceleration, in the frequency domain.

Considering a discrete linear dynamic system in Fig 1.

The relationship in Fig. 1 defines the equation of motion in the frequency domain as:

\[ \{Y(\omega)\} = [H(\omega)] \{F(\omega)\}. \]  

(1)
where \( F(\omega) \) is the excitation force, \( H(\omega) \) is the FRF and \( Y(\omega) \) is the response vector (Vishwakarma et al., 2010).

If the number of unknown forces and the response data are equal, and FRF matrix is square and non-singular, the excitation force is given by rearranging Eq. 1:

\[
\{F(\omega)\} = [H(\omega)]^{-1} \{Y(\omega)\}. \tag{2}
\]

where superscript -1 represents the inverse.

3 MOORE-PENROSE PSEUDO-INVERSE

Moore-Penrose pseudo-inverse is used when the FRF matrix is rectangular and singular. If more responses than applied loads are known, the problem is over-determined. If has the opposite definition to over-determined, the problem is under-determined.

The forces can be estimated through (Vishwakarma et al., 2010):

\[
\{F_{est}(\omega)\} = [H^+(\omega)] [Y(\omega)]. \tag{3}
\]

where for an over-determined problem:

\[
[H^+(\omega)] = [H^T H]^{-1} H^T. \tag{4}
\]

And for an under-determined problem:

\[
[H^+(\omega)] = H^T [H^T H]^{-1}. \tag{5}
\]

The superscript + denotes the pseudo-inverse and \( H^T \) is the Hermitian transpose of \( H \).

4 CASE OF STUDY

The methodology consists essentially in applying discrete dynamic loads on a cantilever beam finite element model in the time domain and retrieve estimates of the loads at selected points. Since the number of points of interest in general differs from the number of points measured, the FRF matrix results rectangular, then the need of pseudo-inverses. The load reconstruction technique used in this paper is illustrated schematically in flow diagram in Fig. 2.

In order to obtain a FRF matrix was run a full harmonic analysis in OptiStruct/HyperWorks, applying a unit load at predetermined points on a finite element model and thereafter measuring the acceleration response in nodes and directions desired for all frequencies in the range established.

Then, if the load is to be reconstructed in the y-direction at the points selected, a unit frequency dependent load was applied to the point and the responses measured. The responses would therefore form a column in the FRF matrix.

In order to obtain the acceleration data was run a full transient analysis. For this analysis, a load curve is segmented into suitable load steps. Consequently, by using a time step in seconds, the load step files were composed. By solving the load step files, the acceleration in time data can be extracted from nodes and directions desired.
A study on dynamic load history reconstruction using pseudo-inverse methods

A regularization procedure is used in acceleration data to smooth the reconstructed load history, aiming the removal of spurious peaks. Thus, the load reconstruction process in Fig. 2, converts the acceleration data into the frequency domain. Finally, the procedure utilizes the pseudo-inverse method to reconstruct the initial load curve.

![Diagram of Load Reconstruction Process](image)

Figure 2 - Load Reconstruction Process adapted from Vishwakarma et al. (2010).

Intending to clarify the relationship between the matrices displayed in Fig. 2, Eq. (6) and Eq. (7) demonstrate the matrices in expanded form.

\[
\{F(\omega)\}_{(m \times 1)} = [H^+(\omega)]_{(n \times m)} \{Y(\omega)\}_{(n \times 1)}.
\]

\[
\begin{pmatrix}
F_1(\omega) \\
F_2(\omega) \\
\vdots \\
F_n(\omega)
\end{pmatrix} = 
\begin{pmatrix}
H^+_{11}(\omega) & H^+_{12}(\omega) & \cdots & H^+_{1m}(\omega) \\
H^+_{21}(\omega) & H^+_{22}(\omega) & \cdots & H^+_{2m}(\omega) \\
\vdots & \vdots & \ddots & \vdots \\
H^+_{n1}(\omega) & H^+_{n2}(\omega) & \cdots & H^+_{nm}(\omega)
\end{pmatrix}
\begin{pmatrix}
Y_2(\omega) \\
Y_3(\omega) \\
\vdots \\
Y_n(\omega)
\end{pmatrix}.
\]

where \(n\) and \(m\) are total number of responses and forces respectively.
In Fig. 3 is illustrated the case study that will be accomplished on a cantilever beam, also can be visualized the site of force application $F(t)$ and the points 1 and 2 to indicate the measurement location of responses.

![Figure 3. Representation of a cantilever beam.](image)

The dynamic load to be reconstructed will be applied in the Y-direction in the form shown in Fig. 4.

![Figure 4. Load initially applied.](image)

## 5 CONCLUSIONS

This paper presented an ongoing work, which demonstrated a brief presentation about dynamic load reconstruction focused on use of the pseudo-inverse method directly.

By using pseudo-inverse method, the problem over-determined (i.e. more responses than applied loads are known) is sufficient to identify unknown force. Therefore, load reconstruction can be done by using remote responses that are a certain distance from the force excitation. Thus, this work has potential for allow obtaining time load history on locations that cannot be measured straightforwardly.
However, expected to implement the method based on the methodology mentioned in different cases of loading and set an amount sufficient points to the response can be considered reliable. In addition, it intends to analyze the influence of the time step, the frequency amplitude and frequency step.

REFERENCES


